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NOVEL UNIAXIAL BIANISOTROPIC MATERIALS: REFLECTION AND TRANSMISSION IN PLANAR STRUCTURES

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1. Introduction

Recently, a novel concept of artificial composite materials with Ω-shaped metal elements was introduced in [1]. The idea originated from wide and intensive studies of isotropic chiral media and their electromagnetic properties in the microwave regime. Artificial chiral materials for applications in microwave engineering possess microstructures with small helical wire inclusions, which provide additional interaction between parallel high-frequency electric and magnetic fields. The helices obviously lack mirror symmetry, and that is the reason for the name "chiral". In isotropic chiral materials, chiral inclusions are randomly distributed and the constitutive equations depend on scalar coupling coefficients [2].

A typical chiral inclusion — a small wire helix — can be considered as an electric dipole connected with a magnetic dipole in such a fashion that high-frequency electric field induces in space magnetic field parallel to the original electric field component, and *vice versa*. This causes optical or microwave activity — rotation of the polariza-

tion plane of a linearly polarized propagating wave. Such chiral structures most effectively interact with circularly polarized waves, since the right and left circular polarizations are the eigenpolarizations in unbounded bii-sotropic media (see, e.g., [3, 4]). However, one may suggest other geometrical configurations which can provide stronger wave-material interaction in other circumstances we deal with in microwave engineering. One of such modifications was introduced in [1], where it was suggested to use particles shaped like the capital Greek letter Ω instead of chiral helical particles to ensure first-order effect on the propagation factor in a partially filled rectangular waveguide. Such a structure possesses mirror symmetry and is then non-chiral. The name "pseudochiral" has been proposed by Saadoun and Engheta [1]. In a regular microstructure with Ω-shaped conductive inclusions, there exists some interaction between electric and magnetic fields which lay in orthogonal planes, and the material can be modeled by bianisotropic constitutive equations.

In the present paper, we study another modification of such microstructure configurations which can be better suited for use in plane non-reflecting coverings and antenna radomes.

We focus the analysis on linearly polarized electromagnetic waves in plane layers. Consider a plane dielectric layer and let the z-axis of a rectangular co-ordinate system be normal to its boundaries, and the x- and y-axes lie in the plane of the interfaces. Let us introduce Ω-particles inside the slab and position them so that their stems stretch along the x-axis and the loops are in the $(x-z)$ plane. In a regular two-dimensional set of that configuration, the x-component of high-frequency electric fields induces, in addition to dielectric polarization, a y-directed high-frequency magnetic field component and *vice versa*. With one such set, the structure is bianisotropic, and a y-directed electric field component causes only electric polarization in the dielectric matrix, but no magnetic field component.

To provide uniform operation for linearly polarized waves with any electric field direction (or for unpolarized plane waves), we suggest to introduce a second ensemble of Ω particles, with the stems along the y-axis and the loops in the $(y-z)$ plane. This set of particles interacts with the electric field component polarized along the y-axis and the magnetic field component directed along the x-axis. As a result, the structure will interact more effectively with linearly polarized waves of any polarization direction. Such a medium can be named as uniaxial

omega medium, because there exists only one particular direction — that one normal to the interfaces. The size of Ω-shaped elements is assumed to be small enough, so that the medium can be described by effective averaged material parameters and the material is modeled by uniaxial bianisotropic constitutive relations which couple electric and magnetic fields. The idea of the uniaxial omega structures was put forward in the letter [5]. Eigenwaves in still more general uniaxial chiral omega structures were studied in [6, 7]. Another modification of uniaxial bianisotropic media was considered in [8], where the coupling dyadics were assumed to be symmetric.

In the following we develop the general theory of plane wave propagation in our novel media and study reflection and transmission in plane uniaxial bianisotropic layered structures. To the knowledge of the authors, in the literature only numerical techniques suitable for calculation of the reflection and transmission in general bianisotropic slabs have been suggested [9, 10]. Here we construct analytical solutions for the proposed structures and analyze their general properties.

As an example interesting for applications, we consider in detail reflection from a plane metal surface covered with a lossy omega layer. The example demonstrates that with the additional Ω-shaped wire elements absorption can be enhanced in a wide frequency range. The additional material parameter can help to manage properties of anti-reflection coatings, in a way similar to the effect provided by the chirality parameter of bii-sotropic materials [11–13]. Another example is the reflection and transmission through a plane slab. Here it is seen that the material is prospective for potential use in antenna radomes since a nearly transparent and non-reflecting covering can be designed. Lossy slabs can be designed to serve as non-reflecting absorbers.

2. Eigenwaves in Uniaxial Omega Media

Constitutive equations for composite materials with two orthogonal sets of Ω-shaped elements arranged as explained in the Introduction we write for the harmonic time dependence ($e^{j\omega t}$) as

$$\bar{D} = \bar{\bar{\epsilon}} \cdot \bar{E} + j\sqrt{\epsilon_0 \mu_0}\, \bar{\bar{K}}_{em} \cdot \bar{H}$$

$$\bar{B} = \overline{\overline{\mu}} \cdot \bar{H} - j\sqrt{\epsilon_0 \mu_0}\, \overline{\overline{K}}_{me} \cdot \bar{E} \tag{1}$$

Here we assume that the medium is reciprocal, so that the coupling dyadics satisfy the reciprocity condition for bianisotropic media [14]

$$\overline{\overline{\epsilon}} = \overline{\overline{\epsilon}}^T \qquad \overline{\overline{\mu}} = \overline{\overline{\mu}}^T \qquad \overline{\overline{K}}_{em} = \overline{\overline{K}}_{me}^T \tag{2}$$

where T denotes the transpose operation, and the two sets of omega particles may be different (in size or in number of the wire elements). This corresponds to two different scalar coupling coefficients K_x and K_y:

$$\begin{aligned} \overline{\overline{K}}_{em} &= -K_x \bar{x}_0 \bar{y}_0 + K_y \bar{y}_0 \bar{x}_0 \\ \overline{\overline{K}}_{me} &= K_y \bar{x}_0 \bar{y}_0 - K_x \bar{y}_0 \bar{x}_0 \end{aligned} \tag{3}$$

If one of the coupling coefficients K_x or K_y is zero (i.e., if there is only one ensemble of Ω particles), the constitutive equations (1) are equivalent to that given in [1], although the material parameters are defined in [1] in a somewhat different fashion.

In the following we assume that the two ensembles are identical, then $K_x = K_y = K$, and both the coupling dyadics are anti-symmetric and proportional to the 90 degree rotator $\overline{\overline{J}} = \bar{y}_0 \bar{x}_0 - \bar{x}_0 \bar{y}_0$ in the $x-y$ plane:

$$\begin{aligned} \bar{D} &= \overline{\overline{\epsilon}} \cdot \bar{E} + jK\sqrt{\epsilon_0 \mu_0}\, \overline{\overline{J}} \cdot \bar{H} \\ \bar{B} &= \overline{\overline{\mu}} \cdot \bar{H} + jK\sqrt{\epsilon_0 \mu_0}\, \overline{\overline{J}} \cdot \bar{E} \end{aligned} \tag{4}$$

In this case $\overline{\overline{\epsilon}}$ and $\overline{\overline{\mu}}$ are uniaxial dyadics with transverse (t) and normal components (n):

$$\overline{\overline{\epsilon}} = \epsilon_0(\epsilon_t \overline{\overline{I}}_t + \epsilon_n \bar{z}_0 \bar{z}_0) \qquad \overline{\overline{\mu}} = \mu_0(\mu_t \overline{\overline{I}}_t + \mu_n \bar{z}_0 \bar{z}_0) \tag{5}$$

where $\bar{z}_0$ stands for the unit vector along the z-axis (normal to the interfaces). $\overline{\overline{I}}_t = \bar{x}_0 \bar{x}_0 + \bar{y}_0 \bar{y}_0$ is the transverse unit dyadic and the rotator $\overline{\overline{J}}$ can be expressed also as $\overline{\overline{J}} = \bar{z}_0 \times \overline{\overline{I}}_t$. The coupling provided

by the omega particles is proportional to the dimensionless parameter K. In lossless media all the material parameters $\epsilon_{t,n}$, $\mu_{t,n}$ and K in (4) must be real.

To study electromagnetic waves in the uniaxial omega media we Fourier transform the Maxwell equations in the plane of the interfaces and eliminate the field components normal to the interfaces. Splitting the fields into normal and transverse parts

$$\bar{E} = E_n \bar{z}_0 + \bar{E}_t \qquad \bar{H} = H_n \bar{z}_0 + \bar{H}_t \tag{6}$$

the Maxwell equations take the form

$$\begin{aligned} -j\bar{k}_t \times \bar{E} + \frac{\partial}{\partial z}\bar{z}_0 \times \bar{E}_t &= -j\omega(\bar{\bar{\mu}} \cdot \bar{H} + jK\sqrt{\epsilon_0\mu_0}\,\bar{z}_0 \times \bar{E}_t) \\ -j\bar{k}_t \times \bar{H} + \frac{\partial}{\partial z}\bar{z}_0 \times \bar{H}_t &= j\omega(\bar{\bar{\epsilon}} \cdot \bar{E} + jK\sqrt{\epsilon_0\mu_0}\,\bar{z}_0 \times \bar{H}_t) \end{aligned} \tag{7}$$

Here, $\bar{k}_t$ is the two-dimensional Fourier variable. The normal field components can be expressed in terms of the transverse fields:

$$E_n \bar{z}_0 = -\frac{1}{\omega\epsilon_0\epsilon_n}\bar{k}_t \times \bar{H}_t \qquad H_n \bar{z}_0 = \frac{1}{\omega\mu_0\mu_n}\bar{k}_t \times \bar{E}_t \tag{8}$$

and eliminated, which converts (7) into the system of two vector transmission-line equations

$$\begin{aligned} \left(\frac{\partial}{\partial z} - k_0 K\right)\bar{E}_t &= \left(j\omega\mu_0\mu_t\bar{\bar{I}}_t - \frac{j}{\omega\epsilon_0\epsilon_n}\bar{k}_t\bar{k}_t\right) \cdot (\bar{z}_0 \times \bar{H}_t) \\ \left(\frac{\partial}{\partial z} + k_0 K\right)\bar{z}_0 \times \bar{H}_t &= \left(j\omega\epsilon_0\epsilon_t\bar{\bar{I}}_t - \frac{j}{\omega\mu_0\mu_n}\bar{z}_0 \times \bar{k}_t\bar{z}_0 \times \bar{k}_t\right) \cdot \bar{E}_t \end{aligned} \tag{9}$$

where we use the conventional notation for the free-space wave number $k_0 = \omega\sqrt{\epsilon_0\mu_0}$.

Second-order wave equation for the transverse electric field component $\bar{E}_t$ immediately follows from the transmission-line equations (9) and it takes the form

$$\frac{\partial^2}{\partial z^2}\bar{E}_t + \left(\beta_{TM}^2\frac{\bar{k}_t\bar{k}_t}{k_t^2} + \beta_{TE}^2\frac{\bar{z}_0 \times \bar{k}_t\bar{z}_0 \times \bar{k}_t}{k_t^2}\right) \cdot \bar{E}_t = 0 \tag{10}$$

Because the dyadic in the last equation is diagonal, the eigensolutions of (10) are obviously two linearly polarized vectors: one is proportional to $\bar{k}_t$ and another one — to $\bar{z}_0 \times \bar{k}_t$. The first solution corresponds to a TM-polarized wave, with the magnetic field orthogonal to the $\bar{k}_t$ vector, and the second one is a TE-wave. β_{TM} and β_{TE} are the normal components of the propagation factors for the TM- and TE-polarized eigenwaves, respectively:

$$\begin{aligned} \beta_{TM}^2 &= \frac{\epsilon_t}{\epsilon_n}\left(k_0^2\epsilon_n\mu_t - k_t^2\right) - k_0^2K^2 \\ \beta_{TE}^2 &= \frac{\mu_t}{\mu_n}\left(k_0^2\epsilon_t\mu_n - k_t^2\right) - k_0^2K^2 \end{aligned} \tag{11}$$

The transverse fields in these eigenwaves depend on each other through wave impedances or admittances:

$$\bar{E}_t = \mp\overline{\overline{Z}}_{\pm}\cdot\bar{z}_0\times\bar{H}_t \qquad \bar{z}_0\times\bar{H}_t = \mp\overline{\overline{Y}}_{\pm}\cdot\bar{E}_t \tag{12}$$

where the dyadic impedances and admittances are diagonal:

$$\overline{\overline{Z}}_{\pm} = Z_{\pm}^{TM}\frac{\bar{k}_t\bar{k}_t}{k_t^2} + Z_{\pm}^{TE}\frac{\bar{z}_0\times\bar{k}_t\bar{z}_0\times\bar{k}_t}{k_t^2} \qquad \overline{\overline{Y}}_{\pm} = \overline{\overline{Z}}_{\pm}^{-1}$$

and the upper and lower signs correspond to the waves propagating in the positive and the negative directions of the z-axis, correspondingly.

The characteristic impedances and admittances can be found from the vector transmission-line equations (9) after substitution of plane linearly polarized solutions of the wave equation (10), which leads to

$$Z_{\pm}^{TM} = \sqrt{\frac{\mu_0\mu_t}{\epsilon_0\epsilon_t}}\left(\sqrt{1-\frac{k_t^2}{k_0^2\epsilon_n\mu_t}-K_n^2}\pm jK_n\right) \tag{13}$$

$$Z_{\pm}^{TE} = \sqrt{\frac{\mu_0\mu_t}{\epsilon_0\epsilon_t}}\frac{1}{1-\dfrac{k_t^2}{k_0^2\epsilon_t\mu_n}}\left(\sqrt{1-\frac{k_t^2}{k_0^2\epsilon_t\mu_n}-K_n^2}\pm jK_n\right) \tag{14}$$

$$Y_{\pm}^{TM} = \sqrt{\frac{\epsilon_0\epsilon_t}{\mu_0\mu_t}}\frac{1}{1-\dfrac{k_t^2}{k_0^2\epsilon_n\mu_t}}\left(\sqrt{1-\frac{k_t^2}{k_0^2\epsilon_n\mu_t}-K_n^2}\mp jK_n\right) \tag{15}$$

$$Y_{\pm}^{TE} = \sqrt{\frac{\epsilon_0 \epsilon_t}{\mu_0 \mu_t}} \left(\sqrt{1 - \frac{k_t^2}{k_0^2 \epsilon_t \mu_n} - K_n^2} \mp jK_n \right) \tag{16}$$

Here, the normalized coupling parameter K_n has been introduced as

$$K_n = \frac{K}{\sqrt{\epsilon_t \mu_t}} \tag{17}$$

It is most important for the following that the impedances and admittances are different for the waves traveling in the opposite z-directions. As is seen from the above results, electromagnetic waves in uniaxial bianisotropic Ω structures can be modeled by the non-symmetric vector transmission-line equations (9), with the normal components of the propagation factors (11) and the characteristic impedances (13), (14) or admittances (15), (16). The normal components of the propagation factors do not change when the propagation direction is reversed, but the impedances and admittances are non-symmetric.

Similar properties can be observed for waves in magnetized ferrites or plasmas. For example, in a special case when a plane wave propagates in a magnetized ferrite in a direction orthogonal to the bias field, the propagation factors are symmetric but the wave impedances are not. Another example is the Tellegen medium [15–18], i.e. the non-reciprocal bii-sotropic material with zero chirality parameter. Moreover, comparing with the Tellegen media we see that the way how the impedances and admittances depend on the material parameters in the present case is similar to that in the Tellegen media. The normalized coupling parameter appears in the position of the normalized non-reciprocity parameter in the equations governing wave propagation in the Tellegen media. Of course, this analogy is rather formal, and the physical effects in these two specific complex media are quite different.

The previous analysis implies that the transverse electric and magnetic field components $\bar{E}_t$ and $\bar{H}_t$ are both non-zero. Two special cases of the wave propagation when either $\bar{E}_t$ or $\bar{H}_t$ vanish must be treated separately. Assuming $\bar{E}_t = 0$, so that the electric field is longitudinal ($\bar{E} = \bar{z}_0 E_n$), we see from the Maxwell equations (7) that the magnetic field is purely transverse with respect to the geometrical axis, and it is related to the electric field as

$$\bar{H} = \bar{H}_t = \frac{1}{\omega\mu_0\mu_t}\bar{k}_t \times \bar{z}_0 E_n \tag{18}$$

The components of the propagation factor satisfy

$$k_t^2 = k_0^2\epsilon_n\mu_t \qquad \beta^2 = k_0^2 K^2 \tag{19}$$

It is interesting to note that for this specific wave the normal component of the propagation factor β depends only on the coupling coefficient but not on the other material parameters.

In analogy, in the dual case when the transverse magnetic field is zero, $\bar{H} = \bar{z}_0 H_n$, the electric field is transverse, and it reads in terms of the magnetic field

$$\bar{E} = \bar{E}_t = -\frac{1}{\omega\epsilon_0\epsilon_t}\bar{k}_t \times \bar{z}_0 H_n \tag{20}$$

For this polarization, $k_t^2 = k_0^2\epsilon_t\mu_n$, and the normal component of the propagation factor is the same as in the previous case (19).

Since the eigenwaves in uniaxial omega media are the linearly polarized plane waves, and waves of the different eigenpolarizations do not couple at plane interfaces, it is possible to consider the TM- and TE-waves separately and to employ the scalar non-symmetric transmission-line theory [19]. This contrasts with the situation we encounter in layered biisotropic media [17, 18], where two circularly polarized eigenwaves couple on interfaces, hence the vector circuit theory [17] or the vector transmission-line theory [18] must be used.

Uniaxial omega media possess some novel properties as compared to the above examples because they are reciprocal, while ferrites, magnetized plasmas and Tellegen media are not. Another feature important for applications is the uniaxial symmetry of the suggested microstructure.

3. Reflection and Transmission in Slabs

In this Section we concentrate on reflection and transmission properties of plane layers filled with uniaxial Ω materials. As is seen from (9) and (10), the transverse electric field components of linearly polarized TM and TE waves satisfy non-symmetric scalar trans-

mission-line equations. The normal component of the propagation factor is given by (11) and the non-symmetric impedances and admittances by (13)–(16). Generalized scalar transmission-line theory has been introduced in [19], and the basic results of the non-reciprocal and non-symmetric transmission-line theory are applicable to the present problem. Considering either TM - or TE -waves, let us introduce scalar equivalent voltages and currents as the amplitudes of the transverse field components:

$$\bar{E}_t = \bar{e}_0 u \qquad -\bar{z}_0 \times \bar{H}_t = \bar{e}_0 i \tag{21}$$

where $\bar{e}_0$ is the unit vector in the direction of the transverse electric field component.

The general solution of the equation (10) for linearly polarized eigenwaves can be expressed as a sum of two plane waves propagating in the opposite directions:

$$u = Ae^{-j\beta z} + Be^{j\beta z} \qquad i = Y_+ Ae^{-j\beta z} - Y_- Be^{j\beta z} \tag{22}$$

with scalar amplitude coefficients A, B and non-symmetric characteristic admittances $Y_\pm$. The superindices TM or TE have been dropped, since the theory applies to any eigenpolarization.

Consider a uniaxial omega slab of the thickness d, excited by a plane linearly polarized wave which comes from an isotropic half space $z < 0$:

$$u^{inc} = e^{-j\beta_1 z} \qquad i^{inc} = Y_1 u^{inc} \tag{23}$$

The propagation factor β_1 and the characteristic admittance Y_1 can be found as special cases of (11) and (15), (16) by replacing the material parameters of the slab with that of the isotropic half space. The total transverse field components in the half space $z < 0$ we write, defining the reflection coefficient R:

$$u = e^{-j\beta_1 z} + Re^{j\beta_1 z} \qquad i = Y_1 e^{-j\beta_1 z} - RY_1 e^{j\beta_1 z} \tag{24}$$

Assume that the slab is backed by another isotropic half space $z > d$ or the structure is terminated with a boundary, behind which there are no fields. In the transmission-line theory both types of termination can be modelled by a load admittance Y_2. At an interface with an isotropic half space (or with another uniaxial bianisotropic half space

as well), the equivalent load admittance equals to the characteristic admittance of plane waves in the corresponding medium, see [20, 21]. In the half space $z > d$ the fields are proportional to the transmission coefficient T:

$$u = Te^{-j\beta_2 z} \qquad i = Y_2 u \tag{25}$$

where β_2 and Y_2 follow from (11) and (15), (16) after replacing the material parameters by that of the medium filling the half space $z > d$.

Demanding the transverse fields to be continuous on the interfaces, one can determine the reflection and transmission coefficients:

$$R = \frac{(Y_1 - Y_+)(Y_2 + Y_-) - (Y_1 + Y_-)(Y_2 - Y_+)e^{-2j\beta d}}{(Y_1 + Y_+)(Y_2 + Y_-) - (Y_1 - Y_-)(Y_2 - Y_+)e^{-2j\beta d}} \tag{26}$$

$$T = \frac{2(Y_- + Y_+)Y_1 e^{-j\beta d}}{(Y_1 + Y_+)(Y_2 + Y_-) - (Y_1 - Y_-)(Y_2 - Y_+)e^{-2j\beta d}} \tag{27}$$

An alternative reflection formula can be obtained from (26) by changing the sign of (26) and by replacing the admittances with the corresponding impedances. Transmission formula in terms of impedances follows from (27) after changing Y to Z and the index 1 to 2 in the numerator. In the special case when $d \to \infty$ or $Y_2 \to Y_+$ (the matched load admittance) (26) gives the reflection coefficient at a junction which coincides with that obtained in [19].

For a slab on an ideally conducting surface with $Y_2 \to \infty$ or $Z_2 \to 0$ the reflection coefficient reads

$$\begin{aligned} R_e &= \frac{Y_1 - Y_+ - (Y_1 + Y_-)e^{-2j\beta d}}{Y_1 + Y_+ - (Y_1 - Y_-)e^{-2j\beta d}} \\ &= -\frac{(Z_1 - Z_+)Z_- + (Z_1 + Z_-)Z_+ e^{-2j\beta d}}{(Z_1 + Z_+)Z_- + (Z_1 - Z_-)Z_+ e^{-2j\beta d}} \end{aligned} \tag{28}$$

and for a slab backed by a magnetic wall ($Y_2 \to 0$ or $Z_2 \to \infty$) we have

$$R_m = -\frac{Z_1 - Z_+ - (Z_1 + Z_-)e^{-2j\beta d}}{Z_1 + Z_+ - (Z_1 - Z_-)e^{-2j\beta d}}$$
$$= \frac{(Y_1 - Y_+)Y_- + (Y_1 + Y_-)Y_+ e^{-2j\beta d}}{(Y_1 + Y_+)Y_- + (Y_1 - Y_-)Y_+ e^{-2j\beta d}} \tag{29}$$

As a check, one can easily see that with symmetric parameters $Y_- = Y_+$, $Z_- = Z_+$ the above results simplify to the well-known formulas of the conventional transmission-line theory. Multilayered structures can be treated with the present technique as well, using the generalized impedance transmission procedure discussed in [19].

4. Analysis and Numerical Results

The novel uniaxial Ω microstructures suggested and studied above are expected to enhance interaction of linearly polarized electromagnetic waves with plane shields. Let us now examine the reflection and transmission formulas for layers of these composite materials in view of potential applications in low-reflecting screens or radomes. Considering plane-wave reflection from an uniaxial omega slab in air ($Z_1 = Z_2$, $Y_1 = Y_2$) one can easily see that the reflection coefficient (26) equals zero when the impedance Z_+ equals the free-space impedance Z_1 (or, what is equivalent, $Y_+ = Y_1$). This means that the characteristic impedance for the waves traveling in the slab in the direction of the incident plane wave matches the impedance of the waves in free space. Obviously, there is then no reflection on both the interfaces. For the TM-polarized fields the equation $Y_+ = Y_1$ is satisfied when

$$K = \frac{j}{2\cos\phi}\left(\mu_t - \epsilon_t \cos^2\phi - \frac{1}{\epsilon_n}\sin^2\phi\right) \tag{30}$$

and for the TE-polarized fields the no-reflection condition reads

$$K = \frac{j}{2\cos\phi}\left(\mu_t \cos^2\phi - \epsilon_t + \frac{1}{\mu_n}\sin^2\phi\right) \tag{31}$$

Here, ϕ is the incidence angle. For normal incidence $\phi = 0$ and both (30) and (31) simplify to

$$K = \frac{j}{2}\left(\mu_t - \epsilon_t\right) \tag{32}$$

For layers filled with simple dielectrics ($K = 0$), the last relation means $\mu_t = \epsilon_t$, which is the well-known condition for parameters of non-reflecting dielectric slabs.

The transmission coefficient for a slab with the parameters satisfying (30) or (31) is an exponential function

$$T = e^{-j\beta d} \tag{33}$$

We can conclude that for a slab with given arbitrary permittivity parameters $\epsilon_{t,n}$ and any permeabilities $\mu_{t,n}$ there exists a specific value of the coupling parameter K, such that the reflection coefficient is zero for any given polarization and an arbitrary incidence angle. The effect is specific for materials modeled by non-symmetric transmission-line equations. Indeed, for any symmetric admittances (30)–(32) reduce to the known formulas for simple dielectrics with $K = 0$.

Starting the analysis from the case of the normal incidence, let us consider an example of a slab with magnetic losses, so that $\mu_t = \mu' - j\mu''$, but with no dielectric losses (ϵ_t is real). Assuming for simplicity that the real part of the magnetic permeability and the dielectric permittivity are equal, $\epsilon_t = \mu'$, the requirement (32) is settled when K is real and $K = \mu''/2$.

The effect of the extra material parameter K on the reflection from a slab at normal incidence is shown in Fig. 1. The absolute value of the reflection coefficient is presented as a function of the normalized frequency $(f - f_0)/f_0$ and the magnetic loss ratio μ''/μ'. The slab thickness is $d = \lambda_0/5$, where the free-space wavelength λ_0 corresponds to the frequency f_0. The material parameters are $\mu_t = \mu' - j\mu''$ with $\mu' = 5$, $\epsilon_t = 5$ and the coupling parameter $K = 5$. The reference picture for the corresponding simple isotropic slab with $K = 0$ is given in Fig. 1b. Obviously, there is no reflection from the simple isotropic slab when the imaginary part of the magnetic permeability is zero, since $\mu' = \epsilon_t$ and the admittances match. With increasing magnetic losses in the isotropic slab, transmission is decreasing but the admittances mismatch and the reflection increases. For the uniaxial Ω layer with the parameters taken in the present example, the input admittance match

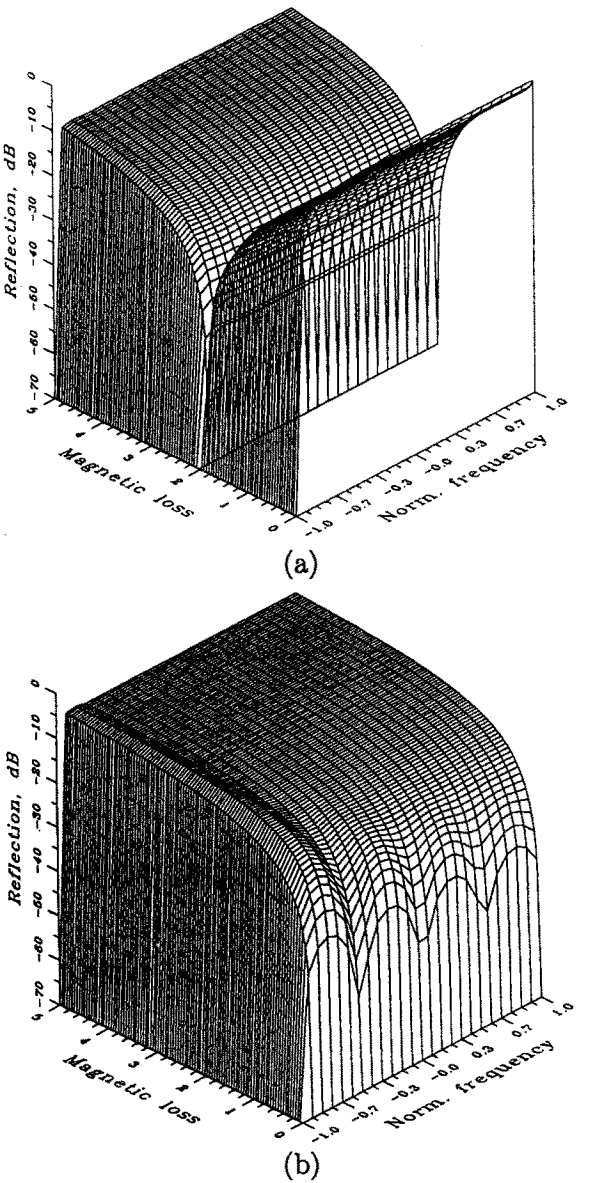


Figure 1. Absolute values of the reflection coefficient from a uniaxial omega slab in air as a function of the normalized frequency $(f - f_0)/f_0$ and the magnetic loss ratio μ''/μ'. The slab thickness is $d = \lambda_0/5$, and the free space wavelength λ_0 corresponds to the central frequency f_0. $\mu_t = \mu' - j\mu''$ with $\mu' = 5$, $\epsilon_t = 5$, and $K = 5$. Figure 1b gives the reference picture for the isotropic slab with $K = 0$.

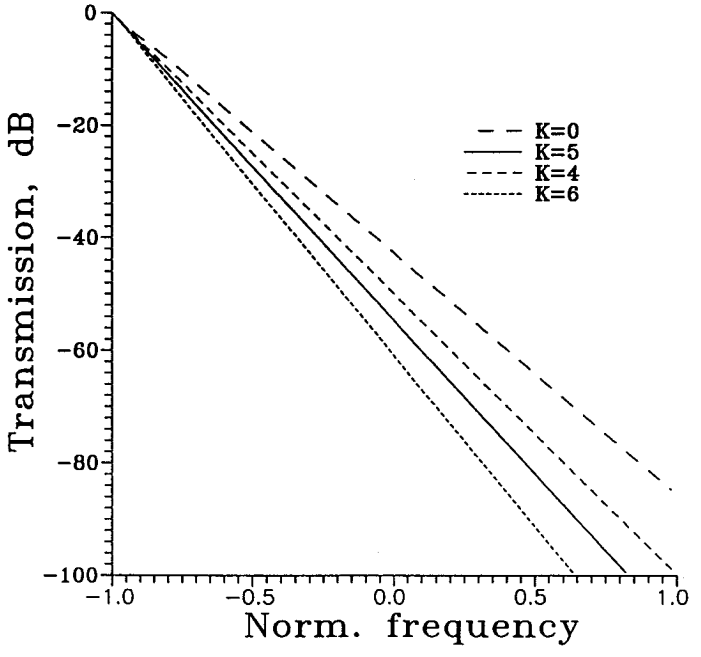
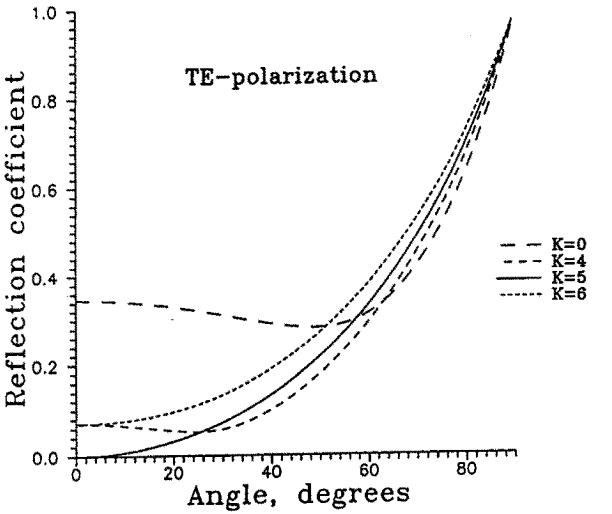


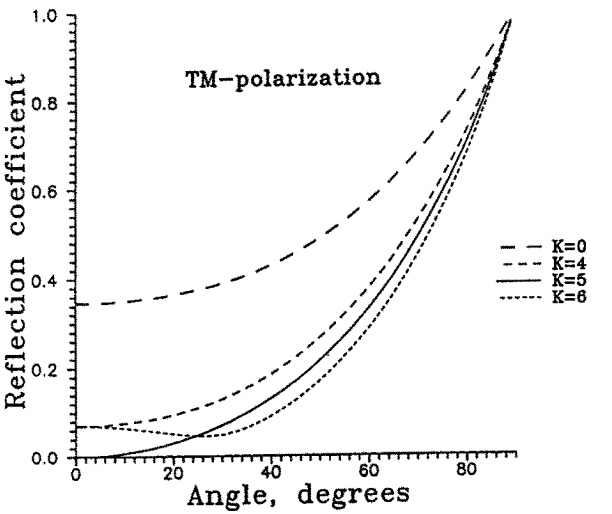
Figure 2. Absolute values of the transmission coefficient through a uniaxial omega slab in air as a function of the normalized frequency $(f-f_0)/f_0$. The slab thickness $d = \lambda_0/5$, the magnetic loss ratio $\mu''/\mu' = 2$, $\epsilon_t = 5$ and the coupling parameter K varies. Normal incidence.

that of free space when $\mu'' = 2K$ (see (32)) or the magnetic loss ratio $\mu''/\mu' = 2$. This effect appears to be useful for potential application, as it allows one to provide low reflection from a lossy layer when the permittivity and permeability are not equal. The transmission coefficient can be either small if the slab is lossy or it can be close to unity in the absolute value for a small loss ratio and (or) small thickness. For extremely low frequencies, when $(f - f_0)/f_0 \to -1$, i.e. $f \to 0$ the reflection is, of course, always zero.

Figure 2 displays the transmission coefficient at normal incidence for an uniaxial Ω slab in comparison with a simple isotropic lossy slab. The parameters are the same as in Figure 1, with the magnetic loss ratio $\mu''/\mu' = 2$. The reflection coefficient for $K = 5$ is zero, and we

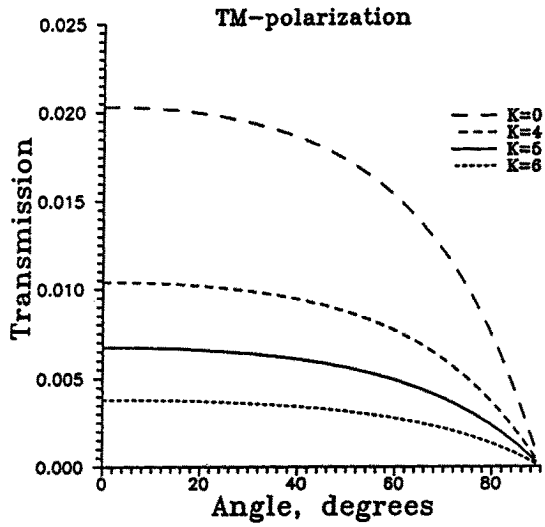


(a)

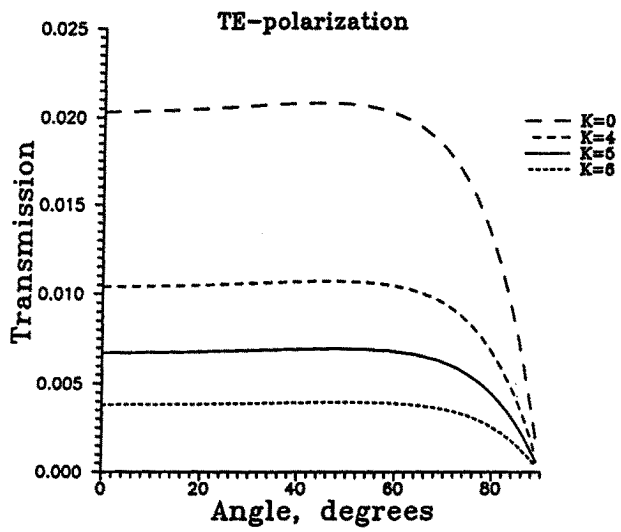


(b)

Figure 3. Absolute values of the reflection coefficient from a uniaxial omega slab in air as a function of the incidence angle. The material parameters are $\mu_t = 5 - j10$, $\mu_n = 5$, $\epsilon_t = \epsilon_n = 5$, the normalized thickness $k_0 d = 1.0$ and K varies. (a). TM-polarization. (b). TE-polarization.



(a)



(b)

Figure 4. Absolute values of the transmission coefficient through an uniaxial omega slab in air as a function of the incidence angle. The parameters are same as in Figure 3. (a). TM-polarization. (b). TE-polarization.

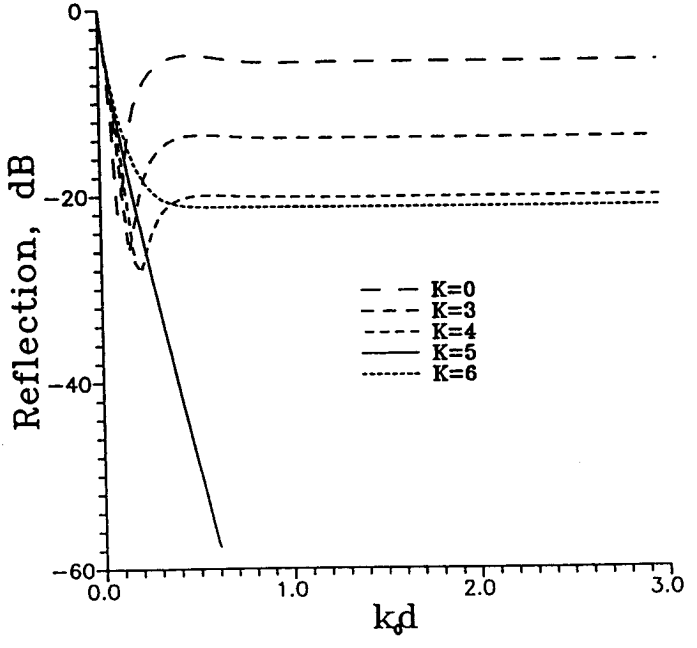
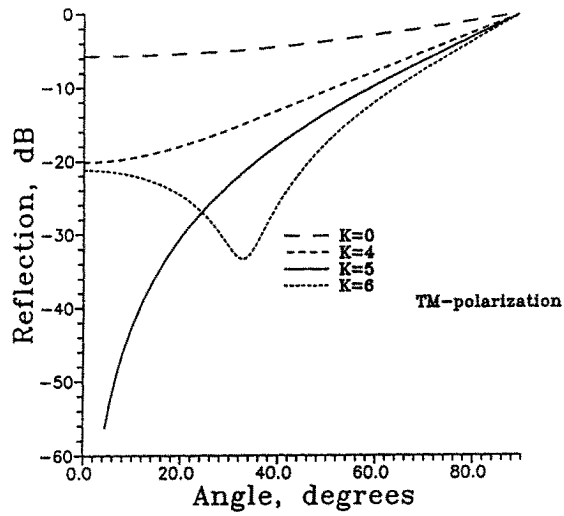


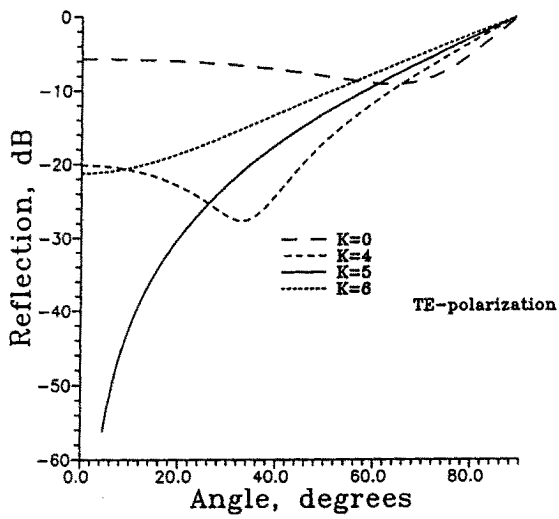
Figure 5. Absolute values of the reflection coefficient for a metal screen covered with a uniaxial omega slab (normal incidence). $\mu_t = 2 - j10$, $\epsilon_t = 2$, the coupling parameter K varies.

see that the transmission coefficient can be small as well, even for thin slabs. For example, it is below 50 dB for the thickness $d = \lambda_0/5$ (the point where the normalized frequency equals zero). It can be observed that the additional field interaction reduces transmission not only for the special coupling parameter value (32).

Angular dependence of the reflection coefficient from omega slabs in air is demonstrated in Fig. 3. The material parameters are again the same as in Fig. 1 and the normalized thickness $k_0 d = 1.0$. It is seen that the reflection coefficient remains small in a wide range of the incidence angles. Transmission properties of the same slab are illustrated by Fig. 4. Transmission decreases with the increasing coupling parameter for all incidence angles and, naturally, converges to zero for very oblique angles.



(a)



(b)

Figure 6. Same coefficient as in Figure 5 as a function of the incidence angle. The normalized slab thickness $k_0 d = 1.0$, $\mu_n = 2$, $\epsilon_n = 2$.

Next we study reflection properties of the uniaxial omega layers on an ideally conducting surface and on a magnetic screen. Reflection from a boundary covered by a lossy layer can be reduced if, in analogy with the previous case, the coming wave does not reflect from the air-slab interface, but only from the boundary. In this case, i.e., when $Z_1 = Z_+$, the reflected wave suffers attenuation due to losses in the layer. Provided that (30) or (31) is hold, the reflection coefficient from a metal-backed slab is

$$R_e = -\frac{(Y_1 + Y_-)e^{-2j\beta d}}{2Y_1 - (Y_1 - Y_-)e^{-2j\beta d}} \tag{34}$$

and for a slab on a magnetic wall we find

$$R_m = \frac{(Z_1 + Z_-)e^{-2j\beta d}}{2Z_1 - (Z_1 - Z_-)e^{-2j\beta d}} \tag{35}$$

where the characteristic admittances for the TM- and TE-polarized waves traveling in the negative z-direction are

$$Y_-^{TM} = \sqrt{\frac{\epsilon_0}{\mu_0}}\frac{\epsilon_t\epsilon_n\cos\phi}{\epsilon_n\mu_t - \sin^2\phi} \qquad Y_-^{TE} = \sqrt{\frac{\epsilon_0}{\mu_0}}\frac{\epsilon_t\mu_n - \sin^2\phi}{\mu_t\mu_n\cos\phi} \tag{36}$$

In Fig. 5 the reflection coefficient from a metal screen covered with an Ω slab is depicted as a function of the normalized thickness $k_0 d$ for normal incidence. The permeability equals $\mu_t = 2 - j10$ and the permittivity is $\epsilon_t = 2$. Curves are shown for several values of the coupling parameter K. As is established by the above analysis, the reflection becomes extremely small even for rather thin slabs, provided the condition (32) holds. With the increasing frequency (i.e., the normalized thickness $k_0 d$), the reflection coefficient for $K = \mu''/2$ sharply decreases. Also, for other values of the coupling parameter, reflection can be reduced considerably. In contrast, for real but negative coupling parameter, the reflection coefficient increases and the structure is a nearly perfect reflector when $K \to -\mu''/2$. The sign of the coupling coefficient depends on the orientation of the wire loops with respect to the z-axis direction.

Figure 6 shows the angular dependence of the reflection coefficient for the same structure and the same material parameter values as in Fig. 5. The normalized thickness $k_0 d = 1.0$. It is seen that the absorption is increased in a wide range of the incidence angles.

The effect of increasing absorption is well known for lossy chiral composites [11–13]. It appears that the uniaxial omega structures can probably offer more practical possibilities as compared with the isotropic chiral materials. In the assumption that the material parameters are constants, the reflection coefficient can be small in extremely wide frequency bands. Also, only one condition imposed on three material parameters has to be satisfied to have a low level of reflection.

5. Conclusion

A new configuration of composite microwave materials has been suggested, which offers novel possibilities in applications. Electromagnetic properties of the material are described by uniaxial bianisotropic constitutive relations. The general theory of eigenwaves in uniaxial Ω media has been addressed, and it has been established that the wave phenomena can be modelled by reciprocal but non-symmetric scalar transmission lines for any linearly polarized fields. The transmission-line model leads to simple analytical expressions for reflection and transmission coefficients for plane layers. The analysis reveals and the numerical examples demonstrate possible prospective application. It appears that the input impedance of a lossy layer on an ideally conducting surface or in free space can be matched with the free-space wave impedance by tuning the additional coupling parameter. The impedance matching condition is frequency-independent (provided the material parameter values can be assumed to be constants) and that suggests a wide frequency band for prospective anti-reflection coverings and antenna radomes.

In the numerical examples given here to demonstrate basic novel features of the uniaxial omega materials we have assumed for simplicity that the slabs have magnetic losses but the other two material parameters are real. In practice, all three parameters are complex for lossy media, however, the impedance match requirement can be satisfied for all complex parameters as well. Practical possibility of achieving the desired values of the material parameters and their frequency dependence have not been considered in the present study. Based on the similarity of the physical processes of the field coupling in the omega composites and in the chiral structures, one can expect that the number of particles and their sizes will be close to that in the chiral com-

posites design, for the same degrees of chirality or omega coupling. In the fabrication of the omega composites, the printed circuit technology can be employed, since the omega structures comprise regular arrays of plane conducting elements. Uniaxial composites can be realized as multilayered structures.

We can also comment that it is not necessary to have a high degree of additional coupling to design a low-reflecting layer. In the given examples, one can compromise between the coupling parameter value, the magnetic losses and the slab thickness. If the impedances match, the thickness is essential for the transmission properties, but not for the reflection. Comparing with the requirements known for the chiral low-reflection screens, it seems easier to achieve the desired effects with the omega composites, because in the chiral screens rather high degree of chirality is required.

Acknowledgment

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