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EIGENWAVES IN THE GENERAL UNIAXIAL BIANISOTROPIC MEDIUM WITH SYMMETRIC PARAMETER DYADICS

I. V. Lindell and A. J. Viitanen

1. Introduction

The most general linear, or bianisotropic, medium is characterized by constitutive equations of the general form [1]

$$\mathbf{D} = \overline{\overline{\epsilon}} \cdot \mathbf{E} + \overline{\overline{\xi}} \cdot \mathbf{H}, \tag{1}$$

$$\mathbf{B} = \overline{\overline{\zeta}} \cdot \mathbf{E} + \overline{\overline{\mu}} \cdot \mathbf{H}. \tag{2}$$

The uniaxial medium considered here, with the z axis defined along their axis of preference, is defined by the four symmetric parameter dyadics

$$\overline{\overline{\epsilon}} = \epsilon_z \mathbf{u}_z \mathbf{u}_z + \epsilon_t \overline{\overline{I}}_t, \qquad \overline{\overline{\mu}} = \mu_z \mathbf{u}_z \mathbf{u}_z + \mu_t \overline{\overline{I}}_t, \tag{3}$$

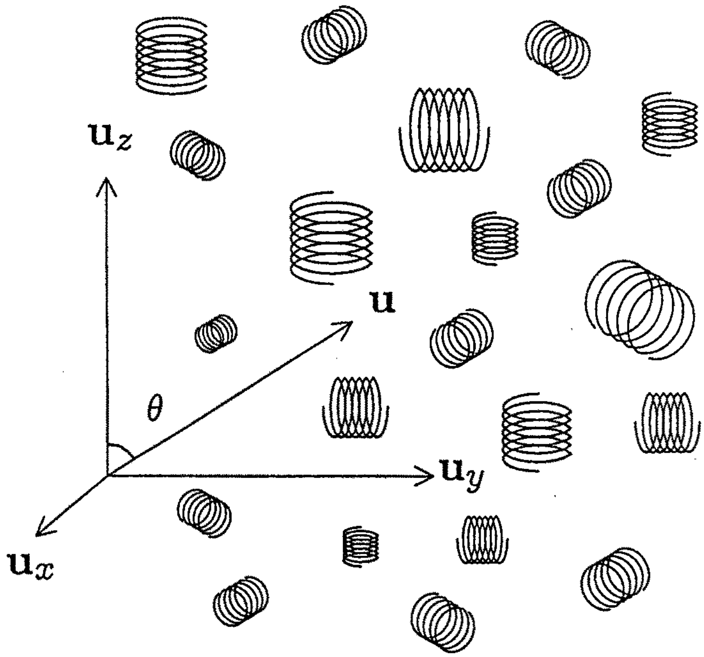

Figure 1. The uniaxial medium can be fabricated by mixing copper helices in a host medium with their axes either parallel or perpendicular to the z axis but otherwise randomly oriented.

$$\overline{\overline{\xi}} = \xi_z \mathbf{u}_z \mathbf{u}_z + \xi_t \overline{\overline{I}}_t, \qquad \overline{\overline{\zeta}} = \zeta_z \mathbf{u}_z \mathbf{u}_z + \zeta_t \overline{\overline{I}}_t, \tag{4}$$

where $\overline{\overline{I}}_t = \overline{\overline{I}} - \mathbf{u}_z \mathbf{u}_z$ is the unit dyadic transverse to the z axis.

Instead of four parameters describing the bi-isotropic medium we now have eight scalar parameters which have complex values in general. Special cases with $\xi_t = \zeta_t = 0$ (axial bianisotropy) or $\xi_z = \zeta_z = 0$ (transverse bianisotropy) have been considered previously by these and other authors [2–6].

An example of the present kind of medium can be obtained if metal helices are scattered on a surface layer before covering them by host material, because the helices will usually not be randomly oriented but tend to be either parallel or perpendicular to the surface as illustrated in Figure 1, where the surface is perpendicular to the z axis. This fabrication process gives a medium which is reciprocal, i.e., satisfies the condition $\overline{\overline{\xi}} = -\overline{\overline{\zeta}}$ [1,7]. The present analysis, however, covers a more general uniaxial medium which may also be nonreciprocal.

If the medium is assumed lossless, the corresponding conditions can be written as [1,7] $\overline{\overline{\epsilon}}^T = \overline{\overline{\epsilon}}^*$, $\overline{\overline{\mu}}^T = \overline{\overline{\mu}}^*$ and $\overline{\overline{\xi}}^T = \overline{\overline{\zeta}}^*$, which means that the off-diagonal scalar parameters in this case satisfy $\xi_z = \zeta_z^*$ and $\xi_t = \zeta_t^*$. Writing

$$\xi_t = (\chi_t - j\kappa_t)\sqrt{\mu_o\epsilon_o}, \qquad \xi_z = (\chi_z - j\kappa_z)\sqrt{\mu_o\epsilon_o}, \tag{5}$$

$$\zeta_t = (\chi_t + j\kappa_t)\sqrt{\mu_o\epsilon_o}, \qquad \zeta_z = (\chi_z + j\kappa_z)\sqrt{\mu_o\epsilon_o}, \tag{6}$$

the parameters $\chi_t, \chi_z, \kappa_t, \kappa_z$ must then all be real. The present analysis does not make the assumption of losslessness, whence the parameters need not be real.

2. Theory

2.1 Field Equations

Considering plane-wave propagation in the general uniaxial bianisotropic medium with the electric and magnetic field functions

$$\begin{pmatrix} \mathbf{E}(\mathbf{r}) \\ \mathbf{H}(\mathbf{r}) \end{pmatrix} = e^{-j\mathbf{k}\cdot\mathbf{r}} \begin{pmatrix} \mathbf{E}(0) \\ \mathbf{H}(0) \end{pmatrix}, \tag{7}$$

after inserting in the Maxwell equations we have the algebraic equations

$$\mathbf{J}\mathbf{k} \times \begin{pmatrix} \mathbf{E} \\ \mathbf{H} \end{pmatrix} = \omega \mathbf{M}_t \begin{pmatrix} \mathbf{E}_t \\ \mathbf{H}_t \end{pmatrix} + \omega \mathbf{u}_z \mathbf{M}_z \begin{pmatrix} \mathbf{E}_z \\ \mathbf{H}_z \end{pmatrix}, \tag{8}$$

with $_z$ and $_t$ denoting components parallel and perpendicular to the z axis, respectively and the matrices

$$\mathbf{J} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{M}_t = \begin{pmatrix} \epsilon_t & \xi_t \\ \zeta_t & \mu_t \end{pmatrix}, \quad \mathbf{M}_z = \begin{pmatrix} \epsilon_z & \xi_z \\ \zeta_z & \mu_z \end{pmatrix}. \tag{9}$$

Denoting $\mathbf{k} = k\mathbf{u}$, where $\mathbf{u} = \mathbf{u}_z c + \mathbf{u}_y s$ is a unit vector pointing in the direction of propagation and $c = \cos\theta$, $s = \sin\theta$, Fig. 1, we can write after eliminating the y and z components of the fields and assuming that the inverse matrices exist,

$$\left[\frac{s^2k^2}{\omega^2}\mathbf{J}\mathbf{M}_z^{-1}\mathbf{J} + \frac{c^2k^2}{\omega^2}\mathbf{J}\mathbf{M}_t^{-1}\mathbf{J} + \mathbf{M}_t\right]\begin{pmatrix} E_x \\ H_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{10}$$

or, in an equivalent form,

$$\left[\frac{s^2k^2}{K_z^2}\mathbf{M}_z^T + \frac{c^2k^2}{K_t^2}\mathbf{M}_t^T - \mathbf{M}_t\right]\begin{pmatrix} E_x \\ H_x \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \tag{11}$$

with T denoting the transpose of a matrix and

$$K_t^2 = \omega^2(\mu_t\epsilon_t - \xi_t\zeta_t), \qquad K_z^2 = \omega^2(\mu_z\epsilon_z - \xi_z\zeta_z). \tag{12}$$

The eigenvalue equation for k is obtained by requiring the determinant of the matrix factor in (11) to be zero. This leads to the following equation

$$\begin{aligned}&\left(\frac{\mu_t}{K_t^2}c^2 + \frac{\mu_z}{K_z^2}s^2 - \frac{\mu_t}{k^2}\right)\left(\frac{\epsilon_t}{K_t^2}c^2 + \frac{\epsilon_z}{K_z^2}s^2 - \frac{\epsilon_t}{k^2}\right)\\ &= \left(\frac{\xi_t}{K_t^2}c^2 + \frac{\xi_z}{K_z^2}s^2 - \frac{\zeta_t}{k^2}\right)\left(\frac{\zeta_t}{K_t^2}c^2 + \frac{\zeta_z}{K_z^2}s^2 - \frac{\xi_t}{k^2}\right),\end{aligned} \tag{13}$$

which is the dispersion equation for the most general uniaxial bianisotropic medium with symmetric parameter dyadics.

The equation can be written in a simpler-looking form

$$\left(M - \frac{\mu_t}{k^2}\right)\left(E - \frac{\epsilon_t}{k^2}\right) = \left(X - \frac{\zeta_t}{k^2}\right)\left(Z - \frac{\xi_t}{k^2}\right), \tag{14}$$

by introducing the notation

$$M = \frac{\mu_t}{K_t^2}c^2 + \frac{\mu_z}{K_z^2}s^2, \qquad E = \frac{\epsilon_t}{K_t^2}c^2 + \frac{\epsilon_z}{K_z^2}s^2, \tag{15}$$

$$X = \frac{\xi_t}{K_t^2}c^2 + \frac{\xi_z}{K_z^2}s^2, \qquad Z = \frac{\zeta_t}{K_t^2}c^2 + \frac{\zeta_z}{K_z^2}s^2. \tag{16}$$

It can be shown that, by setting $\xi_t = \zeta_t = 0$, the dispersion equation (13) or (14) reduces to that of the axially bianisotropic uniaxial medium, in a form given before in [2,3]. Similarly, for another special case $\xi_z = \zeta_z = 0$, the corresponding equation for the transversely bianisotropic uniaxial medium results, also derived previously [6]. In the simple anisotropic case with $\xi_t = \xi_z = \zeta_t = \zeta_z = 0$, the right-hand sides of (13) and (14) vanish and the solutions are those of the well-known TM and TE polarized eigenwaves [8,9,7,3]

$$k_{TM}^2 = \frac{\mu_t}{M} = \frac{k_t^2}{c^2 + \frac{\epsilon_t}{\epsilon_z}s^2}, \qquad k_{TE}^2 = \frac{\epsilon_t}{E} = \frac{k_t^2}{c^2 + \frac{\mu_t}{\mu_z}s^2}. \tag{17}$$

2.2 Wavenumber Solutions

The two solutions for (14) can be written in the following form:

$$k_\pm^2 = \frac{1}{2\omega^2(ME - XZ)}\Bigg[\omega^2(M\epsilon_t + E\mu_t - X\xi_t - Z\zeta_t)$$
$$\pm\sqrt{\omega^4(M\epsilon_t + E\mu_t - X\xi_t - Z\zeta_t)^2 - 4K_t^2\omega^2(ME - XZ)}\Bigg]. \tag{18}$$

By inserting the expressions of the quantities M, E, X and Z, the two solutions are obtained in terms of the original medium parameters but in a more complicated looking form. Defining two more wavenumbers by

$$K_{tz}^2 = k_z^2\frac{1}{2}\left(\frac{\mu_t}{\mu_z} + \frac{\epsilon_t}{\epsilon_z}\right) - (\chi_t\chi_z - \kappa_t\kappa_z)k_o^2, \tag{19}$$

$$K_{tt}^2 = k_t^2 - (\chi_t^2 - \kappa_t^2)k_o^2, \tag{20}$$

(note that K_{tt} equals K_{tz} with z replaced by t), with

$$k_t^2 = \omega^2\mu_t\epsilon_t, \qquad k_z^2 = \omega^2\mu_z\epsilon_z, \tag{21}$$

we obtain the solutions in the form

$$k_\pm^2 = \frac{K_t^2}{a \mp \sqrt{a^2 - b}} \tag{22}$$

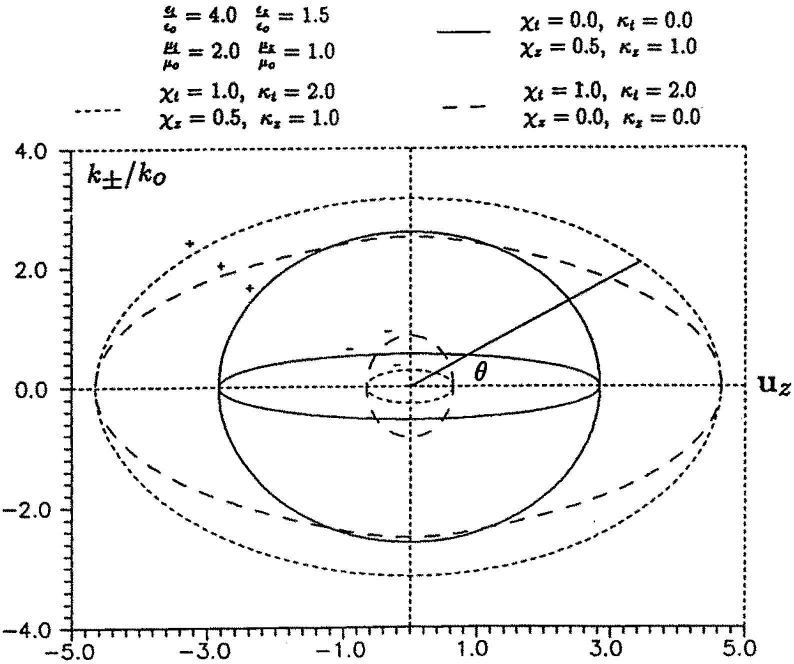

Figure 2. Examples of wavenumber diagrams for lossless general uniaxial bianisotropic media. The diagram is rotationally symmetric around the z axis.

with

$$a = \frac{K_{tt}^2}{K_t^2}\cos^2\theta + \frac{K_{tz}^2}{K_z^2}\sin^2\theta, \tag{23}$$

$$b = \cos^4\theta + 2\frac{K_{tz}^2}{K_z^2}\cos^2\theta\sin^2\theta + \frac{K_t^2}{K_z^2}\sin^4\theta. \tag{24}$$

The expression (18) or (22) is valid for complex values of the parameters and thus represents the wavenumber solution for the general uniaxial bianisotropic medium. It can be checked by considering some special cases with previously known solutions. Examples of wavenumber surfaces are displayed in Figure 2 for axial, transverse and more general bianisotropic media with some values corresponding to lossless material parameters.In the axial bianisotropic case (solid line) there exists an optic axis (the z axis), along which there is only one wavenumber solution, while in the transverse and general bianisotropic case there are none. From the figure it is also seen that in axial propagation ($\theta = 0$), the wavenumbers for transverse and general case coincide, which reflects the fact that the wave is a TEM wave.

2.3 Special Cases

2.3.1 Axial bianisotropy

Considering the special case of axial bianisotropy with $\xi_t = \zeta_t = 0$, the expression (22) can be written in the simpler form

$$k_\pm^2 = \frac{k_t^2 K_z^2}{\cos^2\theta K_z^2 + \sin^2\theta (K_{tz}^2 \mp \sqrt{K_{tz}^2 - k_t^2 K_z^2})}$$
$$= \frac{k_t^2(\mu_z\epsilon_z - \xi_z\zeta_z)}{\cos^2\theta(\mu_z\epsilon_z - \xi_z\zeta_z) + \sin^2\theta\left[\frac{\mu_t\epsilon_z + \mu_z\epsilon_t}{2} \mp \sqrt{\left(\frac{\mu_t\epsilon_z - \mu_z\epsilon_t}{2}\right)^2 - \mu_t\epsilon_t\xi_z\zeta_z}\right]} \tag{25}$$

which coincides with the expressions previously given in [2–4].

2.3.2 Transverse bianisotropy

Likewise, for transverse bianisotropy with $\xi_z = \zeta_z = 0$, after writing

$$\frac{K_{tz}^2}{K_z^2}\sin^2\theta = \frac{1}{2}\left(\frac{\mu_t}{\mu_z} + \frac{\epsilon_t}{\epsilon_z}\right)\sin^2\theta$$
$$= \frac{1}{2}\left(\frac{k_t^2}{k_{TE}^2} + \frac{k_t^2}{k_{TM}^2}\right) - \cos^2\theta, \tag{26}$$

(22) reduces to the form

$$\frac{K_t^2}{k_\pm^2} = \frac{1}{2}\left(\frac{k_t^2}{k_{TE}^2} + \frac{k_t^2}{k_{TM}^2} + \frac{4\kappa_t^2 k_o^2}{K_t^2}\cos^2\theta\right)$$

$$\mp \frac{1}{2}\sqrt{\left(\frac{k_t^2}{k_{TE}^2} + \frac{k_t^2}{k_{TM}^2} + \frac{4\kappa_t^2 k_o^2}{K_t^2}\cos^2\theta\right)^2 - 4\frac{k_t^4}{k_{TE}^2 k_{TM}^2} + 4\frac{k_o^2}{k_z^2}(\chi_t^2 + \kappa_t^2)\sin^4\theta} \tag{27}$$

This can also be shown to coincide with the corresponding expression given in [6].

2.3.3 Axial propagation

For plane eigenwaves propagating along the z axis $(\theta = 0)$, (18) gives the wave numbers

$$k_\pm = \sqrt{K_t^2 + \kappa_t^2 k_o^2} \pm \kappa_t k_o = \sqrt{k_t^2 - \chi_t^2 k_o^2} \pm \kappa_t k_o. \tag{28}$$

These are actually the wavenumbers of two plane eigenwaves in a bi-isotropic medium whose parameters are ϵ_t, μ_t, χ_t, κ_t. A similar result is also obtained for a medium with just transverse bianisotropy [6].

Actually, from the axial symmetry of the medium we can deduce that the two eigenwaves propagating along the z axis must be transversely polarized (TEM) with respect to the z axis. Since in this case the fields do not see the axial medium parameters, they propagate like in a bi-isotropic medium, in which the two eigenwaves are circularly polarized, the '$+$' wave in the right-hand direction and the '$-$' wave in the left-hand direction.

If the medium has no transverse chirality, $\kappa_t = 0$, the two wave numbers coincide:

$$k_\pm = \sqrt{k_t^2 - k_o^2\chi^2}, \tag{29}$$

in which case there is an optic axis along the axis of symmetry.

2.3.4 Transverse propagation

In the transverse propagation $\theta = \pi/2$, from (18) or (22) we can write the following expression for the two eigenwaves:

$$k_\pm^2 = K_{tz}^2 \pm \sqrt{K_{tz}^4 - K_t^2 K_z^2}. \tag{30}$$

If the medium is only axially bianisotropic, i.e., if $\xi_t = \zeta_t = 0$, this is simplified to

$$k_\pm^2 = k_z^2 \frac{1}{2} \left(\frac{\mu_t}{\mu_z} + \frac{\epsilon_t}{\epsilon_z} \pm \sqrt{\left(\frac{\mu_t}{\mu_z} - \frac{\epsilon_t}{\epsilon_z} \right)^2 + 4(\chi_z^2 + \kappa_z^2) \frac{k_t^2 k_o^2}{k_z^4}} \right), \tag{31}$$

which coincides with earlier results [2,3]. On the other hand, if the medium is only transversely biansiotropic, i.e., if $\xi_z = \zeta_z = 0$, (30) has the form

$$k_\pm^2 = k_z^2 \frac{1}{2} \left(\frac{\mu_t}{\mu_z} + \frac{\epsilon_t}{\epsilon_z} \pm \sqrt{\left(\frac{\mu_t}{\mu_z} - \frac{\epsilon_t}{\epsilon_z} \right)^2 + 4(\chi_t^2 + \kappa_t^2) \frac{k_o^2}{k_z^2}} \right), \tag{32}$$

which is the same result as given in [6]. Note the slight difference in form between (31) and (32). For simple anisotropic media both expressions reduce to

$$k_+ = \omega\sqrt{\mu_t \epsilon_z}, \qquad k_- = \omega\sqrt{\mu_z \epsilon_t}, \tag{33}$$

which is a well-known result [10,11].

3. Polarization Properties

3.1 Eigenvectors

The eigenvectors of the electric and magnetic fields could be obtained directly from the equation (8). Instead, let us start from the equation (11) giving us the relation between the x-components of the electric and magnetic field, from which the admittance quantities $Y_{x\pm}$ for the two eigenwaves can be solved in the form

$$Y_{x\pm} = \frac{H_x}{E_x} = -\frac{k_\pm^2 X - \zeta_t}{k_\pm^2 M - \mu_t}. \tag{34}$$

Inserting $H_{x\pm} = Y_{x\pm}E_{x\pm}$ into the equation (8), the field components $E_{y\pm}$ and $E_{z\pm}$ are obtained:

$$E_{y\pm} = -\frac{\omega c k_\pm}{K_t^2}(\xi_t + \mu_t Y_{x\pm})E_{x\pm}, \tag{35}$$

$$E_{z\pm} = \frac{\omega s k_\pm}{K_z^2}(\xi_z + \mu_z Y_{x\pm})E_{x\pm}. \tag{36}$$

Thus, the eigenvectors giving the polarizations of the electric field for the two eigenwaves can be written in the form

$$\mathbf{e}_\pm = \mathbf{u}_x - \frac{\omega c k_\pm}{K_t^2}(\xi_t + \mu_t Y_{x\pm})\mathbf{u}_y + \frac{\omega s k_\pm}{K_z^2}(\xi_z + \mu_z Y_{x\pm})\mathbf{u}_z. \tag{37}$$

The eigenvectors corresponding to the magnetic field can be obtained by using the duality transformation: $\overline{\overline{\epsilon}} \to \overline{\overline{\mu}}$, $\overline{\overline{\mu}} \to \overline{\overline{\epsilon}}$, $\overline{\overline{\xi}} \to -\overline{\overline{\zeta}}$ and $\overline{\overline{\zeta}} \to -\overline{\overline{\xi}}$ in the form

$$\mathbf{h}_\pm = \mathbf{u}_x + \frac{\omega c k_\pm}{K_t^2}(\zeta_t + \epsilon_t Z_{x\pm})\mathbf{u}_y - \frac{\omega s k_\pm}{K_z^2}(\zeta_z + \epsilon_z Z_{x\pm})\mathbf{u}_z, \tag{38}$$

with

$$Z_{x\pm} = \frac{1}{Y_{x\pm}} = -\frac{k_\pm^2 Z - \xi_t}{k_\pm^2 E - \epsilon_t}. \tag{39}$$

The electric and magnetic fields composed of eigenwaves propagating in the same or opposite direction can now be written in the form

$$\mathbf{E}(\mathbf{r}) = A_+ e^{-j\mathbf{k}_+\cdot\mathbf{r}}\,\mathbf{e}_+ + A_- e^{-j\mathbf{k}_-\cdot\mathbf{r}}\,\mathbf{e}_- + B_+ e^{j\mathbf{k}_+\cdot\mathbf{r}}\,\mathbf{e}_+ + B_- e^{j\mathbf{k}_-\cdot\mathbf{r}}\,\mathbf{e}_-, \tag{40}$$

$$\mathbf{H}(\mathbf{r}) = C_+ e^{-j\mathbf{k}_+\cdot\mathbf{r}}\,\mathbf{h}_+ + C_- e^{-j\mathbf{k}_-\cdot\mathbf{r}}\,\mathbf{h}_- + D_+ e^{j\mathbf{k}_+\cdot\mathbf{r}}\,\mathbf{h}_+ + D_- e^{j\mathbf{k}_-\cdot\mathbf{r}}\,\mathbf{h}_- \tag{41}$$

and the coefficients $A_\pm$, $B_\pm$, $C_\pm$ and $D_\pm$ can be determined through boundary conditions in question. Let us consider two special cases.

Axial and transverse propagation

For axial propagation with $\theta = 0$, the eigenvectors (37) become right and left circularly polarized

$$\mathbf{e}_{\pm} = \mathbf{u}_x - \frac{\omega k_{\pm}}{k_{\pm}^2 - K_t^2}(\zeta_t - \xi_t)\mathbf{u}_y = \mathbf{u}_x \mp j\mathbf{u}_y, \tag{42}$$

corresponding to the axial wavenumbers (28). In the transverse propagation, $\theta = \pi/2$, the eigenvectors become

$$\begin{aligned}\mathbf{e}_{\pm} =&\mathbf{u}_x + \frac{\omega k_{\pm}}{k_{\pm}^2 - \frac{\mu_t}{\mu_z}K_z^2}(\zeta_t - \frac{\mu_t}{\mu_z}\xi_z)\mathbf{u}_z \\ =&\mathbf{u}_x + \frac{k_o k_{\pm}}{k_{\pm}^2 - \frac{\mu_t}{\mu_z}K_z^2}\left[(\chi_t - \frac{\mu_t}{\mu_z}\chi_z) + j(\kappa_t + \frac{\mu_t}{\mu_z}\kappa_z)\right]\mathbf{u}_z,\end{aligned} \tag{43}$$

with the corresponding propagation wavenumbers substituted from (30). (43) shows the effect of the medium on eigenpolarizations for the general uniaxial medium. In the lossless case, when all the parameters are real, it is seen that the chirality of the medium is responsible for the ellipticity of the eigenpolarizations. In fact, for $\kappa_z = \kappa_t = 0$ (nonchiral medium), the eigenpolarizations are linear, because the last term in (43) is then real. By changing the material parameters we can control the ellipticity and the orientation of the ellipse of the eigenwaves.

The length of the polarization vector [9]

$$\mathbf{p}(\mathbf{E}) = \frac{\mathbf{E} \times \mathbf{E}^*}{j\mathbf{E} \cdot \mathbf{E}^*} = p\mathbf{u}_y \tag{44}$$

gives information of the polarization of the complex vector $\mathbf{E}$. The direction of $\mathbf{p}(\mathbf{E})$ is perpendicular to the ellipse and points in the right-hand direction of the rotation. For a TEM wave propagating in the $\mathbf{u}_y$ direction and defining $p = \mathbf{u}_y \cdot \mathbf{p}(\mathbf{E})$, for $p = 0$ the field is linearly polarized, for $p = +1$ right circularly polarized and for $p = -1$ left circularly polarized. Values $0 < |p| < 1$ mean that the field has elliptical polarization and there is one-to-one dependence on the axial ratio of the ellipse and value of p [9].

The polarization of the eigenwaves corresponding to the media in Fig. 2 is depicted in Fig. 3 in terms of the p value, i.e., the y component of the polarization vector $\mathbf{p}(\mathbf{e}_{\pm})$ defined above. The polarization

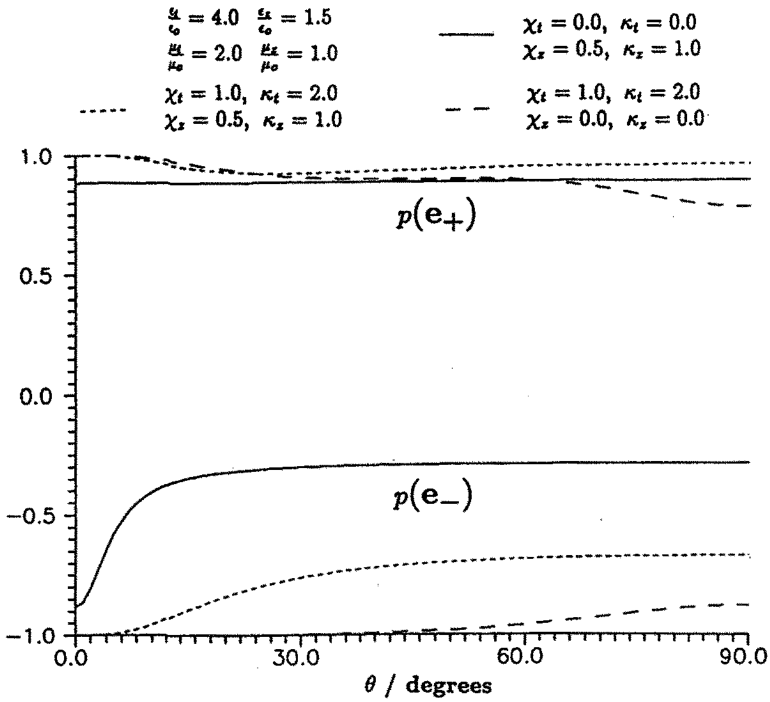

Figure 3. Polarization vector values $p(\mathbf{e}_\pm)$ of eigenwaves in some lossless uniaxial bianisotropic media.

is shown as a function of the propagation angle θ, for certain real values of the parameters, i.e., lossless media. It is seen that the eigenwaves denoted by the $+$ subscript have the right-hand polarization because of $p(\mathbf{e}_+) > 0$, while those denoted by the $-$ subscript have the left-hand polarization because of $p(\mathbf{e}_-) < 0$.

3.2 Polarization Transformation in a Lossless Medium

As an example of the very general theory, let us consider plane wave propagation in transverse direction $(\theta = \pi/2)$ in a lossless and reciprocal uniaxial medium, characterized by real parameters μ_z, μ_t, ϵ_z, ϵ_t, κ_z and κ_t. We consider a linearly polarized wave at $y = 0$ propagating in the direction of the positive y axis. The angle between the polarization of the field and the x axis is denoted by α at $y = 0$, as seen

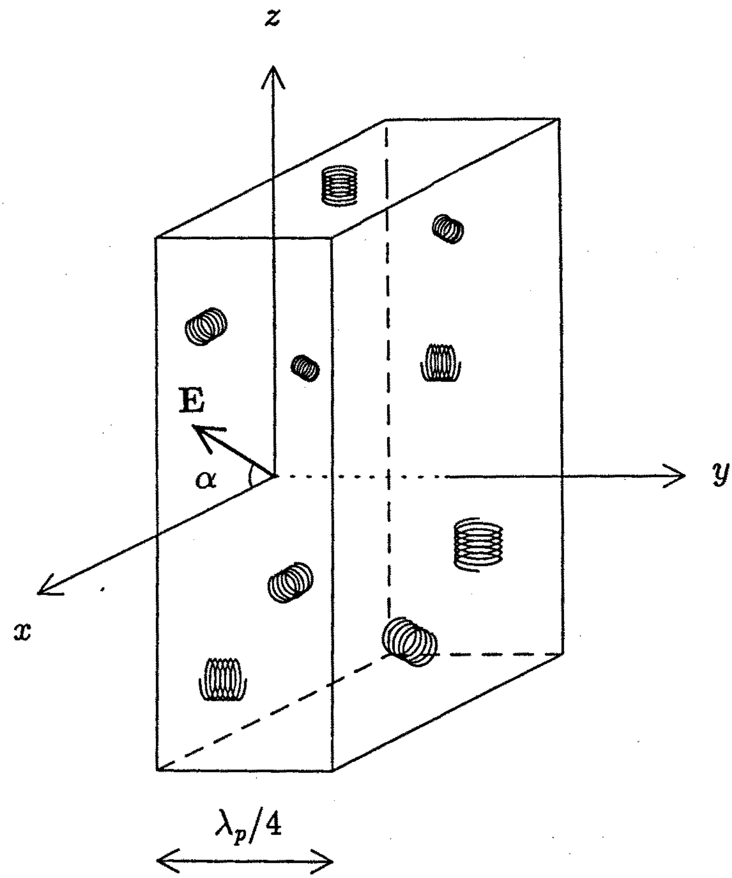

Figure 4. Plane wave propagation in a uniaxial bianisotropic slab.

in Figure 4. The electric field propagating in y direction obeys the expression

$$\mathbf{E}(y) = E_o e^{-j(k_+ + k_-)y/2} \left\{ \left[\sin\alpha \left(e^{j(k_+ - k_-)y/2} - e^{-j(k_+ - k_-)y/2} \right) \right.\right.$$

$$\left. -j(\kappa_t + \frac{\mu_t}{\mu_z}\kappa_z) k_o \cos\alpha \left(\frac{k_+ e^{j(k_+ - k_-)y/2}}{k_+^2 - \frac{\mu_t}{\mu_z} K_z^2} - \frac{k_- e^{-j(k_+ - k_-)y/2}}{k_-^2 - \frac{\mu_t}{\mu_z} K_z^2} \right) \right] \mathbf{u}_x$$

$$+\left[\frac{(\kappa_t+\frac{\mu_t}{\mu_z}\kappa_z)^2k_o^2k_+k_-}{(k_+^2-\frac{\mu_t}{\mu_z}K_z^2)(k_-^2-\frac{\mu_t}{\mu_z}K_z^2)}\cos\alpha\left(e^{j(k_+-k_-)y/2}-e^{-j(k_+-k_-)y/2}\right)\right.$$

$$\left.\left.+j(\kappa_t+\frac{\mu_t}{\mu_z}\kappa_z)k_o\sin\alpha\left(\frac{k_-e^{j(k_+-k_-)y/2}}{k_-^2-\frac{\mu_t}{\mu_z}K_z^2}-\frac{k_+e^{-j(k_+-k_-)y/2}}{k_+^2-\frac{\mu_t}{\mu_z}K_z^2}\right)\right]\mathbf{u}_z\right\} \quad (45)$$

The polarization of the field is seen to repeat itself after integer multiples of the distance $\lambda_p/2$, where the polarization wavelength in the lossless medium is defined by

$$\lambda_p=\frac{4\pi}{|k_+-k_-|}. \quad (46)$$

The polarization of the field is maximally changed from linear to elliptic at midpoints of the previous distances, i.e., at

$$y=\lambda_p/4+n\lambda_p/2, \quad n=0,1,2,... \quad (47)$$

Let us consider the polarization at $y=\lambda_p/4$. The expression of the propagating field (45) is then simplified to

$$\mathbf{E}(\lambda_p/4)=jE_oe^{-j\frac{k_++k_-}{k_+-k_-}\pi/2}$$

$$\left\{2\left[\sin\alpha\mathbf{u}_x+\frac{(\kappa_t+\frac{\mu_t}{\mu_z}\kappa_z)^2k_o^2k_+k_-}{(k_+^2-\frac{\mu_t}{\mu_z}K_z^2)(k_-^2-\frac{\mu_t}{\mu_z}K_z^2)}\cos\alpha\mathbf{u}_z\right]\right.$$

$$\left.-j(\kappa_t+\frac{\mu_t}{\mu_z}\kappa_z)k_o\left(\frac{k_+}{k_+^2-\frac{\mu_t}{\mu_z}K_z^2}+\frac{k_-}{k_-^2-\frac{\mu_t}{\mu_z}K_z^2}\right)[\cos\alpha\mathbf{u}_x-\sin\alpha\mathbf{u}_z]\right\}. \quad (48)$$

3.3 Quarter-Wave Polarization Transformer

In wishing to have a polarization transformation effect with the present medium, similar to what was obtained with a uniaxially bianistropic medium [2,3,12], let us aim at the case when the polarization at $y=\lambda_p/4$ is circular. To this end, by choosing the material parameters properly, the magnitudes of the real and imaginary parts of the electric field vector (48) are required equal in length and orthogonal in direction. Because the polarization of the field is changed continuously from linear to circular in the medium in the interval $0 < y < \lambda/4$, it must have had all polarization states between linear and circular, with the same handedness, in the interval.

Requiring circular polarization for $\mathbf{E}(\lambda_p/4)$ leads to the conditions

$$K_t^2 = K_z^2 \tag{49}$$

and

$$\left|(\kappa_t + \frac{\mu_t}{\mu_z}\kappa_z)k_o(\frac{k_+}{k_+^2 - \frac{\mu_t}{\mu_z}K_z^2} + \frac{k_-}{k_-^2 - \frac{\mu_t}{\mu_z}K_z^2})\right| = 2. \tag{50}$$

With six parameters and only two conditions, there is a great mathematical freedom to choose values for the parameters to obtain this effect. For example, with the material parameters $\mu_t = 2\,\mu_o$, $\epsilon_t = 1.4\,\epsilon_o$, $\mu_z = \mu_o$, $\epsilon_z = 3\,\epsilon_o$, $\kappa_t = \pm 0.5$ and $\kappa_z = \pm 0.66$ the above conditions are fullfilled. The obvious question of realizability cannot, of course, be answered without a thorough basic numerical analysis of helices or other chiral particles in a host medium or, alternatively, a lot of experimental results with different mixtures. This, however, is far beyond the object of the present paper, which gives the basic analysis of waves and demonstrates some properties typical of the medium in question.

With the above choice of material parameters, the polarization vector at $y = \lambda_p/4$ can be written in the simple form

$$\mathbf{p}(\alpha) = -\mathbf{u}_y \cos 2\alpha. \tag{51}$$

This corresponds to the axial ratio of the polarization ellipse [9]

$$e = \tan\left(\frac{\pi}{4} - \alpha\right), \tag{52}$$

which shows us that, for $\alpha = \pm\pi/4$, $\pm 3\pi/4$ the outcoming polarization is linear while for $\alpha = 0$, π it is right circular and, for $\alpha = \pm\pi/2$, left circular. Thus, the $\lambda_p/4$ slab operates as a polarization transformer for a linearly polarized wave propagating in the direction transverse to the z axis.

To see the continuous change in polarization, the component $p = \mathbf{u}_y \cdot \mathbf{p}(\mathbf{E})$ of the polarization vector is displayed in Figure 5, for parameter values satisfying the conditions (49) - (50), as a function of normalized distance along the y axis. When $y = \lambda_p/4$, corresponding

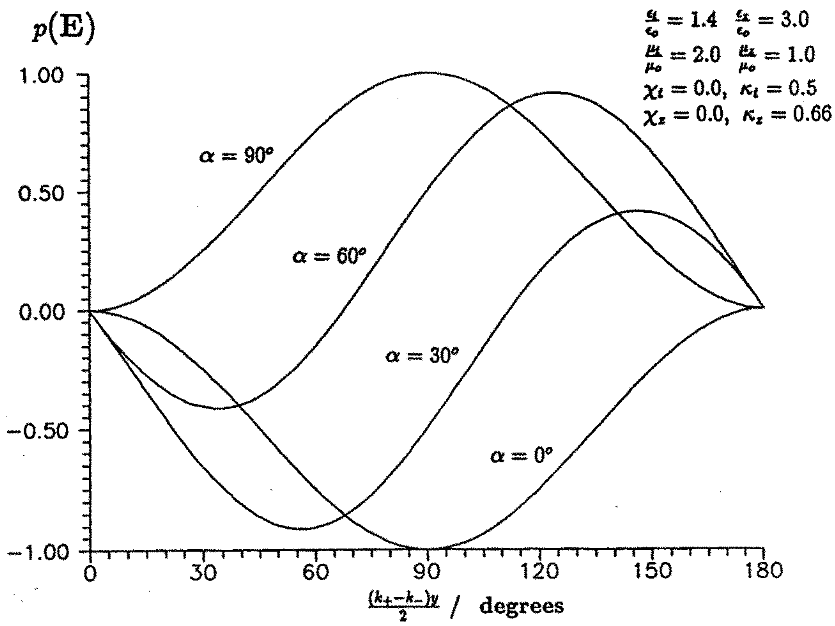

Figure 5. Polarization vector values $p(\mathrm{E})$ of the propagating electric field in a lossless uniaxial bianisotropic medium.

to the value of 90^o on the normalized distance on the horizontal axis of Figure 5, all polarizations can be obtained by changing the angle α of the incoming linearly polarized field. Again, we conclude that a slab of such a medium could be applied as a polarization transformer: by rotating the slab, all polarization ellipses could be produced at the output.

Since there are six medium parameters to choose from, this kind of a polarization transformer can be obtained through many other choices. For example, we may restrict to the transversely uniaxial chiral medium [13] with the material parameter values $\kappa_z = 0$, $\epsilon_t/\epsilon_z = 1,13$, $\mu_t/\mu_z = 2$ and $\kappa_t\sqrt{\mu_o\epsilon_o/\mu_t\epsilon_t} = 0,75$. Also, we can restrict to the uniaxial chiral medium [2,3,12] with the material parameter values $\kappa_t = 0$, $\epsilon_z/\epsilon_t = 3$, $\mu_z/\mu_t = 3$ and $\kappa_z\sqrt{\mu_o\epsilon_o/\mu_z\epsilon_z} = 0,943$ [12], except that in this case we have $\mathbf{p} = +\mathbf{u}_y \cos 2\alpha$, which means that the handedness is opposite to that of the previous two examples.

4. Conclusion

Plane wave propagation in a general uniaxial bianisotropic medium was considered in the present paper. The dispersion equation was solved and the corresponding eigenfields given. The two eigenfields labeled by + and − were seen to reduce to the well-known TM and TE fields in the special case of the simple uniaxial anisotropic medium. Examples of wavenumber diagrams were shown for lossless uniaxial media. For the general uniaxial medium there are no optical axes in contrast to the axially bianisotropic medium considered earlier in [2,3,12].

To see some of the properties of the medium in question, the expressions valid for the general medium were applied to the lossless and reciprocal special case with six real medium parameters. It was seen that, the quarter-wave polarization transformer which changes linear polarization to elliptical with given axial ratio and direction of rotation, discussed earlier in terms of a uniaxially bianisotropic medium with five real parameters, can be realized with the present six-parameter medium. The added freedom may help in finding a more realizable solution.

The uniaxial chiral medium described in the paper may possess applications, e.g., in antenna engineering when polarization transformations are of major interest. Of course, the effect of losses has to be found out before making more practical conclusions. This involves, however, either basic numerical or experimental work, which is outside the scope of the present paper.

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