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ANALYSIS OF SIGNAL DISTORTION ON COUPLED MICROSTRIP LINES WITH AN OVERLAY AND A NOTCH

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1. INTRODUCTION

One of the major problems in designing closely placed interconnections is the distortion due to the difference between the propagation phase velocities of the dominant modes. This phenomenon can be explored by considering parallel microstrip transmission lines. Several techniques were presented to overcome the distortion that arises due to the difference between the propagating phase velocities of the even and odd modes. Quasi-static method of moment based techniques are reported in [1–3]. These techniques are based on controlling the dielectric constant of the notch, the height and width of the notch, dielectric substrate, and overlay materials. Although the reported data are useful in reducing the distortion, it's application is limited due to the quasi-static solution which provides a valid data for the lower band of the frequency of interest. The transmission line being investigated here consists of two symmetric thin perfectly conducting strips with a rectangular notch between the strips, two layers of dielectric substrate, and a dielectric overlay as proposed in [1]. In this work, the method of lines (MoL) is used to compute the effective dielectric constants of the even and odd modes as functions of frequency. The details of the method of lines technique are briefly mentioned here since this technique is fully described in [4].

2. FORMULATION OF THE PROBLEM

The transmission line geometry investigated here is basically the same as that was investigated before using the quasi-static method of moment approach [1] as shown in Fig. 1. In this analysis a PEC boundary is placed around the geometry. The dimensions of the metallic box are chosen large enough such that they do not affect the performance of the original open structure transmission line. The semi-analytical technique, method of lines (MoL) [4], will be used to investigate this transmission line geometry. The method of lines has been found suitable to analyze planar microstrip lines on multilayers and inhomogeneous dielectric substrate [4, 5]. Since the transmission line geometry is enclosed in a perfectly conducting box as shown in Fig. 1, all dielectric layers are partially filled with air, thus they are inhomogeneous dielectric layers in the x-direction.

Due to the inhomogeneity of the transmission line structure, the procedure of this full wave analysis requires solving the Helmholtz and the Sturm-Liouville partial differential equations [4]. They are given, respectively, as follows:

$$\frac{\partial^2 \psi_h}{\partial x^2} + \frac{\partial^2 \psi_h}{\partial y^2} + \left[\varepsilon_r(x) k_0^2 - k_z^2\right] \psi_h = 0 \tag{1a}$$

$$\varepsilon_r(x) \frac{\partial}{\partial x} \left(\frac{1}{\varepsilon_r(x)} \frac{\partial \psi_e}{\partial x} \right) + \frac{\partial^2 \psi_e}{\partial y^2} + \left[\varepsilon_r(x) k_0^2 - k_z^2\right] \psi_e = 0 \tag{1b}$$

where ψ_h and ψ_e are the scalar magnetic and electric potentials, respectively. The wave is assumed to propagate in the z-direction with

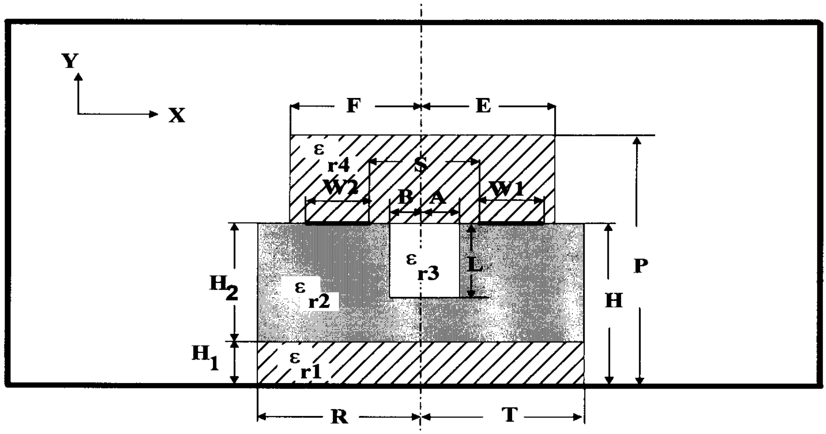


Figure 1. Geometrical Model of the two microstrip transmission line enclosed in a metallic box.

propagation constant $k_z = k_0\sqrt{\varepsilon_{re}}$, in which ε_{re} is the effective dielectric constant of the propagating mode. The free space wave number is $k_0 = \sqrt{\mu_0\varepsilon_0}$. The relative dielectric constant $\varepsilon_r(x)$ in each horizontal layer is inhomogeneous in the x-direction which is obvious from Fig. 1. The magnetic and electric scalar potentials should fulfill the boundary conditions [4]:

$$\text{On electric walls}: \quad \psi_h = 0, \quad \frac{\partial \psi_e}{\partial x} = 0 \tag{2a}$$

$$\text{On magnetic walls}: \quad \psi_e = 0, \quad \frac{\partial \psi_h}{\partial x} = 0 \tag{2b}$$

Upon applying the MoL technique, the x-dimension of the geometry is divided into a number of electric and magnetic lines parallel to the y-direction with discretization distance h. The electric lines are shifted from the magnetic lines by $h/2$. The first order forward finite difference approximation is used to replace the first and the second derivatives with respect to the x-variable as follows:

$$h\frac{\partial \psi_h}{\partial x} \approx D\psi_h \ , \text{ and } \ h^2\frac{\partial^2 \psi_h}{\partial x^2} \approx -D^t D\psi_h \tag{3a}$$

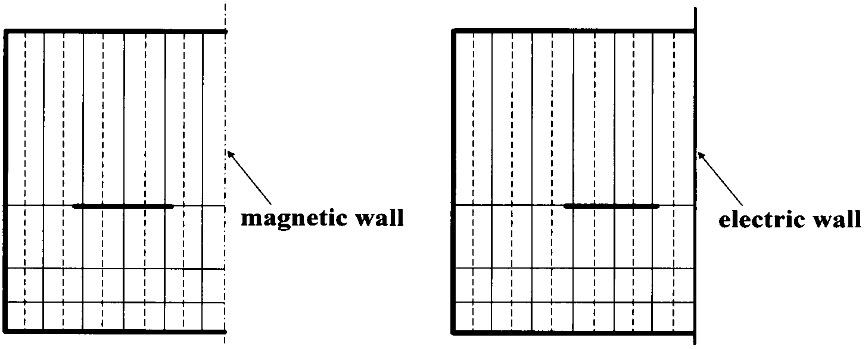
$$h\frac{\partial \psi_e}{\partial x} \approx -D^t\psi_e \ , \text{ and } \ h^2\varepsilon_r(x)\frac{\partial}{\partial x}\left(\frac{1}{\varepsilon_r(x)}\frac{\partial \psi_e}{\partial x}\right) \approx -\varepsilon_e D\varepsilon_h^{-1}D^t D\psi_e \tag{3b}$$

where D is a difference operator in the matrix form that takes into account the boundary conditions of the scalar electric and magnetic potentials given in equations (2a, b). The matrix D^t is the transpose of the matrix D. Thus the partial differential equations (1a, b) are converted to ordinary differential equations in the y-variable which can be solved analytically for ψ_h and ψ_e [4].

For symmetric microstrip line configurations, only one half of the transmission line cross-section will be considered. The x-dimension will be divided into a number of electric and magnetic lines, $N1$ and N, respectively. For the even mode, the line of symmetry is a magnetic wall, and the boundary conditions of the configuration will be Dirichlet-Neuman. However, for the odd mode, the line of symmetry is an electric wall, and the boundary conditions of the configuration will be Dirichlet-Dirichlet. Thus, for the even and odd modes, the matrices $[D]_{even}$ and $[D]_{odd}$ are given by:

$$
\begin{aligned}
[D]_{even} &= \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & 1 & 0 \\ 0 & \dots & 0 & -1 & 1 \end{pmatrix}_{N\times N} \\
[D]_{odd} &= \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ -1 & 1 & \ddots & \ddots & \vdots \\ 0 & -1 & 1 & \ddots & 0 \\ 0 & \ddots & -1 & 1 & 0 \\ \vdots & \ddots & \ddots & -1 & 1 \\ 0 & \dots & 0 & 0 & -1 \end{pmatrix}_{N1\times N}
\end{aligned}
\tag{4}
$$

where for the even mode, N is the number of magnetic lines or the number of electric lines (in this case they are equal), while for the odd mode, $N1$ is the number of electric lines and N is the number of the magnetic lines (in this case $N1 = N+1$). It is obvious that the use of symmetry reduces the order of the matrix operator D and consequently the order of all matrices that are involved in the calculations. The configurations of the even and odd mode cases are shown in Fig. 2. After some algebraic manipulations, a system of equations is obtained as:



a b

Figure 2. (a) Geometrical model for the even mode, number of electric lines (dashed) = 5 and number of magnetic lines (solid) = 5. (b) Geometrical model for the odd mode, number of electric lines (dashed) = 6 and number of magnetic lines (solid) = 5.

$$[Z][J] = [E] \tag{5a}$$

in which each of the matrix $[Z]$, the vector $[J]$, and the vector $[E]$ is of order $(N1 + N)$. The elements of the vector $[J]$ are the tangential (x and z-direction) current densities at each electric and magnetic line on the interface that has the metallic strip. The elements of the vector $[E]$ are the tangential electric field on each electric and magnetic line on that interface. Upon applying the boundary conditions on the dielectric interfaces and the metallic strip, the system of equations presented in (5a) can be reduced and given by:

$$[Z]_{red.}[J]_{red.} = [0] \tag{5b}$$

in which $[J]_{red.}$ contains the tangential current densities on each electric and magnetic line only on the metallic strip. The order of the matrix $[Z]_{red.}$ is $(M_e + M_m)$, where M_e and M_m are the number of the electric and magnetic lines, respectively, on the metallic strip. The elements of the matrix $[Z]_{red.}$ are functions of the frequency, the propagation constant, and the characteristics of every dielectric layer in the geometry (dielectric constant and dimensions). The eigenvalues of equation (5b) are the propagation constants (k_z) of the modes and their eigenvectors are the associated tangential current densities (J_x and J_z) on the metallic strip.

3. NUMERICAL RESULTS

3.1 Convergence of MoL

In the MoL technique the geometry of the transmission line structure is discretized into a number of lines in the x-direction where the dielectric substrate might be inhomogeneous in this direction. Moreover, the discretization line that is separating two different dielectric regions should be a magnetic line that has dielectric constant equal to the arithmetic average of the two different values [4]. As the difference of the neighboring dielectric constants becomes large, more discretization lines should be used especially if that inhomogeneous region has strong electric and magnetic fields such as the notch between the two strips. Three cases are tested for the convergence of the MoL in this work and the results are shown in Figs. 3a and 3b for the even and the odd modes, respectively. The dimensions of Fig. 1 are $W1 = W2 = H = 1mm$, $A = B = .99mm$, $T = R = 6mm$, $L = 0.48mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, and $\varepsilon_{r4} = 1$. In Fig. 3a, the three cases for the even modes are case 1 $(N = 50,\ h = 0.18mm,\ M_e = 5,\ M_m = 6)$, case 2 $(N = 87,\ h = 0.104mm,\ M_e = 9,\ M_m = 10)$, and case 3 $(N = 133,\ h = 0.068mm,\ M_e = 14,\ M_m = 15)$, where N is the total number of magnetic lines, h is the discretization distance, M_e is the number of electric lines on the strip, and M_m is the number of magnetic lines on the strip.

In Fig. 3b, the three cases for the odd modes are case 1 $(N = 54,\ h = 0.165mm,\ M_e = 6,\ M_m = 7)$, case 2 $(N = 91,\ h = 0.099mm,\ M_e = 10,\ M_m = 11)$, and case 3 $(N = 136,\ h = 0.066mm,\ M_e = 15,\ M_m = 16)$. The results show that the MoL technique converges as the number of discretization lines increases as shown in Figs. 3a and b, where the dielectric constant of the notch ε_{r3} changes from 1 to 30.

The MoL technique involves a number of matrix operations such as inversion and multiplication. The order of these matrices is the total number of magnetic lines plus the total number of electric lines. Thus these operations become time consuming as the total number of discretization lines increases (cases 2 and 3). In Figs. 4–5, the transmission line structure does not contain a notch or an overlay and it

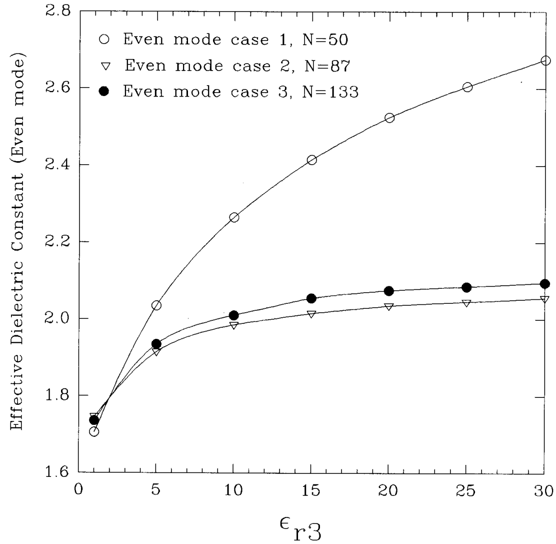


Figure 3a. Effective dielectric constant for the even mode versus ε_{r3} with $W1 = W2 = H = 1mm$, $T = R = 6mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, $\varepsilon_{r4} = 1$, $A = B = 0.99mm$, $L = 0.48mm$, and $f = 1GHz$.

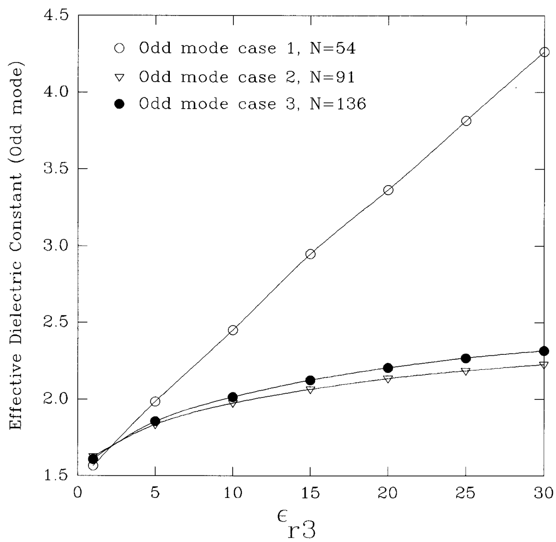


Figure 3b. Effective dielectric constant for the odd mode versus ε_{r3} with $W1 = W2 = H = 1mm$, $T = R = 6mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, $\varepsilon_{r4} = 1$, $A = B = 0.99mm$, $L = 0.48mm$, and $f = 1GHz$.

is inhomogeneous in a region that is far from the strips (metallic enclosure). In this case, the total number of magnetic lines is chosen to be $N = 50$ with discretization distance $h = 0.2mm$ (similar to but not exactly as case 1). In Figs. 6–9, the effect of electric and geometric properties of both the overly and the notch are investigated. The transmission line structure becomes more inhomogeneous by increasing the dielectric constant of the overlay or the notch up to 30. Thus in Figs. 6–9, the number of the magnetic lines, the discretization distance, and the number of magnetic and electric lines on the strip are chosen to be the same as in case 3 for both the even and odd modes.

3.2 Comparison with Reported Results

The developed MoL computer code is verified by comparing the numerical results with the previously reported data for a simpler transmission line geometry [6]. The configuration chosen from [6] to compare the results with, consists of two symmetric microstrip lines located on two dielectric substrates. The width of the metallic strips are $W1 = W2 = 1.5mm$, the separation between the strips is $S = 3.0mm$, the height of the lower substrate is $H1$ and its dielectric constant is $\varepsilon_{r1} = 2.2$, the height of the top substrate is $H2$ and its dielectric constant is $\varepsilon_{r2} = 9.7$, and the total height of the two substrates is $H = H1 + H2 = 1.5mm$. For the even mode, the number of magnetic lines (or the number of electric lines) is $N = 50$, the discretization distance is $h = 0.2mm$, the number of electric lines on the strip is $M_e = 7$, and the number of magnetic lines on the strip is $M_m = 8$. For the odd mode, the total number of magnetic lines is $N = 50$, the total number of electric lines is $N1 = 51$ with the same discretization distance and number of electric and magnetic lines on the strips as in the even mode. A comparison between our results and the results obtained in [6] is shown in Fig. 4. The effective dielectric constants of the even and the odd modes are plotted as functions of the relative height of the lower substrate $H1/H$ at $f = 10GHz$. Good agreement between our results and those reported in [6] is observed in Fig. 4. The calculations are repeated at $f = 1GHz$ for the configuration shown in Fig. 1 with $W1 = W2 = 1.5mm$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r2} = \varepsilon_{r3} = 9.7$, $\varepsilon_{r4} = 1$, $T = R = 7.5mm$, $H = H1 = 1.5mm$. The results are plotted in Fig. 5a, and are compared with the results obtained in [1] using the quasi-static moment method technique. The results in Fig. 5a show good agreement between the MoL at $f = 1GHz$ and the results published

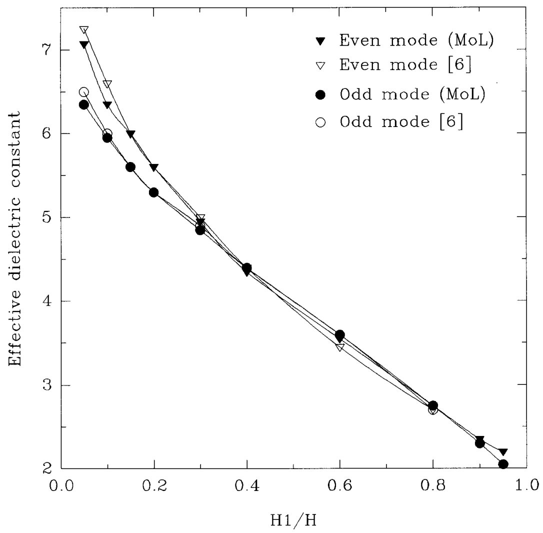


Figure 4. Effective Dielectric constant versus $H1/H$ with $W1 = W2 = 1.5mm$, $S = 3mm$, $H = 1.5mm$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r2} = \varepsilon_{r3} = 9.7$, $\varepsilon_{r4} = 1$, $T = R = 7.5mm$, and $f = 10GHz$.

in [1]. From Fig. 5a, at $H1/H = 0.3$ or $H1/H = 0.85$, the effective dielectric constants of the even and odd modes become equal. In Fig. 5b, the relative height of the lower substrate is fixed at $H1/H = 0.3$, and the effective dielectric constants of the even and odd modes are plotted as functions of frequency from $1GHz$ to $100GHz$. The results in Fig. 5b show that the effective dielectric constants of the even and odd modes are almost equal at all frequencies from $f = 1GHz$ up to $f = 100GHz$.

3.3 Effect of the Overlay

In Fig. 6a, the normalized phase velocities of the even and odd modes are plotted as functions of the relative dielectric constant of the overlay (ε_{r4}) at $f = 1GHz$ for the configuration shown in Fig. 1. The dimensions used to produce the results in Fig. 6a are $W1 = W2 = 1mm$, $H = 1mm$, $S = 2mm$, $P = 1.5mm$, $H1 = 0.7mm$, $T = R = 5mm$, $E = F = 3mm$, $A = B = 0.99mm$, $L = 0.25mm$, $\varepsilon_{r1} = 9.7$, $\varepsilon_{r2} = 2.2$, and $\varepsilon_{r3} = 1.0$. For the even and odd modes, the total number of magnetic lines is $N = 133$ with discretization

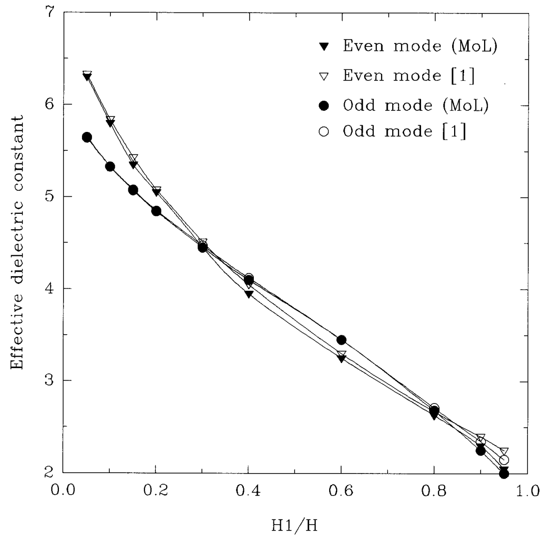


Figure 5a. Effective dielectric constant versus $H1/H$ with $W1 = W2 = 1.5mm$, $S = 3mm$, $H = 1.5mm$, $\varepsilon_{r1} = 2.2$, $\varepsilon_{r2} = \varepsilon_{r3} = 9.7$, $\varepsilon_{r4} = 1$, $T = R = 7.5mm$, and $f = 1GHz$.

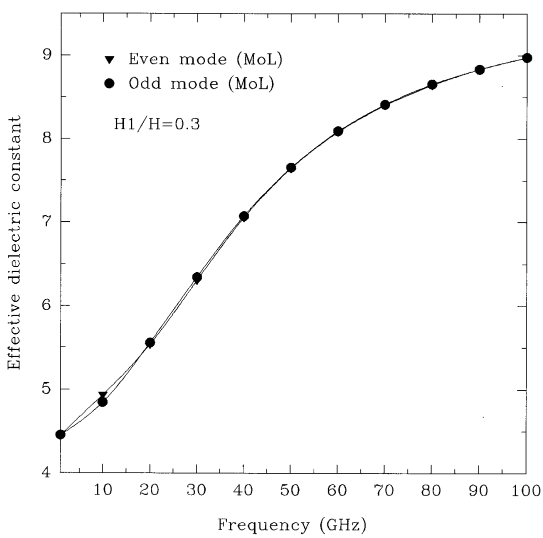


Figure 5b. Effective dielectric constant versus frequency with $H1/H = 0.3$ and the same dimensions for Fig. 5a.

distance $h = 0.068mm$, and $N = 136$ with $h = 0.066mm$, respectively. The results based on the MoL technique are compared with those obtained by the quasi static moment method [1]. The comparison shows good agreement especially when the dielectric constant of the overlay is not very large. As the dielectric constant of the overlay is equal to $\varepsilon_{r4} = 7.2$, the normalized phase velocities of the even and odd modes become equal. In Fig. 6b, the effective dielectric constants of the even and odd modes are plotted as functions of frequency with $\varepsilon_{r4} = 7.2$. The results show that the difference between the effective dielectric constants of the even and odd modes is not significant as the frequency increases from $1GHz$ up to $100GHz$.

3.4 Effect of the Notch

The effect of the geometrical and electrical parameters of the notch will be discussed in this section. The effective dielectric constants of the even and odd modes are plotted in Fig. 7a versus the dielectric constant of the notch ε_{r3}. The MoL based results are compared with those reported in [1]. Good agreement is observed in Fig. 7a especially for the odd mode. For the even and odd modes, the total number of magnetic lines is $N = 133$ with discretization distance $h = 0.068mm$, and $N = 136$ with $h = 0.066mm$, respectively. The dimensions of Fig. 1 are $W1 = W2 = H = 1mm$, $S = 2$, $A = B = .99mm$, $T = R = 6mm$, $L = 0.48mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$ and $\varepsilon_{r4} = 1$ [1]. The results shown in Fig. 7a indicates that as $\varepsilon_{r3} = 9.5$, the effective dielectric constants of the even and odd modes become equal. Upon fixing the dielectric constant of the notch equal to 9.5, the normalized phase velocities of the even and odd mode are computed and plotted as functions of frequency up to $100GHz$ as shown in Fig. 7b. Even though the normalized phase velocities of the even and odd modes started with equal values at $f = 1GHz$, they become totally different at frequencies higher than $30GHz$ as shown in Fig. 7b.

The relative dielectric constant of the notch is then chosen to be $\varepsilon_{r3} = 15$ while the height of the notch is varied from $L/H = 0.04$ to $L/H = 0.87$. The effective dielectric constants of the even and odd modes are plotted versus the notch height (L/H) as shown in Fig. 8a. The results show that as $L/H = 0.117$ or $L/H = 0.7$, the effective dielectric constants of the even and odd modes become equal. No such equality can be obtained if the notch is filled with air. Upon choosing $L/H = 0.7$, the normalized phase velocities of the even and

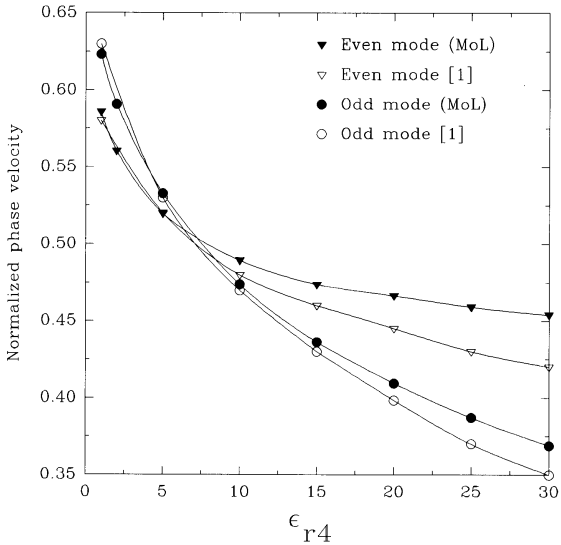


Figure 6a. Normalized phase velocity versus ε_{r4} with $W1 = W2 = 1mm$, $S = 2mm$, $H = 1mm$, $P = 1.5mm$, $H1 = 0.7mm$, $T = R = 5mm$, $E = F = 3mm$, $A = B = 0.99mm$, $L = 0.25mm$, $\varepsilon_{r1} = 9.7$, $\varepsilon_{r2} = 2.2$, $\varepsilon_{r3} = 1$, and $f = 1GHz$.

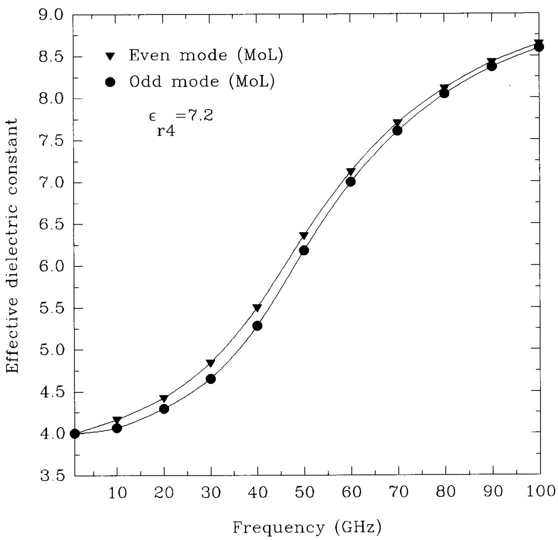


Figure 6b. Effective dielectric constant versus frequency with $\varepsilon_{r4} = 7.2$ and the same dimensions for Fig. 6a.

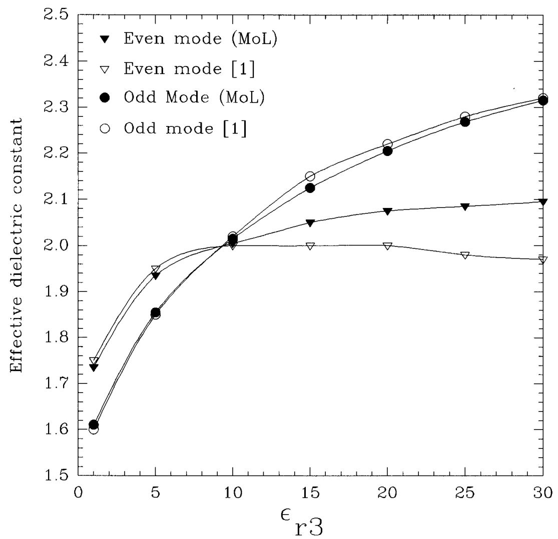


Figure 7a. Effective dielectric constant versus ε_{r3} with $W1 = W2 = H = 1mm$, $S = 2mm$, $A = B = 0.99mm$, $T = R = 6mm$, $L = 0.48mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, $\varepsilon_{r4} = 1$, and $f = 1GHz$.

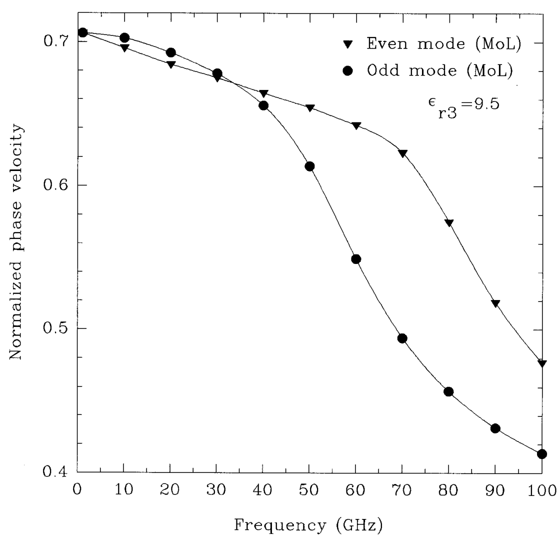


Figure 7b. Normalized phase velocity versus frequency with $\varepsilon_{r3} = 9.5$ and the same dimensions for Fig. 7a.

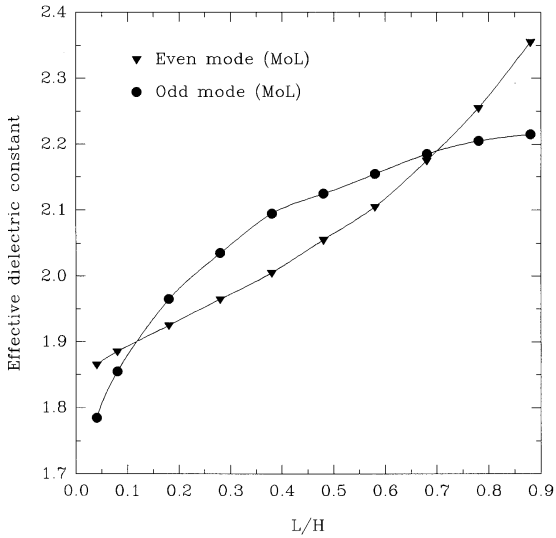


Figure 8a. Effective dielectric constant versus notch height L/H with $\varepsilon_{r3} = 15$, $W1 = W2 = H = 1mm$, $S = 2mm$, $A = B = 0.99mm$, $T = R = 6mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, $\varepsilon_{r4} = 1$, and $f = 1GHz$.

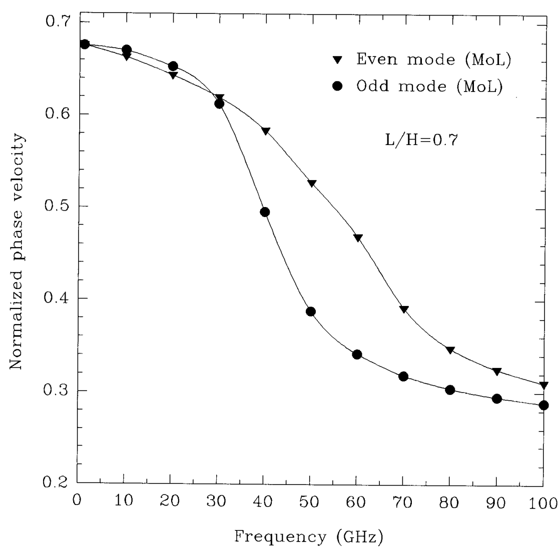


Figure 8b. Normalized phase velocity versus frequency with $L/H = .7$ and the same dimensions for Fig. 8a.

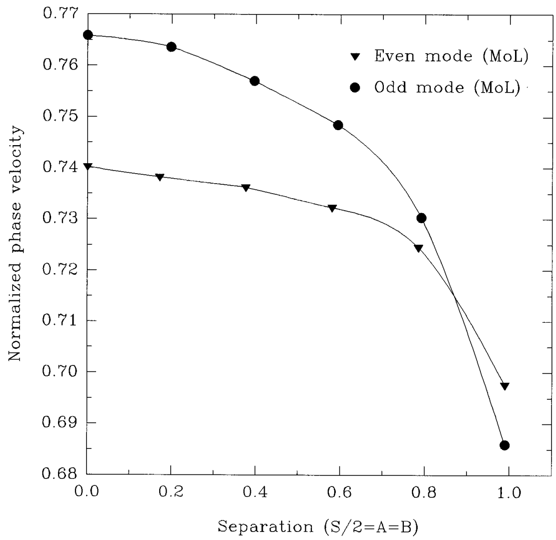


Figure 9a. Normalized phase velocity versus $S/2$ with $W1 = W2 = H = 1mm$, $T = R = 6mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, $\varepsilon_{r3} = 15$, $\varepsilon_{r4} = 1$, $L = 0.48mm$, and $f = 1GHz$.

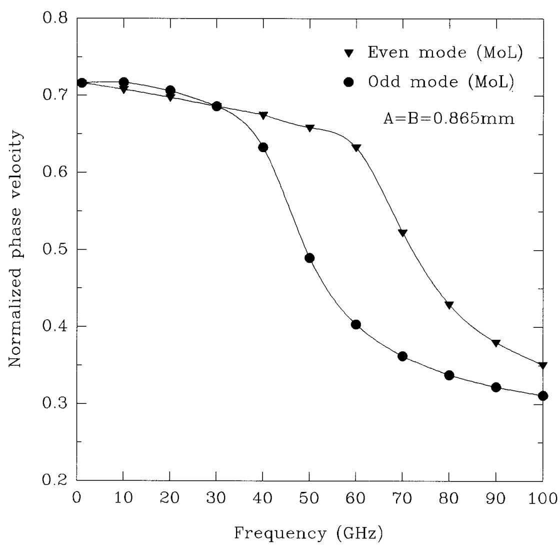


Figure 9b. Normalized phase velocity versus frequency with $A = B = .865mm$ and the same dimensions for Fig. 9a.

odd modes are plotted versus the frequency and shown in Fig. 8b. The results show that the normalized phase velocities of the even and odd modes are deviating from each other at higher frequencies although they started with equal values at low frequency.

The separation distance S $(S = A+B)$ between the two symmetric strips is changed to study its effect on the propagation constants of the even and odd modes. In Fig. 9a, the normalized phase velocities of the even and odd modes are plotted versus $A = B = S/2$ at $f = 1GHz$. The dimensions of Fig. 1 are $W1 = W2 = H = 1mm$, $T = R = 6mm$, $L = 0.48mm$, $\varepsilon_{r1} = \varepsilon_{r2} = 2.2$, and $\varepsilon_{r4} = 1$. As $A = B = 0.865mm$ $(S = 1.73mm)$, the normalized phase velocities of both the even and odd modes become equal. The separation S is kept equal to $1.73mm$ and the normalized phase velocities of the even and odd modes are computed and plotted versus the frequency up to $100GHz$. The results are shown in Fig. 9b where the normalized phase velocities of the even and odd modes are equal up to $f = 30GHz$ but they start to rapidly deviate from each other at higher frequencies.

4. CONCLUSIONS

The distortion of signals propagating over a coupled planar microstrip lines is investigated in this paper with special attention to the behavior at high frequencies. For microstrip lines without a notch, or with a notch having low dielectric constant, the numerical results show that the parameters for distortionless lines obtained from the low frequency solution can be used at high frequencies up to $100GHz$. However, for transmission line structures with more inhomogeneous notch between the lines, the parameters obtained at low frequency are found to be appropriate only up to $30GHz$ for the geometry considered in this paper.

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REFERENCES

1. Elsherbeni, A. Z., Smith, C. E., Golestanian, H., and He, S., "Quasi-static characteristics of a two-conductor multilayer microstrip transmission line with dielectric overlay and a notch between the strips," *Journal of Electromagnetic Waves and Applications,* Vol. 7, 769–789, 1993.
2. Smith, C. E., and Chang, R. S., "Microstrip transmission lines with finite-width dielectric," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-28, No. 2, 90–94, 1980.
3. He, S., Elsherbeni, A. Z., and Smith, C. E., "Decoupling between two conductor microstrip transmission line," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-41, No. 1, 53–61, 1993.
4. Pregla, R., Pascher, W., *Numerical Techniques for Microwave and Millimeter Wave Passive Structures,* edited by: T. Itoh, Ch.6, New York, Wiley, 1989.
5. El-Shenawee, M., "Effect of inhomogeneous dielectric layers on dispersion of microstrip lines using MoL," *IEEE AP-S Proceedings,* Baltimore, 2206–2209, 1996.
6. Gilb, J. P., and Balanis, C. A., "Pulse distortion on multilayer coupled microstrip lines," *IEEE Trans. Microwave Theory Tech.,* Vol. MTT-37, No. 10, 1620–1627, 1989.