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DECOMPOSITION THEOREMS APPLIED TO RANDOM AND STATIONARY RADAR TARGETS

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1. INTRODUCTION

When a radar target is time-dependent, a deterministic scattering matrix does not suffice to describe the target. The elements of the scattering matrix become stochastic variables and one scattering matrix is related to one statistical representation of the target. Thus a time average must be performed. Other matrices are used to describe time-dependent targets: the Kennaugh matrix [1], the target coherency matrix [2] and the covariance matrix [3]. Their elements are linear functions of the time-averaged cross-products of the time-dependent scattering matrix elements $\langle S_{hh}S_{hv}{}^*\rangle$, $\langle S_{hh}S_{vv}{}^*\rangle$, $\langle S_{hv}S_{vv}{}^*\rangle$ and their time-averaged modulus square $\langle S_{hh}S_{hh}{}^*\rangle$, $\langle S_{hv}S_{hv}{}^*\rangle$ and $\langle S_{vv}S_{vv}{}^*\rangle$ where $\langle\ \rangle$ indicates a time average and * the complex conjugate. Five real parameters describe polarimetrically a stationary target when the scattering matrix with relative phase only is symmetric. The time-dependent target is now described with nine real elements. In order to analyze further the time-dependent target, decomposition techniques are applied.

The purpose of a decomposition in radar polarimetry is to provide means for interpretation and optimum utilization of polarimetric scattering data. The objective for any decomposition is to combine or manipulate the scattering matrix elements in order to obtain more descriptive and discriminative target parameters, which is of decisive importance in applications of radar polarimetry.

Decomposition techniques have traditionally been related to incoherent processing in order to describe time-dependent targets. The first decomposition was proposed by Huynen in [1]. The Kennaugh matrix is decomposed into a sum of an average stationary target and a residual N-target (non-symmetric "noise"-target). Incoherent decompositions other than Huynen's have been presented by notably Cloude ([2, 4]), and Holm and Barnes [5], based on the target coherency matrix. Unlike the Huynen decomposition, decompositions of the target coherency matrix are usually based on eigenvalue analysis. Other decomposition approaches exist [6, 7]. They have been developed for SAR images classification and they focus on the decomposition in different types of scattering (odd bounce, even bounce, ...).

With the Delft Atmospheric Research Radar [8], an experimental study to search for a polarimetric analysis of time-dependent targets with a focus on target decomposition theorems is performed at the Delft University of Technology. For an atmospheric polarimetric radar, the dominant type of scattering is less important than for a SAR image where a lot of different kinds of targets are often present. Therefore the classical incoherent target decomposition theorems ([1, 9]), [2] and [5], are first selected. The utility of these decomposition methods for the purpose of extracting useful "real world information" from the polarimetric data must be investigated.

For this experimental study, the BSA (BackScatter Alignment) convention is used [10] and the absolute phase of the scattering matrix is not considered. Assuming reciprocity of the antenna system of the Delft Atmospheric Research Radar [11], the data used for this analysis are time-dependent symmetric scattering matrices. The three incoherent decompositions differ and they lead to different results at first sight. In this paper, the decompositions are applied on simple targets as illustrations: a stationary and a completely random target. Their results are discussed and compared. This step is important for the interpretation of the decomposition results of partially random targets.

2. OVERVIEW OF THE TARGET DECOMPOSITION THEOREMS

The Cloude decomposition [2] as well as the Holm and Barnes decomposition [5] can be easily applied to the target coherency matrix and to the covariance matrix. The Kennaugh matrix [1] and the target coherency matrix are straightforward decomposed using the Huynen decomposition. For convenient comparison, these decomposition theorems are therefore applied to the target coherency matrix $\langle [T_c] \rangle$. The elements of this target matrix used by Cloude are directly related to the Huynen parameters A_0, B_0, B_ψ ... as given in equation (1). The Huynen parameters are real. It reduces to a 3×3 matrix when the time-dependent scattering matrix is symmetric. This hermitian matrix consists of three real elements on the diagonal and three complex elements off-diagonal. The equation (2) relates the elements of the target coherency matrix to the elements of the time-dependent scattering matrix. The matrix $\langle [T_c] \rangle$ is not directly measured but calculated from the time-averages of the measured scattering matrices.

$$\langle [T_c] \rangle = \begin{bmatrix} 2A_0 & C_\psi + jD_\psi & H_\psi - jG_\psi \\ C_\psi - jD_\psi & B_0 + B_\psi & E_\psi - jF \\ H_\psi + jG_\psi & E_\psi + jF & B_0 - B_\psi \end{bmatrix} \tag{1}$$

$$
\begin{aligned}
2A_0 =& \frac{1}{4} \left\langle |S_{hh}(t) + S_{vv}(t)|^2 \right\rangle \\
B_0 + B_\psi =& \frac{1}{4} \left\langle |S_{hh}(t) - S_{vv}(t)|^2 \right\rangle \\
B_0 - B_\psi =& \left\langle |S_{hv}(t)|^2 \right\rangle \\
C_\psi + jD_\psi =& \frac{1}{4} \left\langle |S_{hh}(t)|^2 - |S_{vv}(t)|^2 + 2jIm(S_{hh}^*(t) S_{vv}(t)) \right\rangle \\
H_\psi - jG_\psi =& \frac{1}{2} \left\langle S_{hh}(t) S_{hv}^*(t) + S_{vv}(t) S_{hv}^*(t) \right\rangle \\
E_\psi - jF =& \frac{1}{2} \left\langle S_{hh}(t) S_{hv}^*(t) - S_{vv}(t) S_{hv}^*(t) \right\rangle
\end{aligned} \tag{2}
$$

2.1 Cloude Decomposition

Cloude introduces the target vector (3) equivalent to the scattering matrix. The general term of the target coherency matrix is straightforward expressed in (4) as a function of the components of the target vector $\overline{k}_c(t)$. This hermitian matrix is diagonalized leading to three eigenvalues λ_i and three eigenvectors $\overline{k}_{ci}$ (5) which are orthogonal when the eigenvalues are different. To each eigenvector corresponds a scattering matrix $[S_i]$ of a stationary target (6), weighted by the related eigenvalue [2].

$$
\overline{k}_c(t) = \begin{bmatrix} \frac{1}{2}(S_{hh}(t) + S_{vv}(t)) \\ \frac{1}{2}(S_{hh}(t) - S_{vv}(t)) \\ S_{hv}(t) \end{bmatrix} \tag{3}
$$

$$
\langle [T_c] \rangle_{l,\ m} = \left\langle k_{c,\ l}(t) k_{c,\ m}^*(t) \right\rangle \qquad l,\ m \in \{0,\ 1,\ 2\} \tag{4}
$$

$$
\langle [T_c] \rangle \, \overline{k_{ci}} = \lambda_i \overline{k_{ci}} \quad \text{with} \ |\overline{k_{ci}}| = 1 \tag{5}
$$

$$
[S_i] = \sqrt{\lambda_i} \begin{bmatrix} k_{ci,\ 0} + k_{ci,\ 1} & k_{ci,\ 2} \\ k_{ci,\ 2} & k_{ci,\ 0} - k_{ci,\ 1} \end{bmatrix} \qquad i = 0,\ 1,\ 2 \tag{6}
$$

To measure the statistical disorder of time-dependent targets, the target entropy H_c is defined from the eigenvalues λ_i (7). H_c varies from 0 for a stationary target to 1 for a random target.

$$
\begin{aligned}
H_c =& -\sum_{i=0}^{2} p_i \log_3 p_i \\
p_i =& \frac{\lambda_i}{\sum_{i=0}^{2} \lambda_i}
\end{aligned} \tag{7}
$$

2.1.1 Diagonalization

Since the target coherency matrix (8) is hermitian, this matrix can always be diagonalized.

$$\langle [T_c] \rangle = \begin{bmatrix} \langle k_{c,0} k^*_{c,0} \rangle & \langle k_{c,0} k^*_{c,1} \rangle & \langle k_{c,0} k^*_{c,2} \rangle \\ \langle k_{c,1} k^*_{c,0} \rangle & \langle k_{c,1} k^*_{c,1} \rangle & \langle k_{c,1} k^*_{c,2} \rangle \\ \langle k_{c,2} k^*_{c,0} \rangle & \langle k_{c,2} k^*_{c,1} \rangle & \langle k_{c,2} k^*_{c,2} \rangle \end{bmatrix} \tag{8}$$

λ_0, λ_1, and λ_2 are the eigenvalues of the target coherency matrix and $\left\langle \left[T_c^D\right] \right\rangle$ is its diagonal representation in the basis consisting of the normalized eigenvectors. Each matrix $\left[T_{ci}^D\right]$ is built with the normalized eigenvector $\overline{k}_{ci}$.

$$\left\langle \left[T_c^D\right] \right\rangle = \lambda_0 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_1 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{9}$$

$$\left\langle \left[T_c^D\right] \right\rangle = \sum_{i=0}^{2} \lambda_i \left[T_{ci}^D\right] \tag{10}$$

To come back to the initial basis $(\bar{h},\ \bar{v})$, an unitary transformation $[U_t]$ is carried out. The columns of the unitary matrix contain the eigenvectors written in the basis $(\bar{h}, \bar{v})$.

$$\langle [T_c] \rangle = \sum_{i=0}^{2} \lambda_i [T_{ci}] \tag{11}$$

$$[T_{ci}] = [U_t] \left[T_{ci}^D\right] [U_t]^\dagger \tag{12}$$

2.2 Holm and Barnes Decomposition

The eigenvalues of the target coherency matrix are calculated and ordered. When λ_0 is the largest eigenvalue and λ_2 the smallest one, then the diagonalized target coherency matrix $\left\langle \left[T_c^D\right] \right\rangle$ is decomposed as follows

$$\left\langle \left[T_c^D\right] \right\rangle = (\lambda_0 - \lambda_1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + (\lambda_1 - \lambda_2) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \lambda_2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{13}$$

The only stationary target is weighted by $\lambda_0 - \lambda_1$. The same stationary target is found in the Cloude decomposition but weighted by λ_0. In the case of a quasi-stationary target where λ_1 and λ_2 are very small compared to λ_0, the averaged target obtained from both decompositions will be the same.

$$[U_t] = \left[\overline{k}_{c0}\ \overline{k}_{c1}\ \overline{k}_{c2}\right] \tag{14}$$

The decomposition in [5] is formulated differently than (13) because of the ordering of the eigenvectors.

2.3 Huynen Decomposition

A time-dependent target is described by the target coherency matrix $\langle [T_c] \rangle$ consisting of nine independent real parameters, whereas a stationary target is determined by five real parameters. Therefore Huynen considered the possibility of decomposing the nine-parameter target coherency matrix, (15), into an average stationary effective target described by $\left[T_c^S\right]$ in (16) with five parameters and a residue part given by $\left\langle \left[T_c^N\right] \right\rangle$ in (17) which contains the four remaining degrees of freedom ([1, 9]). The core of this decomposition is the choice of the residue target.

Huynen chooses the N-target for the following reasons. It is determined by four parameters and it is a roll-invariant target. It means that if the N-target is rotated around the line of sight through an angle ψ or equivalently, another linear orthogonal polarization basis than $(\bar{h}, \bar{v})$ is used, the target coherency matrix remains of the form given in (17).

The target coherency matrix $\left[T_c^S\right]$ consists of five real independent parameters like the scattering matrix. There are thus four real dependent relations between its elements shown in (18). The first two equations are real and the third one is complex. Using the eigenvalues formalism in the preceding paragraphs, only one eigenvalue different from 0 corresponds to a stationary target which means that the determinant of $\left[T_c^S\right]$ as well as the determinant of its 2×2 minors equals zero. The relations (18) are derived from the determinant of some of the 2×2 minors. The scattering matrix related to the target coherency matrix $\left[T_c^S\right]$ is calculated using (2) without the average symbol $\langle\ \rangle$.

$$\langle [T_c] \rangle = \left[T_c^S\right] + \left\langle \left[T_c^N\right] \right\rangle \tag{15}$$

$$\left[T_c^S\right] = \begin{bmatrix} 2A_0 & C_\psi + jD_\psi & H_\psi - jG_\psi \\ C_\psi - jD_\psi & B_0^S + B_\psi^S & E_\psi^S - jF^S \\ H_\psi + jG_\psi & E_\psi^S + jF^S & B_0^S - B_\psi^S \end{bmatrix} \tag{16}$$

$$\left\langle \left[T_c^N\right] \right\rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0^N + B_\psi^N & E_\psi^N - jF^N \\ 0 & E_\psi^N + jF^N & B_0^N - B_\psi^N \end{bmatrix} \tag{17}$$

$$\begin{aligned} B_0^S + B_\psi^S &= \frac{C_\psi^2 + D_\psi^2}{2A_0} \\ B_0^S - B_\psi^S &= \frac{G_\psi^2 + H_\psi^2}{2A_0} \\ E_\psi^S - jF^S &= \frac{(C_\psi H_\psi - D_\psi G_\psi) - j(C_\psi G_\psi + D_\psi H_\psi)}{2A_0} \end{aligned} \tag{18}$$

2.3.1 Decomposition of the N-target

The N-target described by the target coherency matrix $\left\langle \left[T_c^N\right] \right\rangle$ can be decomposed (19) to obtain a stationary N-target (20) and a so-called "unpolarizing" N-target (21), [12]. The dependency equation (22) is easily retrieved, calculating the determinant of the appropriate 2×2 minor of $\left[T_c^N{}_S\right]$ and using the property $B_0'^N \geq 0$. The Kennaugh matrix $\left\langle \left[K_u^N\right] \right\rangle$ related to $\left\langle \left[T_c^N{}_u\right] \right\rangle$ when applied to the Stokes vector of a linear polarized wave, fully depolarizes this pure polarization, from which the name of "unpolarizing" N-target for this matrix (23). This second step in the Huynen decomposition mentioned in [12] leads to an additional scattering matrix allowing an easier comparison with the Cloude decomposition.

$$\left\langle \left[T_c^N\right] \right\rangle = \left[T_c^N{}_S\right] + \left\langle \left[T_c^N{}_u\right] \right\rangle \tag{19}$$

$$\left[T_c^N{}_S\right] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0'^N + B_\psi^N & E_\psi^N - jF^N \\ 0 & E_\psi^N + jF^N & B_0'^N - B_\psi^N \end{bmatrix} \tag{20}$$

$$\left\langle \left[T_c^N{}_u\right] \right\rangle = \begin{bmatrix} 0 & 0 & 0 \\ 0 & B_0^N - B_0'^N & 0 \\ 0 & 0 & B_0^N - B_0'^N \end{bmatrix} \tag{21}$$

$$B_0'^N = \sqrt{\left(B_\psi^N\right)^2 + \left(E_\psi^N\right)^2 + \left(F^N\right)^2} \tag{22}$$

$$\underbrace{\begin{bmatrix} B_0^N - B_0'^N & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_0^N - B_0'^N \end{bmatrix}}_{\text{Kennaugh matrix}\,\langle [K_u^N] \rangle} \underbrace{\begin{bmatrix} A^2 \\ A^2\cos 2\theta \\ A^2 \sin 2\theta \\ 0 \end{bmatrix}}_{\substack{\text{Stokes vector} \\ \text{linear polarization}}}$$
$$= B_0^N - B_0'^N \underbrace{\begin{bmatrix} A^2 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\substack{\text{Stokes vector} \\ \text{unpolarized wave}}} \quad (23)$$

2.3.2 Definition of the N-target

The classical Huynen decomposition is based on the choice of the N-target. The N-target is mathematically defined as follows

$$\langle [T_c^N] \rangle \, \bar{q} = 0 \quad (24)$$

with

$$\bar{q} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (25)$$

The vector $\bar{q}$ belongs to the null space of the N-target and the relation (24) is roll invariant, which means

$$[U_N]^\dagger \langle [T_c^N] \rangle [U_N] \, \bar{q} = 0 \quad (26)$$

where the unitary matrix $[U_N]$ is a rotation matrix. The matrix $[U_N]$ is transposed and conjugated (hermitian adjoint) on the left side of the equation since $\langle [T_c^N] \rangle$ is expressed in the reference basis $(\bar{h}, \bar{v})$.

$$[U_N] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 2\psi & \sin 2\psi \\ 0 & -\sin 2\psi & \cos 2\psi \end{bmatrix} \quad (27)$$

The elements of the vector $\bar{q}$ are a combination of the components of the transmitted polarization vector $\bar{e}_t$ and the receiving polarization vector $\overline{e}_r$ (28). The vector $\bar{q}$ is obtained with a left-handed circular polarization (L) for transmission and a right-handed circular polarization (R) for reception or conversely. Any target with a LR or RL null is thus a Huynen N-target. This is the physical definition of the Huynen N-target.

$$\bar{q} = \begin{bmatrix} e_{rh}e_{th} + e_{rv}e_{tv} \\ e_{rh}e_{th} - e_{rv}e_{tv} \\ e_{rh}e_{tv} + e_{rv}e_{th} \end{bmatrix}^* \tag{28}$$

This kind of decomposition is not unique. Two other Huynen-like decompositions can be defined keeping the roll-invariant property of the residue target [5]. The two other possibilities lead to respectively a residue target LL null and RR null. Summarizing, choosing for the roll-invariant property for the residue target is choosing for targets with circular polarization nulls.

2.3.3 ψ-Dependency of the Target Coherency Matrix

When the linear polarization orthonormal basis $(\bar{h},\ \bar{v})$ is rotated through an angle ψ around the line of sight, which means a change of linear basis, then the scattering matrix of a stationary target is unitary transformed by a rotation matrix of angle ψ. The same transformation occurs when the target is rotated through an angle $-\psi$ around the line of sight.

When the scattering matrix is unitary transformed by a rotation of angle ψ, then the target coherency matrix is unitary transformed by a rotation matrix of angle 2ψ as can be seen in (27).

3. EXPERIMENTAL SET-UP

The time-dependent scattering matrices are measured by the "roof-based" FM-CW polarimetric research radar of the Delft University of Technology, operating in the S-band [8]. The target coherency matrix is calculated from the time-dependent scattering matrices. The bandwidth of the radar can be adjusted from 1 MHz up to 50 MHz corresponding to range resolutions of 150 to 3 meters. The polarimetric calibration of the radar was performed with a rotatable dihedral corner reflector [11] where the reciprocity of the antenna system has been assumed and justified. The measurements are copolar. The polarization is linear and varies sinusoidally between $\pm 90^o$ as a function of time with a periodicity of 40 ms. A N-point discrete Fourier transform of the target signal is performed per sweep or measurement. There are typically 32 sweeps during a polarizer period of 40 ms. The measured signal of the target is given by a linear combination of the three elements of the scattering matrix and at least three measurements are needed to retrieve the scattering matrix [8].

To reduce noise influence on the determination of the scattering coefficients, the calculation, described in [8] and [11], is made over a complete polarizer period (40 ms). This calculation is valid as soon as the decorrelation time of the target is much larger than 40 ms. Each scattering matrix is the least squares solution of a system of 32 linear equations in 3 unknowns. In this way the scattering matrix of a stationary target is calculated each 40 ms.

To calculate the scattering matrix of a random target, three consecutive sweeps are used. Then the scattering matrix is measured within 3.75 ms. If this measurement time is still too long, the minimum sweep time available of 0.625 ms is selected.

3.1 Example of a Random Target

The noise signal U of the radar is used to build artificially time-dependent scattering matrices. The polarizers are not active, there is no transmission of power and the frequency excursion is set to 0 MHz. The elements of the scattering matrix consist of the noise signal of the frequency cell 25 contained in three consecutive sweeps. The choice of the frequency cell as well as the sweeps is arbitrary. Calibration data were not used. The phase of each element of the scattering matrices typically follows an uniform distribution between $-\pi$ and $+\pi$. These elements are equal in power after sufficient averaging (29) and not correlated. In this example, the average is performed on 12.8 s (320 scattering matrices). The averaged relative power of the elements of the scattering matrices tends to -7 dB and there is a weak correlation of 10^{-2} left (30).

$$\begin{aligned}\left\langle |S_{hh}(t)|^2 \right\rangle \approx \left\langle |S_{hv}(t)|^2 \right\rangle \approx \left\langle |S_{vv}(t)|^2 \right\rangle \\ \approx \left\langle |U(t)|^2 \right\rangle \approx -7 \text{ dB}\end{aligned} \tag{29}$$

$$\begin{aligned}\langle S_{hh}(t) S_{hv}^*(t) \rangle \approx \langle S_{hh}(t) S_{vv}^*(t) \rangle \approx \langle S_{hv}(t) S_{vv}^*(t) \rangle \\ \approx 10^{-2}(1+j) \left\langle |U(t)|^2 \right\rangle\end{aligned} \tag{30}$$

3.1.1 Decomposition Results

When the phase of the elements of the scattering matrices is random for a time-dependent target, the target coherency matrix is expressed by (31). The element $B_0 + B_\psi$ tends to $2A_0$ and the elements D_ψ, E_ψ, F, H_ψ, and G_ψ tend to zero.

$$\langle [T_c] \rangle \approx \begin{bmatrix} 2A_0 & C_\psi & 0 \\ C_\psi & 2A_0 & 0 \\ 0 & 0 & B_0 - B_\psi \end{bmatrix} \tag{31}$$

In the chosen example, the copolar elements of the scattering matrices have about the same averaged power (29) which implies C_ψ tending to zero (see definition of C_ψ in (2)). Therefore the target coherency matrix tends to a diagonal matrix with two elements on the diagonal equal and the third one two times larger.

$$\langle [T_c] \rangle \approx \begin{bmatrix} 2A_0 & 0 & 0 \\ 0 & 2A_0 & 0 \\ 0 & 0 & 4A_0 \end{bmatrix} \tag{32}$$

The calculated target coherency matrix is given in Table 1.

Target coherency matrix		
(0.1029, 0.)	(0.0007, 0.0017)	(−0.0031, −0.0035)
(0.0007, −0.0017)	(0.1051, 0.)	(0.0117, −0.0083)
(−0.0031, 0.0035)	(0.0117, 0.0083)	(0.2254, 0.)

Table 1. The Target Coherency Matrix of the Random Target (Noise).

The results of the Cloude decomposition are the scattering matrices $[S_i]$ (6). The results of the Huynen decomposition are the two scattering matrices respectively related to $\left[T_c^S\right]$ (16) and $\left[T_c^N{}_S\right]$ (20) plus the target coherency matrix $\left\langle \left[T_c^N{}_u\right] \right\rangle$ (21). In order to compare the energy contribution of each scattering matrix, resulting from the decomposition, the following quantity is defined

$$Span\left([S]\right) = |S_{hh}|^2 + 2|S_{hv}|^2 + |S_{vv}|^2 \tag{33}$$

When there is an unique relation between the target coherency matrix and the scattering matrix, i.e. in the case of a stationary target, it is found that

$$Trace\left([T_c]\right) = \frac{1}{2}Span\left([S]\right) \tag{34}$$

Therefore the energy contribution of the target coherency matrix is quantified using the trace of the matrix.

3.1.1.1 Huynen Decomposition

The decomposition of the approximated theoretical target coherency matrix is given below.

$$\langle [T_c] \rangle \approx \underbrace{\begin{bmatrix} 2A_0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{[T_c^S]} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2A_0 \end{bmatrix}}_{[T_c^N{}_S]} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2A_0 & 0 \\ 0 & 0 & 2A_0 \end{bmatrix}}_{\langle [T_c^N{}_u] \rangle} \tag{35}$$

The corresponding theoretical results are given in Table 2 and the experimental ones in Table 3.

Stationary Target and Stationary N-target

Huynen	Stationary target	Stationary N-target
$Span$	$4A_0$	$4A_0$
S_{hh}	$\sqrt{2A_0}$	0
S_{hv}	0	$\sqrt{2A_0}$
S_{vv}	$\sqrt{2A_0}$	0

Unpolarizing N-target

Huynen	Unpolarizing N−target target coherency matrix
$2 \times$ trace	$8A_0$
$B_0^N - B'^N_0$	$2A_0$

Table 2. Theoretical Estimation of the Huynen Decomposition of the Random Target (Noise).

Stationary Target and Stationary N-target

Huynen	Stationary target	Stationary N-target
Span(dB)	−6.8	−6.0
$S_{hh}(dB,\ deg)$	−9.8, 0	-27.7, 0
$S_{hv}(dB,\ deg)$	−36.7, 133	−9.1, 35
$S_{vv}(dB,\ deg)$	−9.9, 2	−27.7, −180

Unpolarizing N-target

Huynen	Unpolarizing N−target target coherency matrix
$2 \times$ trace (dB)	−3.8
$B_0^N - B'^N_0\ (dB)$	−9.9

Table 3. Results of the Huynen Decomposition of the Random Target (Noise).

The span of the obtained scattering matrices, in theory $4A_0$, is expected to be about -7 dB. The copolar elements of the stationary target scattering matrix, the crosspolar element of the stationary N-target scattering matrix and the only element different from zero of the unpolarizing N-target coherency matrix are expected to be equal to -10 dB. They are equal respectively to -9.8, -9.9, -9.1 and -9.9 dB. There is no phase difference between the copolar elements of the stationary target scattering matrix theoretically and experimentally 2 degrees are found.

Summarizing, with the Huynen decomposition, the noise signal leads to the scattering matrices of a trihedral and a dihedral rotated through $45°$, and a target coherency matrix of an unpolarizing N-target.

3.1.1.2 Cloude Decomposition

The approximate theoretical target coherency matrix is already diagonal. Thus the eigenvalues are $\lambda_0 = 4A_0$ and $\lambda_1 = \lambda_2 = 2A_0$. The corresponding eigenvectors are expressed in (36).

$$\overline{k}_{c,0} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \overline{k}_{c,i} = \frac{1}{\sqrt{|a|^2 + |b|^2}} \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \quad i = 1,\ 2 \quad \forall\, a, b \text{ complex} \tag{36}$$

In this case where two eigenvalues are identical, the condition of orthogonality of the corresponding eigenvectors is not fulfilled. Therefore a whole plane (dimension 2) instead of one direction (dimension 1) contains the possible eigenvectors.

Using (6) and knowing that the eigenvectors $\overline{k}_{ci}$ are normalized to 1, it is straightforward to derive that the span of the scattering matrices resulting from the Cloude decomposition equals two times the corresponding eigenvalue.

The theoretical results of the decomposition are given in Table 4 and the experimental ones in Table 5.

The entropy equals 0.93. The eigenvalues with the corresponding stationary targets are sorted in descending order, i.e. the target 0 described by the scattering matrix $[S_0]$ (6) corresponds to the largest eigenvalue λ_0, etc ... The crosspolar element S_{hv} is estimated to be -7 dB for the stationary target 0 and is found to be -6.5 dB. Concerning the stationary targets 1 and 2, the energy tends to split up into the copolar elements S_{hh} and S_{vv}. The scattering matrix element S_{vv} is about half S_{hh} in power for the stationary target 1 and it is vice versa for the stationary target 2, ($a = c$ and $b = -d$).

Cloude	Stationary target 0	Stationary target 1	Stationary target 2
Eigenvalues	$4A_0$	$2A_0$	$2A_0$
Span	$8A_0$	$4A_0$	$4A_0$
S_{hh}	0	$\sqrt{2A_0}\frac{a+b}{\sqrt{\lvert a\rvert^2+\lvert b\rvert^2}}$	$\sqrt{2A_0}\frac{c+d}{\sqrt{\lvert c\rvert^2+\lvert d\rvert^2}}$
S_{hv}	$\sqrt{4A_0}$	0	0
S_{vv}	0	$\sqrt{2A_0}\frac{a-b}{\sqrt{\lvert a\rvert^2+\lvert b\rvert^2}}$	$\sqrt{2A_0}\frac{c-d}{\sqrt{\lvert c\rvert^2+\lvert d\rvert^2}}$

Table 4. Theoretical Estimation of the Cloude Decomposition of the Random Target (Noise).

Cloude	Stationary target 0	Stationary target 1	Stationary target 2
Eigenvalues	0.2273	0.1055	0.1006
$Span(dB)$	-3.4	-6.8	-7.0
$S_{hh}(dB,\ deg)$	$-25.0,\ 0$	$-8.6,\ 0$	$-11.7,\ 0$
$S_{hv}(dB,\ deg)$	$-6.5,\ 53$	$-33.4,\ -172$	$-29.8,\ -87$
$S_{vv}(dB,\ deg)$	$-24.5,\ -146$	$-11.5,\ 99$	$-8.8,\ -80$

Table 5. Results of the Cloude Decomposition of the Random Target (Noise).

3.1.1.3 Holm and Barnes Decomposition

The following decomposition of the target coherency matrix is expected

$$\langle [T_c] \rangle \approx \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2A_0 \end{bmatrix}}_{[T_c^S{}_H]} + \underbrace{\begin{bmatrix} 2A_0 & 0 & 0 \\ 0 & 2A_0 & 0 \\ 0 & 0 & 2A_0 \end{bmatrix}}_{\langle [T_c^D{}_H] \rangle} \tag{37}$$

It can be performed straightforward in the $(\bar{h},\ \bar{v})$ basis since the matrix $\langle [T_c] \rangle$ can be assumed diagonal. The stationary target resulting from the Holm and Barnes decomposition is the stationary Huynen N-target (dihedral rotated through $45°$). The theoretical expected results are given in Table 6 and the experimental ones in Table 7.

Stationary Target

Holm and Barnes stationary target			
$Span$	S_{hh}	S_{hv}	S_{vv}
$4A_0$	0	$\sqrt{2A_0}$	0

Diagonal Target Coherency Matrix

Holm and Barnes diagonal target coherency matrix	
$2 \times$ trace	$12A_0$
diagonal element	$2A_0$

Table 6. Theoretical estimation of the Holm and Barnes decomposition of the Random Target (Noise).

Stationary Target

Holm and Barnes stationary target			
$Span(dB)$	$S_{hh}(dB,\ deg)$	$S_{hv}(dB,\ deg)$	$S_{vv}(dB,\ deg)$
-6.1	$-27.7,\ 0$	$-9.2,\ 53$	$-27.2,\ -146$

Non-Stationary Target Coherency Matrices

Target coherency matrix $\langle [T^I_{c\ H}] \rangle$ $2 \times \text{trace} = -17.1\ dB$		
$(0.0021,\ 0.)$	$(0.0008,\ 0.0022)$	$(-0.0003,\ -0.0002)$
$(0.0008,\ -0.0022)$	$(0.0028,\ 0.)$	$(0.0003,\ -0.0002)$
$(-0.0003,\ 0.0002)$	$(0.0003,\ 0.0002)$	$(0.0048,\ 0.)$

Target coherency matrix $\langle [T^D_{c\ H}] \rangle$ $2 \times \text{trace} = -2.2\ dB$		
$(0.1006,\ 0.)$	$(0.,\ 0.)$	$(0.,\ 0.)$
$(0.,\ 0.)$	$(0.1006,\ 0.)$	$(0.,\ 0.)$
$(0.,\ 0.)$	$(0.,\ 0.)$	$(0.1006,\ 0.)$

Table 7. Results of the Holm and Barnes Decomposition of the Random Target (Noise).

The cross-polar element of the Holm and Barnes scattering matrix, expected to be -10 dB, equals -9.2 dB. The span, estimated to be -7 dB, is found to be -6.1 dB. The trace of the calculated diagonal target coherency matrix leads to the expected power of -2.2 dB. The other target coherency matrix, neglected in theory, is present with a power of -17.1 dB.

3.1.2 Discussion

There is a good agreement between theoretical estimates and measurements. The differences are caused by a weak correlation left between the elements of the time-dependent scattering matrices (30). The average of these elements and their averaged power are tending (but are not equal) respectively to zero and -7 dB. A larger averaging time may decrease these differences. Results of the decomposition were also investigated for an averaging time of 1 s (25 scattering matrices). The results of the average on 12.8 s (320 scattering matrices) are much nearer to the theoretical predictions. For such a random target, 25 averages are not sufficient.

The Huynen scattering matrices differ from the Cloude ones. Nevertheless both decompositions show a decomposition in copolar and cross-polar elements. There is no predominance of one scattering matrix. The three Cloude scattering matrices or the two Huynen scattering matrices with the target coherency matrix of the "unpolarizing" N-target are needed to describe the noise signal. The Cloude scattering matrix related to the largest eigenvalue is comparable to the stationary N-target scattering matrix even though they differ in span.

The scattering matrix obtained by the Holm and Barnes decomposition is identical, relatively, to the Cloude decomposition scattering matrix corresponding to the largest eigenvalue. They differ in span. The Holm and Barnes scattering matrix has a span equivalent to the stationary N-target obtained with the Huynen decomposition.

In [2] and [4] the Cloude decomposition is performed on a target coherency matrix 4×4 since the scattering matrix is considered non symmetric (general case). Therefore the energy is split up equally into the 4 scattering matrices obtained after decomposition of a random target. The span of each scattering matrix would be -7 dB in this example. Concerning the target coherency matrix $\langle [T_c] \rangle$, instead of having 4 eigenvalues identical, there are 3 with one being two times larger than the two equal other ones.

3.2 Example of a Stationary Target

3.2.1 Measurement

The chimney of a power plant was measured in presence of light rain during a period of 58 s. The range is 1186 m and the range resolution is 15 m. The main lobe of the antenna system illuminates a surface of 30×30 m^2 at this range. About 15×30 m^2 of the top of the chimney is illuminated by the main lobe. The signal to noise ratio is 50 dB. The processed data supply a scattering matrix each 40 ms. The copolar elements of the scattering matrices of the chimney are correlated. The scattering matrices are calibrated [11]. The averaged calibrated scattering matrix of the chimney is given in Table 8. The average is performed on 1450 scattering matrices which is equivalent to 58 s of measurement.

Calibrated scattering matrix	
$Span(dB)$	25.4
$S_{hh}(dB,\ deg)$	23.5, 0
$S_{hv}(dB,\ deg)$	−7.4, 14
$S_{vv}(dB,\ deg)$	20.9, 1

Table 8. The Calibrated Scattering Matrix of the Chimney (Stationary Target).

3.2.2 Decomposition Results

The target coherency matrix is presented in Table 9.

Target coherency matrix		
(169.83, 0.)	(24.51, 1.53)	(5.42, −1.32)
(24.51, −1.53)	(3.56, 0.)	(0.77, −0.24)
(5.42, 1.32)	(0.77, 0.24)	(0.19, 0.)

Table 9. The Target Coherency Matrix of the Chimney (Stationary Target).

The Huynen and Cloude decomposition results are given respectively in Table 10 and 11. The entropy H_c is 3.4×10^{-4}. As said the entropy quantifies the randomness of the target. It tends to 0 for a stationary target and to 1 for a random target. This parameter can be used as a classifier of randomness in SAR images [13].

The averaged scattering matrix of a stationary target like the chimney is identical to the Huynen stationary target scattering matrix and to the Cloude scattering matrix related to the largest eigenvalue after decomposition. So the largest span scattering matrix that results after the decomposition is the averaged scattering matrix of the target when the target is stationary.

Stationary Target and Stationary N-target

Huynen	Stationary target	Stationary N-target
$Span(dB)$	25.4	-31.7
$S_{hh}(dB,\ deg)$	23.5, 0	-37.1, 0
$S_{hv}(dB,\ deg)$	-7.4, 14	-38.4, 148
$S_{vv}(dB,\ deg)$	20.9, 1	-37.1, -180

Unpolarizing N-target

Huynen	Unpolarizing N-target target coherency matrix
$2 \times \text{trace}(dB)$	-19.8
$B_0^N - B'^N_0(dB)$	-25.9

Table 10. Results of the Huynen Decomposition of the Chimney (Stationary Target)

Cloude	Stationary target 0	Stationary target 1	Stationary target 2
Eigenvalues (dB)	173.56 (0)	0.0029 (-47.8)	0.0025 (-48.3)
$Span(dB)$	25.4	-22.4	-23.0
$S_{hh}(dB,\ deg)$	23.5, 0	-29.6, 0	-30.7, 0
$S_{hv}(dB,\ deg)$	-7.4, 14	-28.4, 153	-29.0, -31
$S_{vv}(dB,\ deg)$	20.9, 1	-27.5, 184	-27.7, -182

Table 11. Results of the Cloude Decomposition of the Chimney (Stationary Target).

The other scattering matrices resulting from the decomposition have a span much lower (40 to 50 dB less). The Huynen decomposition leads to a target coherency matrix of the stationary target $\left[T_c^S\right]$ nearly equal to the input target coherency matrix $\langle [T_c] \rangle$ (Table 9). The N-target coherency matrix $\left\langle \left[T_c^N\right] \right\rangle$ is therefore negligible.

Stationary Target

Holm and Barnes stationary target			
$Span(dB)$	$S_{hh}(dB,\ deg)$	$S_{hv}(dB,\ deg)$	$S_{vv}(dB,\ deg)$
25.4	23.5, 0	−7.4, 14	20.9, 1

Non-Stationary Target Coherency Matrices

Target coherency matrix $\langle [T^{I}_{c\,H}] \rangle$ $2 \times$ trace $= -28.5\ dB$		
(0.0003, 0.)	(0., 0.)	(0., 0.)
(0., 0.)	(0.0002, 0.)	(−0.0002, 0.)
(0., 0.)	(−0.0002, 0.)	(0.0002, 0.)

Target coherency matrix $\langle [T^{D}_{c\,H}] \rangle$ $2 \times$ trace $= -18.2\ dB$		
(0.0025, 0.)	(0., 0.)	(0., 0.)
(0., 0.)	(0.0025, 0.)	(0., 0.)
(0., 0.)	(0., 0.)	(0.0025, 0.)

Table 12. Results of the Holm and Barnes Decomposition of the Chimney (Stationary Target).

The Huynen and Cloude decompositions lead to the same largest span scattering matrix. The other scattering matrices (having a span much smaller) are also comparable. Looking at the Cloude scattering matrices related to the small eigenvalues, each of them tends to a N-target scattering matrix (the copolar elements of the N-target scattering matrix are equal in modulus and have a phase difference of $\pm\pi$). Huynen's idea to obtain the "averaged" target plus the "noise" target represented by a N-target is verified with this measurement when applying the Cloude decomposition to a stationary target.

To be complete, the results of the Holm and Barnes decomposition are given in Table 12. Like expected, the Holm and Barnes scattering matrix is the averaged scattering matrix of the chimney.

4. CONCLUSION

When symmetric scattering matrices are measured, assuming reciprocity of the antenna system of the radar, a random target is described by three scattering matrices in the Cloude decomposition or two scattering matrices plus a coherency matrix containing one parameter in the Huynen decomposition. None of these matrices is predominant in energy and one of them has a N-target behavior in both decompositions. The entropy of a random target tends to 1. The decompositions proposed by Huynen and Cloude show both a decomposition in copolar and cross-polar elements.

The Holm and Barnes decomposition leads to a scattering matrix relatively identical to the Cloude scattering matrix corresponding to the largest eigenvalue. Their span differ when the target is non stationary. The Holm and Barnes scattering matrix tends to a N-target scattering matrix when the target is random. One target coherency matrix, the non diagonal one, can be neglected when the smallest and middle eigenvalues are nearly the same. This is the case for a random and stationary target.

A stationary target is described by one scattering matrix and the three decompositions lead to the averaged scattering matrix of the target. In this case the entropy tends to 0. The other matrices, results of the decomposition, have a span (or trace) comparable to the noise level of the radar. When resulting from the Huynen or Cloude decomposition, they show a N-target behavior.

This paper gives a comprehensive overview of three radar target decomposition theorems. The decompositions are then applied to simple targets as illustrations. These examples, random $(H_c \approx 1)$ and stationary target $(H_c \approx 0)$, are limiting statistical cases for time-dependent targets $(0 < H_c < 1)$, H_c is the entropy of the target. The Cloude decomposition, mathematically unique, is appealing since it leads to three orthogonal stationary targets weighted by the eigenvalues, indicating then which stationary target is predominant. The Holm and Barnes decomposition results in a scattering matrix relatively identical to the Cloude scattering matrix corresponding to the largest eigenvalue. Only their span differ. The reason for this decomposition choice is until now not clear to us. The Huynen decomposition states that the time-dependent target can be, most of the time, described by an effective mean target plus a residual target. This decomposition is based on one hypothesis: the residual target is chosen to have a LR or RL nul (definition of the N-target). This hypothesis is a reasonable one. Considering the residual scattering matrices obtained from the Cloude decomposition of the chimney (quasi-stationary target), they tend to N-targets.

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