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COMPLETE SPECTRAL REPRESENTATION FOR THE ELECTROMAGNETIC FIELD OF PLANAR MULTI-LAYERED WAVEGUIDES CONTAINING PSEUDOCHIRAL Ω-MEDIA

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1. INTRODUCTION

Novel electromagnetic features of complex media – namely in the microwave and millimeter-wave regimes – have attracted, in recent years, basic and applied research activities in Electromagnetics. In particular, the pseudochiral Ω-medium [1, 2] has generated considerable attention in the literature [3–8].

As is well-known, chiral artificial materials – which exhibit *optical activity* at microwave frequencies – can be obtained by inserting small wire helices into the isotropic host medium. On the other hand, synthetic pseudochiral Ω-media - which are nonchiral - can be obtained by doping a host isotropic medium with Ω-shaped conducting microstructures where both the loop and stamps lie in the same plane. Although the electric field, in both the wire helices and Ω inclusions, induces not only electric but also magnetic polarizations, a different mutual placement in these polarization vectors is observed: in the wire helices of the chiral medium these interacting vectors are parallel; in the Ω microstructures of the pseudochiral medium they are perpendicular to each other. This distinctive characteristic of Ω-media implies that the orientation of the doping elements in the host isotropic medium cannot be random - as in the chiral medium - but must be parallel to a unique preferred direction. Otherwise, with a random distribution of the Ω-shaped microstructures, the overall electro- magnetic coupling would result in a null average.

The problem of guided electromagnetic wave propagation in pseudochiral Ω-structures was, for some special devices, already analyzed (e.g., [5, 8]). However, a rigorous analysis of guided modes, especially for open Ω-structures, aiming to derive a complete field representation including the surface as well as the radiation modes, is – as far as the authors are aware – still lacking.

In this paper, based on a linear-operator formalism, we present a complete spectral representation for the electromagnetic field of planar multilayered waveguides containing pseudochiral Ω-media. In fact, using the theory of linear operators and adopting a suitable definition of a two-vector transverse mode function, we reduce the problem of electromagnetic wave propagation in planar waveguides, inhomogeneously filled with Ω-media, to an eigenvalue equation related to a 2×2 matrix differential operator. This theoretical framework is similar to the one developed by the authors for planar chirowaveguides [9, 10].

Using the concept of adjoint waveguide, bi-orthogonality relations are derived for the hybrid modes. In order to have a complete field representation in open pseudochiral waveguides, these relations are of utmost importance when choosing an appropriate set of mutually orthogonal radiation modes. As an example of application, a general analysis of the surface and radiation modes of a grounded pseudochiral Ω-slab waveguide is also presented.

One should finally note that, for the case of homogeneous layers, the general formalism reduces to an algebraic 2×2 coupling matrix eigenvalue problem.

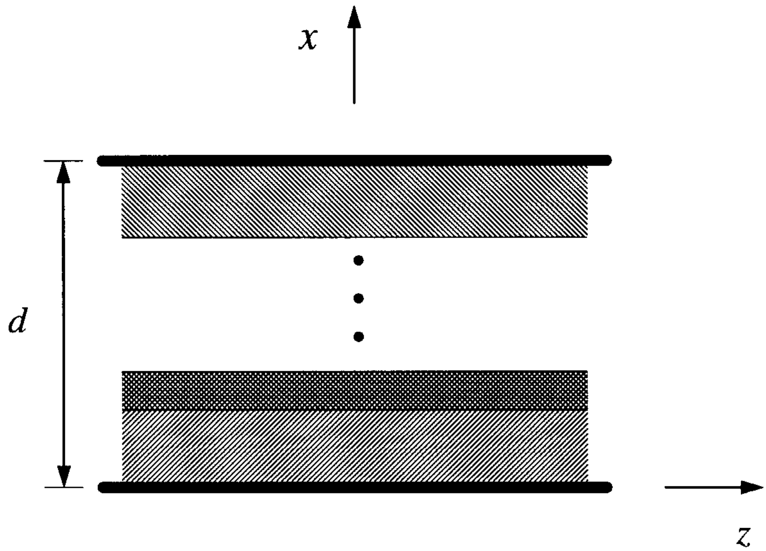

Figure 1. Multilayered pseudochiral waveguide closed by electric and/or magnetic walls placed at $x = 0$ and $x = d$.

2. LINEAR-OPERATOR FORMALISM

The aim of this section is to reduce the problem of guided electromagnetic wave propagation in (open or closed) inhomogeneous pseudochiral waveguides to a linear-operator formalism. Based on the transverse electromagnetic field equations an eigenvalue problem is obtained. For each eigenvalue the corresponding eigenfunction represents a transverse mode function of the waveguide. Hence, the orthogonality properties of these eigenfunctions can be used to represent the electromagnetic field as a superposition of mode functions, as long as completeness is guaranteed.

In this section, the general layered grounded open waveguide depicted in Figure 1 will be considered. It is uniform in the y direction and inhomogeneously filled with spatially nondispersive lossless Ω-medium.

For bianisotropic Ω-media the constitutive relations may be written as

$$\boldsymbol{D} = \varepsilon_0(\overline{\varepsilon} \cdot \boldsymbol{E} + Z_0\overline{\xi} \cdot \boldsymbol{H}) \tag{1a}$$

$$\boldsymbol{B} = \mu_0(Y_0\overline{\zeta} \cdot \boldsymbol{E} + \overline{\mu} \cdot \boldsymbol{H}) \tag{1b}$$

with $Z_0 = Y_0^{-1} = k_0/(\omega\varepsilon_0) = (\omega\mu_0)/k_0$, where $\overline{\varepsilon}$ and $\overline{\mu}$ are the relative dielectric permittivity and relative magnetic permeability dimensionless tensors, and $\overline{\xi}$ and $\overline{\zeta}$ are the magnetoelectric coupling dimensionless tensors. As the medium is considered spatially nondispersive these relations are local. The structure depicted in Figure 1 is uniform and infinite in the y direction (hence $\partial/\partial y = 0$) and can be inhomogeneously filled with Ω-media, i.e., $\overline{\varepsilon}(\omega,\ x)$, $\overline{\mu}(\omega,\ x)$, $\overline{\xi}(\omega,\ x)$ and $\overline{\zeta}(\omega,\ x)$ may be piece-wise continuous functions of x, although, in order to obtain the adjoint operator, one has to assume that $\overline{\xi}$ and $\overline{\zeta}$ are piecewise constant functions along x'.

Introducing normalized distances marked with primes (*e.g.*, $x' = k_0 x,\ y' = k_0 y,\ z' = k_0 z$) and the normalized magnetic field $\mathcal{H}$ such that

$$\mathcal{H} = Z_0\,\boldsymbol{H} \tag{2}$$

then, from Maxwell's curl equations for source free regions together with (1a) and (1b), one may write

$$-j\boldsymbol{\nabla'} \times \mathcal{H} = \overline{\varepsilon}\cdot\boldsymbol{E} + \overline{\xi}\cdot\mathcal{H} \tag{3a}$$

$$j\boldsymbol{\nabla'} \times \boldsymbol{E} = \overline{\zeta}\cdot\boldsymbol{E} + \overline{\mu}\cdot\mathcal{H} \tag{3b}$$

where time-harmonic field variation of the form $\exp(j\omega t)$ was assumed and $\boldsymbol{\nabla'} = \boldsymbol{\nabla}/k_0$. Considering forward plane wave propagation of the form $\exp(-j\beta z')$, where β is the normalized longitudinal wavenumber given by

$$\beta = \frac{k}{k_0} \tag{4}$$

one has

$$\boldsymbol{\nabla'} = \partial_{x'}\,\hat{\boldsymbol{x}} - j\beta\,\hat{\boldsymbol{z}} \tag{5}$$

where $\partial_{x'}$ stands for $\partial/\partial_{x'}$.

In this paper, only the case of Ω-shaped perfectly conducting microstructures oriented as in Figure 2 in a isotropic host material, will be considered. The normal to the planes of the loops points in the x direction while the stamps are aligned along the z direction, and the loops are oriented in the positive y direction. Therefore tensors $\overline{\varepsilon}$, $\overline{\mu}$, $\overline{\xi}$, and $\overline{\zeta}$ have the dyadic representation

$$\overline{\varepsilon} = \varepsilon_{xx}\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \varepsilon_{yy}\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + \varepsilon_{zz}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} \tag{6a}$$

$$\overline{\mu} = \mu_{xx}\hat{\boldsymbol{x}}\hat{\boldsymbol{x}} + \mu_{yy}\hat{\boldsymbol{y}}\hat{\boldsymbol{y}} + \mu_{zz}\hat{\boldsymbol{z}}\hat{\boldsymbol{z}} \tag{6b}$$

$$\overline{\xi} = j\Omega\hat{\boldsymbol{z}}\hat{\boldsymbol{x}} \tag{6c}$$

$$\overline{\zeta} = -j\Omega\hat{\boldsymbol{x}}\hat{\boldsymbol{z}} \tag{6d}$$

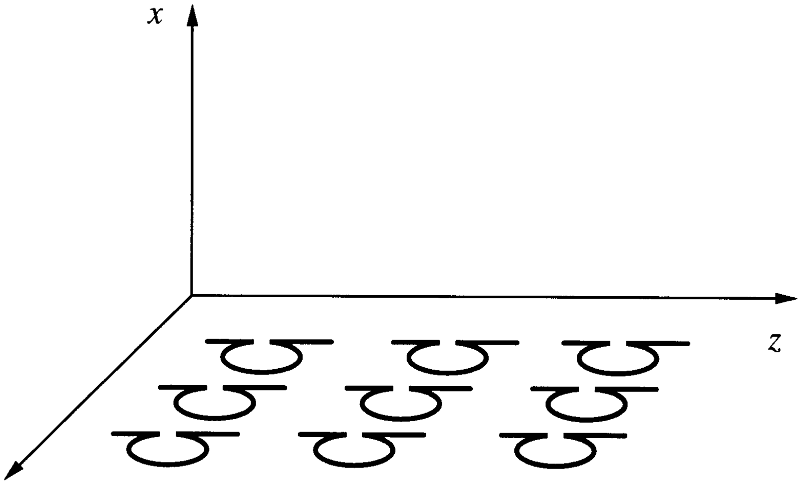

Figure 2. Spatial orientation of planar, Ω-shaped, conducting microstructures in the hosting isotropic material.

where Ω is the dimensionless pseudochiral parameter (which is positive). If the loops were oriented in the negative y direction, one should have $\Omega < 0$.

From (6) one has $\overline{\xi} = -\overline{\zeta}^T$, which shows that the Ω-medium is reciprocal. Moreover, when the medium is lossless the constitutive parameters are all real.

After substituting (6) into Maxwell's equations (3) and eliminating the components directed along x, one obtains the following set of coupled partial differential equations

$$\partial_{x'} \mathbf{f}_t = -j\overline{\mathbf{C}} \cdot \mathbf{f}_t \tag{7}$$

where $\mathbf{f}_t$ is a column vector with the electric and magnetic field components tangential to the yz plane

$$\mathbf{f}_t = [E_y \quad \mathcal{H}_z \quad \mathcal{H}_y \quad E_z]^T \quad , \tag{8}$$

(T stands for transpose), whereas $\overline{\mathbf{C}}$ is a 4×4 coupling matrix given by

$$\overline{\mathbf{C}} = \begin{bmatrix} 0 & \mu_{zz} & 0 & 0 \\ \varepsilon_{yy} - \frac{\beta^2}{\mu_{xx}} & 0 & 0 & j\beta\frac{\Omega}{\mu_{xx}} \\ j\beta\frac{\Omega}{\mu_{xx}} & 0 & 0 & -\varepsilon_{zz} + \frac{\Omega^2}{\mu_{xx}} \\ 0 & 0 & -\mu_{yy} + \frac{\beta^2}{\varepsilon_{xx}} & 0 \end{bmatrix} . \tag{9}$$

The transverse field components may be algebraically expressed in terms of $\mathbf{f}_t$ as follows

$$\mathbf{f}_n = \overline{\mathbf{G}} \cdot \mathbf{f}_t \tag{10}$$

where

$$\mathbf{f}_n = [E_x \quad \mathcal{H}_x]^T . \tag{11}$$

In (10) $\overline{\mathbf{G}}$ is a 2×4 matrix given by

$$\overline{\mathbf{G}} = \begin{bmatrix} 0 & 0 & \frac{\beta}{\varepsilon_{xx}} & 0 \\ -\frac{\beta}{\mu_{xx}} & 0 & 0 & j\frac{\Omega}{\mu_{xx}} \end{bmatrix} . \tag{12}$$

One should not that, according to (8)-(12), only hybrid modes can propagate in the planar structure.

Moreover, one has

$$\mathrm{tr}\ \overline{\mathbf{C}} = 0 \tag{13a}$$

$$\mathrm{tr}\ (\mathrm{adj}\ \overline{\mathbf{C}}) = 0, \tag{13b}$$

and hence the eigenvalues of $\overline{\mathbf{C}}$ are anti-symmetric, therefore allowing (7) to be recast as a 2×2 matrix eigenvalue problem.

A. Eigenvalue Equation for Inhomogeneous Waveguides

In order to recast the electromagnetic field equations in terms of a single eigenvalue equation, the following definition of a state vector mode function (or eigenfunction) is introduced:

$$\mathbf{\Phi} = [E_z \quad \mu_{xx}\mathcal{H}_x]^T . \tag{14}$$

One should note that $\mathbf{\Phi}$ is a continuous function of x' across any interface of the multilayered structure. Hence, from (7)-(12) one obtains the eigenvalue equation

$$\overline{\mathcal{L}} \cdot \mathbf{\Phi} = \beta^2 \overline{\mathcal{W}} \cdot \mathbf{\Phi} \tag{15}$$

Where $\overline{\mathcal{L}}$ is a 2×2 matrix differential operator given by

$$\overline{\mathcal{L}} = \begin{bmatrix} -j\partial_{x'}\frac{1}{\mu_{zz}}\partial_{x'}\Omega - j\varepsilon_{yy}\Omega & \partial_{x'}\frac{1}{\mu_{zz}}\partial_{x'} + \varepsilon_{yy} \\ \partial^2_{x'} + \mu_{yy}\varepsilon_{zz} & \frac{j\mu_{yy}\Omega}{\mu_{xx}} \end{bmatrix} \tag{16}$$

and $\overline{\mathcal{W}}$ is the weight operator

$$\overline{\mathcal{W}} = \begin{bmatrix} 0 & \frac{1}{\mu_{xx}} \\ \frac{\varepsilon_{zz}}{\varepsilon_{xx}} & j\frac{\Omega}{\varepsilon_{xx}\mu_{xx}} \end{bmatrix}. \tag{17}$$

Once the field components E_z and $\mathcal{H}_x$ have been determined through (15), the remaining components can also be determined according to Appendix A.

In everything that follows within this Section, three classes of waveguides will be considered: (1) *closed* waveguides with electric and/or magnetic walls placed at $x' = 0$ and $x' = d'$; (ii) *open* waveguides extending from $x' = -\infty$ to $x' = +\infty$; (iii) *open grounded* waveguides extending from an electric or magnetic wall placed at $x' = 0$ to $x' = +\infty$. Hence, a finite, infinite or semi-infinite interval I on x' will be introduced as follows: (i) $I = [0,\ d']$ for closed waveguides; (ii) $I =]-\infty,\ +\infty[$ for open waveguides; (iii) $I = [0,\ +\infty[$ for open grounded waveguides. In order to define the domain D of $\overline{\mathcal{L}}$, only surface modes will be considered for the two classes (ii) and (iii) of open waveguides. Consequently, E_z and $\mathcal{H}_x$ always have finite energy and hence they belong to the vector space of square integrable functions over I. However, only for closed waveguides (i.e., for regular problems corresponding to finite interval I), a complete spectral representation is possible within D.

B. Bi-Orthogonality Relation

Introducing the following real type inner product

$$\langle \boldsymbol{u},\ \boldsymbol{u}^a \rangle = \int_I (u_1 u_1^a + u_2 u_2^a) dx' \tag{18}$$

it is possible to determine the adjoint operators $\overline{\mathcal{L}}^a$ and $\overline{\mathcal{W}}^a$ of $\overline{\mathcal{L}}$ and $\overline{\mathcal{W}}$, respectively, with $\boldsymbol{\Phi}^a$ satisfying the *same* boundary conditions. In fact, making use of (18) with $\boldsymbol{u}_1 = [u_1,\ u_2]^T \in D$ and $\boldsymbol{u}_1^a = [u_1^a,\ u_2^a]^T \in D^a$, where D^a denotes the domain of $\overline{\mathcal{L}}^a$, one can easily see that

$$\overline{\mathcal{L}}^a = \overline{\mathcal{L}}^T, \quad \overline{\mathcal{W}}^a = \overline{\mathcal{W}}^T \tag{19}$$

according to (16) and (17), inasmuch as Ω is a piecewise constant function of x' (see Appendix B).

At this point it is useful to introduce the concept of adjoint waveguide [11], as the one which has the same geometry and dimensions of the original waveguide, with identical boundaries, and satisfying to the eigenvalue problem

$$\overline{\mathcal{L}}^a \cdot \mathbf{\Phi}^a = \beta_a^2 \overline{\mathcal{W}}^a \cdot \mathbf{\Phi}^a \tag{20}$$

where plane wave propagation of the form $\exp(-j\beta_a z)$ was considered. According to the fact that every eigenvalue β^2 of $\overline{\mathcal{L}}$ is an eigenvalue of $\overline{\mathcal{L}}^a$ [12], one can readily prove that

$$(\beta_m^2 - \beta_n^2) \left\langle \overline{\mathcal{W}} \cdot \mathbf{\Phi}_m, \ \mathbf{\Phi}_n^a \right\rangle = 0 \tag{21}$$

if $\mathbf{\Phi}_m \ni D$ and $\mathbf{\Phi}_n^a \ni D^a$. Hence, after a suitable normalization, the following bi-orthogonality relation holds:

$$\left\langle \overline{\mathcal{W}} \cdot \mathbf{\Phi}_m, \ \mathbf{\Phi}_n^a \right\rangle = \delta_{mn} \tag{22}$$

where δ_{mn} is the Kronecker delta.

3. HOMOGENEOUS LAYERS

For the special case of homogeneous layers, the linear operator formalism herein derived is reduced to a 2×2 coupling matrix eigenvalue problem. In fact, for this case, one obtains from (15)–(17)

$$\partial_{x'}^2 \mathbf{\Phi} = -\overline{\boldsymbol{R}} \cdot \mathbf{\Phi} \tag{23}$$

where

$$\overline{\boldsymbol{R}} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \tag{24}$$

with

$$R_{11} = \varepsilon_{zz} \left(\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}} \right) \tag{25a}$$

$$R_{12} = j\Omega\mu_{xx} \left(\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}} \right) \tag{25b}$$

$$R_{21} = j\Omega\varepsilon_{zz} \left(\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}} \right) - j\Omega\varepsilon_{yy}\mu_{zz} \tag{25c}$$

$$R_{22} = \mu_{zz} \left(\varepsilon_{yy} - \frac{\beta^2}{\mu_{xx}} \right) - \frac{\Omega^2}{\mu_{xx}} \left(\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}} \right). \tag{25d}$$

Hence, in a similar way as shown in [9], one may write for each homogeneous pseudochiral layer:

$$\mathbf{\Phi}(x') = \overline{\boldsymbol{M}} \cdot \mathbf{\Psi}(x') \tag{26}$$

where

$$\overline{\boldsymbol{M}} = \begin{bmatrix} 1 & 1 \\ \tau_a & \tau_b \end{bmatrix} \tag{27}$$

is the modal matrix of $\overline{\boldsymbol{R}}$, such that

$$\partial^2_{x'} \mathbf{\Psi} = -\overline{\mathbf{\Lambda}} \cdot \mathbf{\Psi} \tag{28}$$

with $\mathbf{\Psi} = [\Psi_a \;\; \Psi_b]^T$ and $\overline{\mathbf{\Lambda}} = \mathrm{diag}(h_a^2,\; h_b^2)$. Therefore, one has

$$h_s^2 = \frac{R_{11} + R_{22} \pm \sqrt{(R_{11} - R_{22})^2 + 4R_{12}R_{21}}}{2} \tag{29}$$

and

$$\tau_s = \frac{h_s^2 - R_{11}}{R_{12}} = \frac{R_{21}}{h_s^2 - R_{22}} \tag{30}$$

with $s = a,\; b$.

4. GROUNDED PSEUDOCHIRAL SLAB WAVEGUIDE

As an example of application of the previous formalism, the grounded pseudochiral slab waveguide depicted in Figure 3 will be analyzed. Since this waveguide is an *open structure* extending from the perfectly conducting plane at $x' = 0$ to $x' = +\infty$, the operator $\mathcal{L}$ is defined over a semi-infinite interval, and has a discrete spectrum as well as a continuous spectrum. One should stress that, for the sake of completeness, the radiation modes must be included in the analysis. Nevertheless, the radiation modes do not actually belong to the domain of the operator: indeed they are improper eigenfunctions.

Assuming that the Ω-shaped conducting microstructures have a spatial orientation in the slab as in Figure 2, all the modes in the waveguide are hybrid. This problem – as far as the authors are aware – has never been addressed in the literature.

According to (26), (27), (29) and (30), for $0 < x' < t'$, where t' is the normalized thickness of the slab, one has,

$$E_z = \Psi_a + \Psi_b \tag{31a}$$

$$\mu_{xx}\mathcal{H}_x = \tau_a\Psi_a + \tau_b\Psi_b \tag{31b}$$

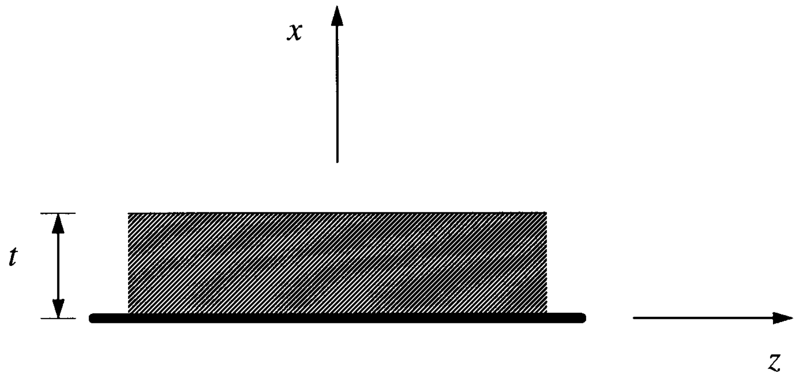

Figure 3. Grounded pseudochiral slab waveguide. The slab of thickness t is an isotropic host medium with Ω-shaped conducting microstructures with spatial orientation as in Figure 2. The upper medium is the air.

where

$$\Psi_a = A[\sin(h_a x') - Q\cos(h_a x')] \tag{32a}$$
$$\Psi_b = A[R\sin(h_b x') + Q\cos(h_b x')] \tag{32b}$$

which automatically guarantees that $E_z = 0$ for $x' = 0$. Imposing the other boundary condition at $x' = 0$, i.e., $E_y = 0$, one obtains from $(A1)$

$$Q = 0 \tag{33}$$

In the air region, i.e., for $x' > 0$, one gets

$$E_z = \alpha_a A\cos[\rho(x' - t')] + B_1 \sin[\rho(x' - t'] \tag{34a}$$
$$\mathcal{H}_x = \alpha_2 A\cos[\rho(x' - t')] + B_2 \sin[\rho(x' - t')] \tag{34b}$$

with

$$\rho^2 = 1 - \beta^2 \tag{35}$$

and where B_1 and B_2 are arbitrary constants determined according to the type of modes to be considered. For example, when considering the hybrid surface modes one should take $B_1 = -j$ and $B_2 = -j$, while $\rho = -j\alpha$ with $\alpha = \sqrt{\beta^2 - 1}$ real for lossless media. In order to satisfy the radiation condition one must have $\alpha > 0$, i.e., $\beta > 1$.

Imposing the continuity of E_y and E_z at $x' = t'$, coefficients α_1 and α_2 can be determined according to $(31a)$ and $(A1)$:

$$\alpha_1 = \sin(h_a t') + R\sin(h_b y') \tag{36}$$

$$\alpha_2 = (\mu_{xx}\tau_a - j\Omega)\sin(h_a t') + (\mu_{xx}\tau_b - j\Omega)R\sin(h_b t'). \tag{37}$$

After enforcing the remaining boundary conditions, i.e., the continuity of $\mathcal{H}_y$ and $\mathcal{H}_z$ at $x' = t'$, the following linear system is obtained:

$$\begin{bmatrix} \eta_a & \eta_b \\ \nu_a & \nu_b \end{bmatrix} \cdot \begin{bmatrix} 1 \\ R \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{38}$$

where

$$\eta_s = (\mu_{xx}\tau_s - j\Omega)\left[\frac{h_s}{\mu_{zz}}\cos(h_s t') - B_2\rho\sin(h_s t')\right] \tag{39a}$$

$$\nu_s = \left(\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}}\right) B_1\sin(h_s t') - \rho h_s\cos(h_s t') \tag{39b}$$

with $s = a,\ b$. In order to have a determined system and hence obtain nontrivial solutions, one has to ensure that

$$\eta_a\nu_b - \nu_a\eta_b = 0. \tag{40}$$

Furthermore, one also obtains from (38)

$$R = -\frac{\eta_a}{\eta_b} = -\frac{\nu_a}{\nu_b}. \tag{41}$$

For the surface modes and since B_1 and B_2 are both defined, (38) becomes the modal equation of the pseudochiral waveguide of Figure 3.

A. Surface Modes

The surface modes, which constitute the discrete spectrum of the linear operator $\overline{\mathcal{L}}$ and define its domain as the set of eigenfunctions $\mathbf{\Phi} = [E_z \ \ \mathcal{H}_x]^T$, must satisfy to the radiation condition. Therefore, one should have $B_1 = -j$ and $B_2 = -j$ in (34), while $\rho = -j\alpha$, α real and positive for lossless media. According to (35), all the surface

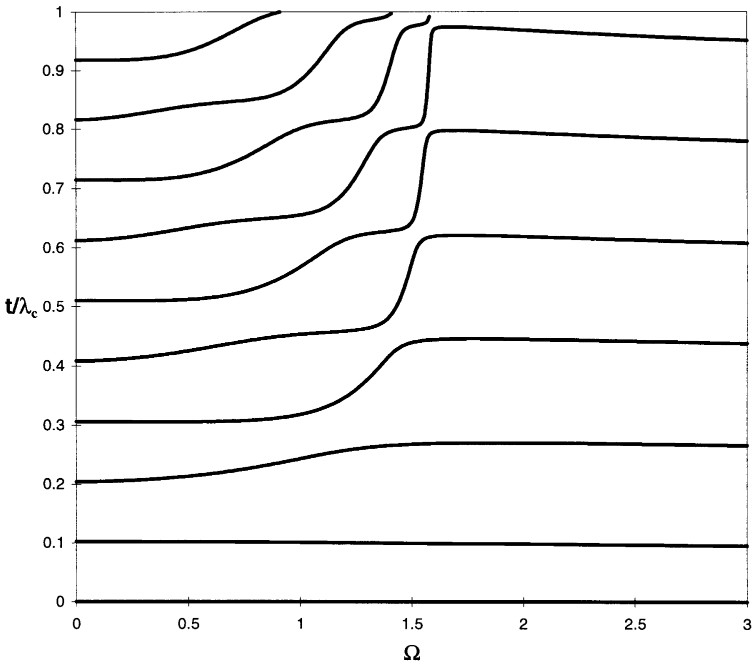

Figure 4. Variation of t/λ_c with the pseudochiral parameter Ω for the first hybrid modes of the grounded pseudochiral slab waveguide of Figure 3: $\varepsilon_{xx} = 2$, $\varepsilon_{yy} = 3$, $\varepsilon_{zz} = 4$, $\mu_{xx} = 1$, $\mu_{yy} = 2$, and $\mu_{zz} = 3$.

modes are slow modes, i.e., $\beta > 1$, and each cutoff when $\alpha = 0$, i.e., for $\beta = 1$.

In Figure 4, the variation of t/λ_c with Ω-space where λ_c denotes the cutoff wavelength - is presented. These curves are easily calculated by making $\alpha = 0$ in the modal equation (40). For numerical results the following values for the dimensionless constitutive parameters were considered: $\varepsilon_{xx} = 2$, $\varepsilon_{yy} = 3$, $\varepsilon_{zz} = 4$, $\mu_{xx} = 1$, $\mu_{yy} = 2$, and $\mu_{zz} = 3$. Hereafter, the descriptor H_p will be used for each hybrid mode, where the subscript p, with $p \geq 0$, indicates the mode order, where all the modes are ordered after increasing cutoff frequencies. The fundamental mode H_0 (i.e., the first propagating mode) has no cutoff ($t/\lambda_c = 0$). For any value of t' where $t' = 2\pi t/\lambda$, one easily obtains from Figure 4 the number of propagating modes.

In Figure 5, the variation of β with t/λ - defined in (4) - is presented for $\Omega = 0.1$. In the high frequency regime, when $t/\lambda \to \infty$,

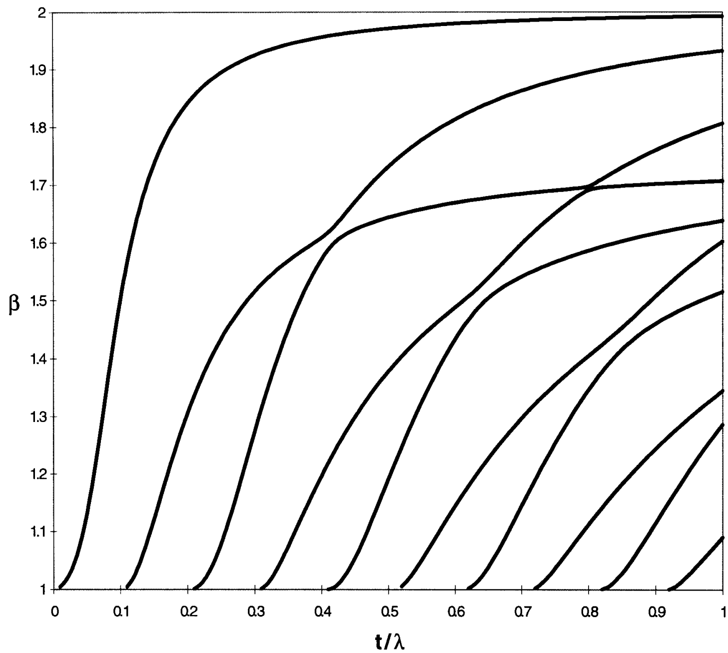

Figure 5. Variation of the normalized longitudinal wavenumber β with t/λ for the hybrid modes of a grounded pseudochiral slab waveguide when $\Omega = 0.1$.

there are two asymptotic values for β (β_a and β_b) corresponding to $h_s \to 0$, with $s = a, b$. In both cases, when $h_s = 0$, one has $\det(\overline{\boldsymbol{R}}) = 0$ in (23). Nevertheless, for every mode, the dispersion curve always converges to the highest of these two values. In the present example, one has

$$\beta_b = \sqrt{\varepsilon_{xx}\mu_{yy}} \tag{42}$$

when $h_b = 0$, while

$$\beta_b = \sqrt{\varepsilon_{yy}(\mu_{xx} - \Omega^2/\varepsilon_{zz})} \tag{43}$$

when $h_a = 0$. Since $\varepsilon_{xx}/\varepsilon_{yy} > \mu_{xx}/\mu_{yy}$, one always has $\beta_b > \beta_a$. Therefore, for all modes, $1 < \beta < \beta_b$. In as much as $t/\lambda \to \infty$, all the dispersion curves converge to $\sqrt{\varepsilon_{xx}\mu_{yy}}$. When $\Omega \to 0$ all the hybrid modes degenerate into the TE and TM surface modes of the biaxial

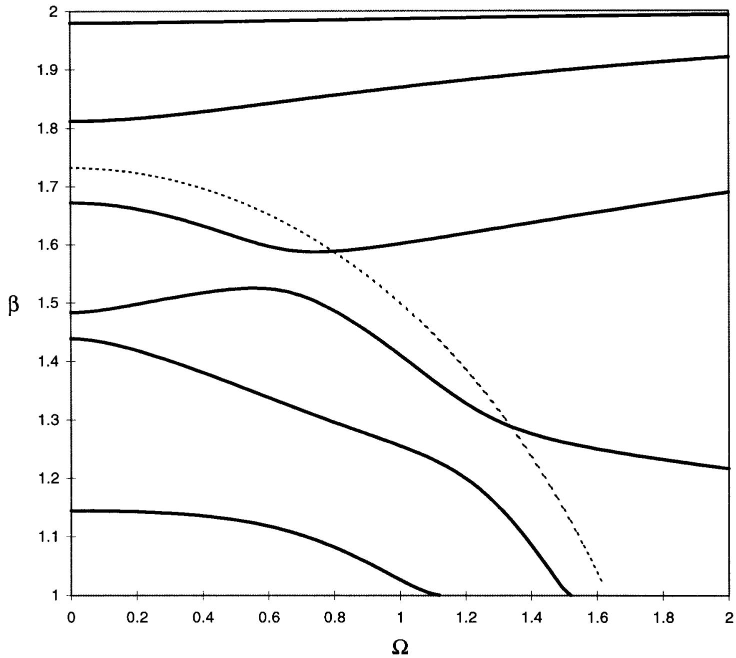

Figure 6. Variation of the normalized longitudinal wavenumber β with the pseudochiral parameter Ω for the hybrid surface modes of a grounded pseudochiral slab waveguide with $t/\lambda = 0.6$. The dashed line corresponds to the asymptotic value $\beta = \beta_a$ given by (43).

anisotropic case, with the dispersion curves crossing each other instead of displaying coupling points.

Finally, Figure 6 shows the variation of β with the dimensionless pseudochiral parameter Ω for all propagating modes when $t/\lambda = 0.6$, where the dashed curved corresponds to $\beta = \beta_a$. One should stress that modes H_4 and H_5 each cutoff with the increase of the pseudochiral parameter Ω.

B. Radiation Modes

The set of modes described in Section A is sufficient to describe any guided field distribution in the slab waveguide provided that there is not any variation along the z direction. However, this set is not sufficient to describe the radiation phenomena. For a complete spectral representation, the analysis must include an infinite number of radiation modes. The fields of the radiation modes do not decay in the outside of the structure, i.e., they are not bound to the slab, which means that they do not satisfy the radiation condition. Unlike the guided modes, each individual radiation mode carries an infinite amount of energy. Therefore, for these type of modes – also called *pseudosurface* modes – the biorthogonality relation (22) must involve the Dirac delta function instead of the Kronecker delta [13]:

$$\left\langle \overline{\mathcal{W}} \cdot \mathbf{\Phi}\,(x',\ \rho),\ \ \mathbf{\Phi}^{\,a}(x',\ \rho') \right\rangle = \delta(\rho - \rho'). \tag{44}$$

This bi-orthogonality relation is necessary for the normalization of the radiation modes.

According to (34) there are two arbitrary constants B_1 and B_2 to be chosen, in order to have a complete set of orthogonal radiation modes. This unique degree of freedom shows that only two types of radiation modes need to be considered for a complete spectral representation. One possible choice is the ITE (*Incident Transverse Electric*) and ITM (*Incident Transverse Magnetic*) hybrid radiation modes [14] which obey to the bi-orthogonality relation (44). In fact, this type of hybrid radiation modes can be seen as a perturbation of the common TE and TM radiation modes of the isotropic waveguide which are known to be mutually orthogonal [15]. Moreover, these hybrid radiation modes have a clear physical interpretation. When a TE (TM) plane wave impinges on the surface of the pseudochiral slab, a hybrid standing-wave generates inside the slab, and a TE along with a TM plane wave is reflected from the slab surface. The TE (TM) plane wave outside the slab is also a standing-wave while the TM (TE) plane wave is a traveling-wave. This discussion is schematically depicted in Figure 7. Therefore, for ITE radiation modes, one must have

$$B_1 = -j \tag{45}$$

in (34a), which leads to

$$E_z = \alpha_1 A \exp[-j\rho(x' - t')] \tag{46}$$

in the air region. For ITM radiation modes, one must have

$$B_2 = -j \tag{47}$$

in (34b), which leads to

$$\mathcal{H}_x = \alpha_2 A \exp[-j\rho(x' - t')] \tag{48}$$

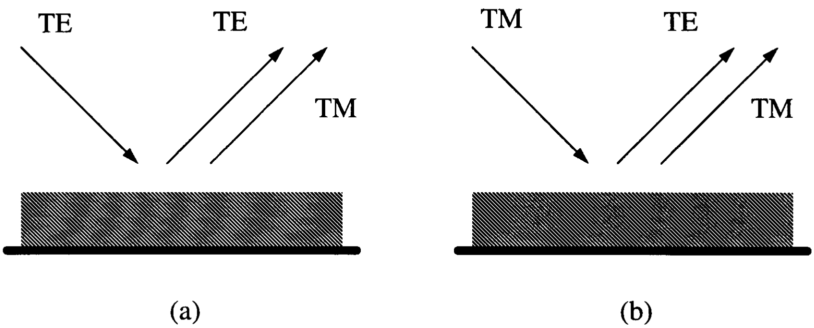

Figure 7. Plane wave decomposition for the (a) Incident Transverse Electric (ITE) radiation modes and for the (b) Incident Transverse Magnetic (ITM) radiation modes.

in the air region. In (46) and (48) ρ may take any value in the interval $0 < \rho < \infty$. Hence, as for the isotropic case, one has propagating radiation modes for $0 < \rho < 1$ and evanescent radiation modes for $1 < \rho < \infty$.

5. CONCLUSION

A linear-operator formalism for the analysis of inhomogeneous pseudochiral multilayered planar waveguides was developed. The original and adjoint waveguides were described by eigenvalue equations related, respectively, to a 2×2 matrix differential operator and its transpose. Accordingly, a bi-orthogonality relation for the hybrid modes, which involves the two-vector eigenfunctions of both the original and adjoint waveguides, was derived. For homogeneous layers the linear-operator formalism is reduced to a 2×2 coupling matrix eigenvalue problem.

As an example of application, a complete spectral representation for the field in a grounded pseudochiral slab waveguide was derived. Namely, two types of mutually orthogonal radiation modes were proposed, for the first time, along with a physical interpretation: the incident transverse electric ITE and the incident transverse magnetic ITM hybrid radiation modes. This framework is a valuable tool in the study of more complex structures involving pseudochiral Ω-media (e.g., step discontinuities in pseudochiral planar waveguides), whenever a mode-matching procedure requires a complete field representation. This subject will be addressed in a forthcoming paper.

APPENDIX A

In this appendix we derive the field components of the hybrid modes in terms of E_z and $\mathcal{H}_x$. According to (7)–(12) and taking E_z and $\mathcal{H}_x$ as the supporting field components, one obtains

$$E_y = -\frac{1}{\beta}(\mu_{xx}\mathcal{H}_x - j\Omega E_z) \qquad (A1)$$

$$\mathcal{H}_y = -j\frac{1}{\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}}}\partial_{x'}E_z \qquad (A2)$$

and

$$\mathcal{H}_z = -j\frac{1}{\beta\mu_{zz}}(\partial_{x'}\mu_{xx}\mathcal{H}_x - j\partial_{x'}\Omega E_z) \qquad (A3)$$

$$E_x = -j\frac{\frac{\beta}{\varepsilon_{xx}}}{\mu_{yy} - \frac{\beta^2}{\varepsilon_{xx}}}\partial_{x'}E_z. \qquad (A4)$$

APPENDIX B

Using definition (18), one can prove that $\overline{\mathcal{L}}^a = \overline{\mathcal{L}}^T$ and $\overline{\mathcal{W}}^a = \overline{\mathcal{W}}^T$ are the adjoint operators [12] of $\overline{\mathcal{L}}$ and $\overline{\mathcal{W}}$, respectively, i.e.,

$$\langle\overline{\mathcal{L}}\cdot\mathbf{u},\mathbf{u}^a\rangle = \langle\mathbf{u},\overline{\mathcal{L}}^T\cdot\mathbf{u}^a\rangle \qquad (B1a)$$

$$\langle\overline{\mathcal{W}}\cdot\mathbf{u},\mathbf{u}^a\rangle = \langle\mathbf{u},\overline{\mathcal{W}}^T\cdot\mathbf{u}^a\rangle \qquad (B1b)$$

Since the proof of $(B1b)$ is trivial, only the proof of $(B1a)$ will be presented here. Therefore, if $\mathbf{u} = [u_1, u_2]^T \in D$ and $\mathbf{u}^a = [u_1^a, u_2^a]^T \in D^a$, one has to prove that

$$J = \langle\overline{\mathcal{L}}\cdot\mathbf{u},\mathbf{u}^a\rangle - \langle\mathbf{u},\overline{\mathcal{L}}^a\cdot\mathbf{u}^a\rangle = 0 \qquad (B2)$$

Due to definition (18) and according to (16) and (19), one obtains for J – after canceling the identical terms – the following expression:

$$J = \sum_{i=1}^{3} J_i \qquad (B3)$$

where

$$J_1 = -j\int_{\mathbf{I}} \left\{ u_1^a \partial_{x'} \left[\frac{1}{\mu_{zz}} \partial_{x'}(\Omega u_1) \right] - u_1 \partial_{x'} \left[\frac{1}{\mu_{zz}} \partial_{x'}(\Omega u_1^a) \right] \right\} dx' \quad (B4)$$

$$J_2 = \int_{\mathbf{I}} \left[u_1^a \partial_{x'} \left(\frac{1}{\mu_{zz}} \partial_{x'} u_2 \right) - u_2 \partial_{x'} \left(\frac{1}{\mu_{zz}} \partial_{x'} u_1^a \right) \right] dx' \quad (B5)$$

$$J_3 = \int_{\mathbf{I}} \left[u_2^a \partial_{x'}^2 u_1 - u_1 \partial_{x'}^2 u_2^a \right] dx'. \quad (B6)$$

Using integration by parts, inasmuch as Ω is a piecewise constant function along x', one gets

$$J_1 = \left[-j\frac{1}{\mu_{zz}} u_1^a \partial_{x'}(\Omega u_1) \right]_I - \left[-j\frac{1}{\mu_{zz}} u_1 \partial_{x'}(\Omega u_1^a) \right]_I \quad (B7)$$

$$J_2 = \left[\frac{1}{\mu_{zz}} u_1^a \partial_{x'} u_2 \right]_I - \left[\frac{1}{\mu_{zz}} u_2 \partial_{x'} u_1^a \right]_I \quad (B8)$$

$$J_3 = [u_2^a \partial_{x'} u_1]_I - [u_1 \partial_{x'} u_2^a]_I \quad (B9)$$

with $[f]_I = f(x_2') - f(x_1')$ and where x_1' and x_2' are, respectively, the lower and upper limits of the interval I. One can easily see that, for any class of I, $J_i = 0$ for $1 \le i \le 3$, and hence, according to $(B3)$, $J = 0$ $(q.e.d.)$. In fact, if an electric wall is placed at x_k' $(k = 1, 2)$, one should have

$$u_1(x_k') = u_2(x_k') = 0 \quad (B10a)$$

$$u_1^a(x_k') = u_2^a(x_k') = 0 \quad (B10b)$$

On the other hand, if a magnetic wall is placed at x_k', one should have instead

$$\partial_{x'} u_1(x_k') = \partial_{x'} u_2(x_k') = 0 \quad (B11a)$$

$$\partial_{x'} u_1^a(x_k') = \partial_{x'} u_2^a(x_k') = 0 \quad (B11a)$$

Finally, if the hybrid mode is a surface wave, one should have $(s = 1, 2)$

$$u_s(\pm\infty) = u_s^a(\pm\infty) = 0 \quad (B10a)$$

$$\partial_{x'} u_s(\pm\infty) = \partial_{x'} u_s^a(\pm\infty) = 0 \quad (B10a)$$

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