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DIFFERENTIAL CROSS SECTION OF MULTI-LAYERED LOSSY CYLINDER OF FINITE LENGTH

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1. INTRODUCTION

The electromagnetic scattering of plane waves by a lossy dielectric cylinder of finite length has been extensively investigated in literature, particularly since the resulting cross section offers many interesting possibilities for applications. Various numerical methods have been developed to treat cylinders of different shapes, sizes and constituencies [1–2]. Bussey and Richmond derived expressions for the scattering electromagnetic fields from dielectric circular infinite-length multilayer cylinders at normal incidence [3].

The present investigation deals with the scattering from circular multilayer finite-length cylinder at oblique incidence and an extension of the first author's previous work [4] to multilayers. The analysis is based on the approximation that the internal fields induced within the finite-length cylinders are the same as those induced within the infinite-length cylinders having the same permittivity, orientation and diameter [4]. These internal fields are then used to calculate the dyadic scattering amplitudes in terms of the physical dimensions, orientation and dielectric properties of the cylinder. The results apply when the minimum dimension of the cylinder's cross section is small compared with both the wavelength and the cylinder length, although the length needs not be small compared with wavelength. The scattered field relations are used to obtain the differential cross sections. The computed results are obtained, compared with the results given in literature and good agreement was obtained for TE and TM polarizations.

The organization of the paper is as follows. In section 2, the problem is formulated and ref. [3] is extended to oblique incidence. Scattering amplitudes are determined in section 3 for finite length multilayered cylinder. In section 4 scattering amplitude for cylinder made of magnetic materials is briefly considered. Numerical results and discussions are presented in section 5. At the end necessary appendixes are included.

A time dependence of $\exp(j\omega t)$ has been assumed and suppressed.

2. BASIC SCATTERING FORMULATION

We consider a planar electromagnetic wave to be oblique incident upon a dielectric multi-layered cylinder of finite length having the m th layer permittivity ε_m, permeability $\mu_m = \mu_o$, radius ρ_m and length ℓ. The geometry of the problem is shown in figures (1) and (2). We represent the incident electric field (assumed to be of unit amplitude) by

$$\underline{\mathbf{E}}_{\mathbf{i}} = \underline{\mathbf{q}}\exp\{-jk_o\underline{\mathbf{i}}\cdot\underline{\mathbf{r}}\} \tag{1}$$

where $\underline{\mathbf{q}}$ is the unit polarization vector, $\underline{\mathbf{i}}$ is the unit vector in the direction of incident wave, $\underline{\mathbf{r}}$ is a vector from the origin to the observation point and $k_o = \omega\sqrt{\mu_o\varepsilon_o}$ is the free space wave number.

The dyadic scattering amplitude $\underline{\underline{\mathbf{f}}}$ describes the far field behavior of the scattered field $\underline{\mathbf{E}}_{\mathbf{s}}$ and is defined implicitly by the relation

$$\underline{\mathbf{E}}_{\mathbf{s}} = \left[\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}},\underline{\mathbf{i}})\cdot\underline{\mathbf{q}}\right]\exp\{-jk_o\underline{\mathbf{o}}\cdot\underline{\mathbf{r}}\}/|\underline{\mathbf{r}}| \tag{2}$$

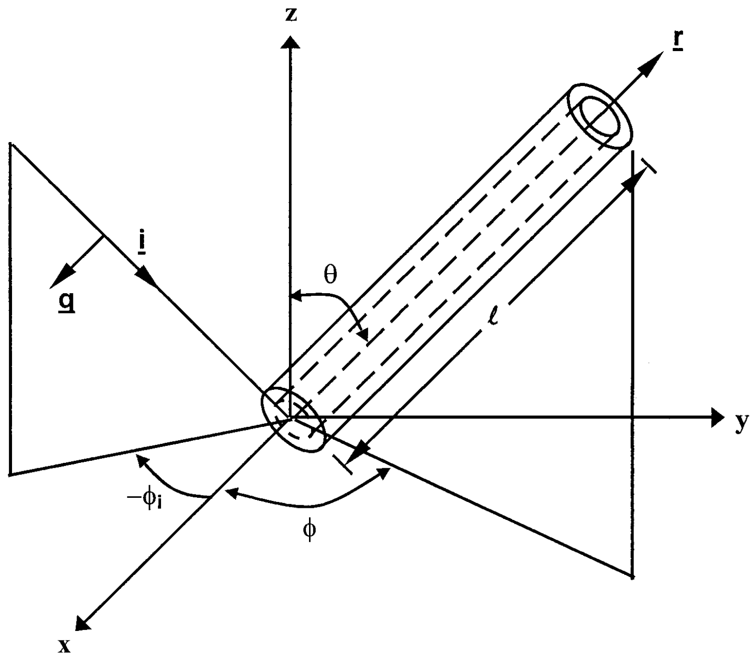


Figure 1. The scattering geometry in the principal coordinate system.

where, in addition to the variables already defined, $\underline{\mathbf{o}}$ is the unit vector in the direction of scattered wave propagation.

The dyadic scattering amplitude $\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}})$ can be related to the electric field induced within the cylinder $\underline{\underline{\mathbf{E}}}_{int}$ by [4].

$$\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = A \int_V \chi_m(\underline{\underline{\mathbf{I}}} - \underline{\mathbf{o}}\,\underline{\mathbf{o}}) \cdot \underline{\underline{\mathbf{E}}}_{int}(\underline{\mathbf{r}}') \cdot e^{jk_o(\underline{\mathbf{o}} \cdot \underline{\mathbf{r}}')} \underline{\mathbf{dr}'} \tag{3}$$

where, in addition to the previously defined variables, $A = k_o^2/4\pi$, χ_m is the relative susceptibility of the m th cylinder layer, $\underline{\underline{\mathbf{I}}}$ is a unit dyadic, $\underline{\underline{\mathbf{E}}}_{int}(\underline{\mathbf{r}}')$ is the internal dyadic electric field within the cylinders at localized coordinate system as seen in fig. (2). The integration is carried out over the volume of the cylinder. ρ_M is the radius of the outermost cylinder and M is the total number of layers beginning from $m = 1$. The localized coordinate system (x', y', z') is related to the principal frame (x, y, z) through the Euler angles (θ, ϕ, γ) of rotation. Because the internal fields within the finite length cylinders due

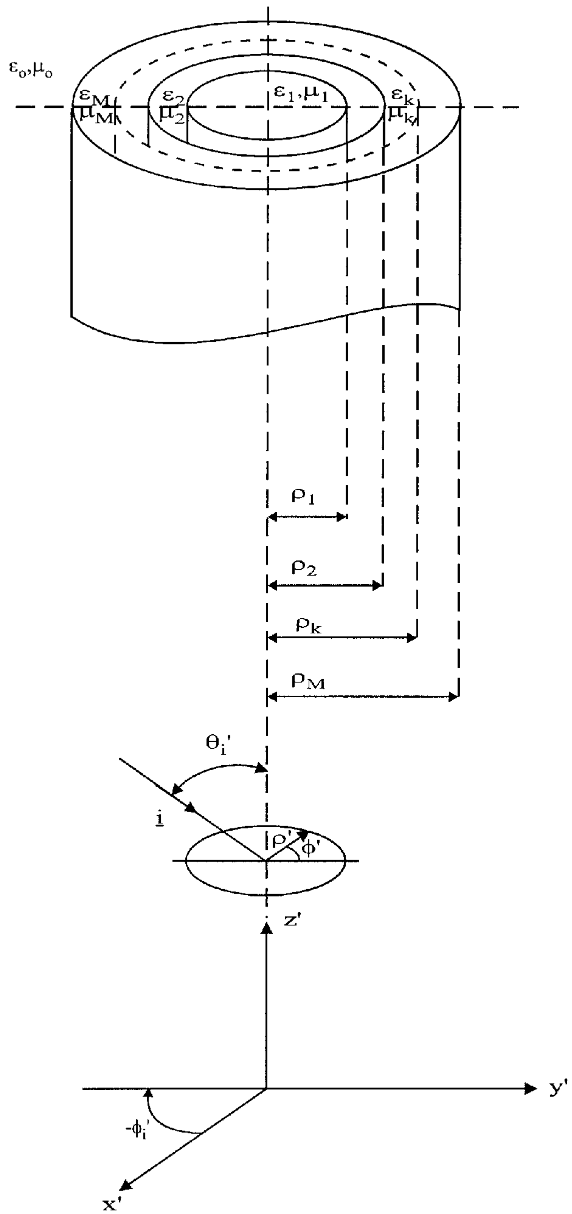


Figure 2. The scattering geometry in the primed coordinate system fixed to the cylinder axis and the cross section of the cylinder.

to an oblique incident are not known exactly, they are approximated by the internal fields within the infinite length cylinders of equal and respective diameters, permittivities and orientation [4]. In order to derive the internal electric field induced within infinite-length dielectric cylinders of circular cross-section due to an oblique incident, we assume that the electromagnetic wave can be expressed using dyadic notation as

$$\underline{\underline{\mathbf{E}}}_{int} = \underline{\mathbf{E}}_{int}^{TE}\underline{\mathbf{h}}' + \underline{\mathbf{E}}_{int}^{TM}\underline{\mathbf{v}}' = \underline{\mathbf{E}}^{h}\underline{\mathbf{h}}' + \underline{\mathbf{E}}^{v}\underline{\mathbf{v}}' \quad (4)$$

where $\underline{\mathbf{E}}_{int}^{TE}$ and $\underline{\mathbf{E}}_{int}^{TM}$ are the TE and TM electric fields induced within the cylinder when the incident electric field (1) is polarized normal to $(\underline{\mathbf{q}} = \underline{\mathbf{h}}')$ and within $(\underline{\mathbf{q}} = \underline{\mathbf{v}}')$, respectively, the local plane of incidence. More explicitly, with reference to the localized coordinate system affixed to the cylinder (x', y', z'), the internal electric fields, $\underline{\mathbf{E}}_{int}^{TE}$ and $\underline{\mathbf{E}}_{int}^{TM}$, are obtained by applying the boundary conditions on each layer boundary of the multi-layered cylinder. The total internal fields, $\underline{\mathbf{E}}_{int}^{TE}$ and $\underline{\mathbf{E}}_{int}^{TM}$ can be expressed by the usual complete set as

$$\underline{\underline{\mathbf{E}}}_{int}\left(\rho'_{m-1} < \rho' < \rho'_m\right) = \sum_q \underline{\underline{\mathbf{E}}}_m^q \quad , \quad \underline{\mathbf{q}} \in \{\underline{\mathbf{h}}, \underline{\mathbf{v}}\} \quad , \quad \rho'_o = 0$$

with

$$\begin{aligned}
\underline{\underline{\mathbf{E}}}_m^q = \sum_{n=-\infty}^{+\infty} \Bigg\{ & \left[A_{mn}^q J_n(\lambda_m\rho') + B_{mn}^q N_n(\lambda_m\rho')\right] \underline{\mathbf{z}}\,\underline{\mathbf{q}} + \\
& + \left[j\frac{k_z}{\lambda_m} A_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}} + j\frac{\omega\mu_m n}{\lambda_m} C_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}}\right] J'_n(\lambda_m\rho')\cos\phi' \\
& + \left[j\frac{k_z}{\lambda_m} A_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}} - j\frac{\omega\mu_m n}{\lambda_m} C_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}}\right] J'_n(\lambda_m\rho')\sin\phi' \\
& + \left[j\frac{k_z}{\lambda_m} B_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}} + j\frac{\omega\mu_m n}{\lambda_m} D_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}}\right] N'_n(\lambda_m\rho')\cos\phi' \\
& + \left[j\frac{k_z}{\lambda_m} B_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}} - j\frac{\omega\mu_m n}{\lambda_m} D_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}}\right] N'_n(\lambda_m\rho')\sin\phi' \\
& + \left[\frac{\omega\mu_m n}{\lambda_m^2} C_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}} - \frac{k_z n}{\lambda_m} A_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}}\right] \frac{1}{\rho'} J_n(\lambda_m\rho')\cos\phi' \\
& + \left[\frac{\omega\mu_m n}{\lambda_m^2} C_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}} - \frac{k_z n}{\lambda_m} A_{mn}^q \underline{\mathbf{x}}\underline{\mathbf{q}}\right] \frac{1}{\rho'} J_n(\lambda_m\rho')\sin\phi'
\end{aligned}$$

$$+\left[\frac{\omega\mu_m n}{\lambda_m^2}D_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}} - \frac{k_z n}{\lambda_m}B_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}}\right]\frac{1}{\rho'}N_n(\lambda_m\rho')\cos\phi'$$
$$\left.+\left[\frac{\omega\mu_m n}{\lambda_m^2}D_{mn}^q \underline{\mathbf{y}}\,\underline{\mathbf{q}} - \frac{k_z n}{\lambda_m}B_{mn}^q \underline{\mathbf{x}}\,\underline{\mathbf{q}}\right]\frac{1}{\rho'}N_n(\lambda_m\rho')\sin\phi'\right\} e^{jn\overline{\phi}_i'}\, e^{jk_z' z'} \tag{5}$$

where $\overline{\phi}_i' = \phi - \phi_i$, $k_z' = k_o \cos\theta_i'$ and

$$\begin{bmatrix} A_{mn}^q \\ B_{mn}^q \\ C_{mn}^q \\ D_{mn}^q \end{bmatrix} = \prod_{k=1}^{m-1}[\beta_{kn}] \begin{bmatrix} A_{1n}^q \\ 0 \\ C_{1n}^q \\ 0 \end{bmatrix} \quad , \quad q \in \{h, v\} \tag{6a}$$

The matrix multiplication is carried on from left to right in $(6a)$. Matrix $[\beta_{kn}]$ is given in Appendix A and the coefficients A_{1n}^q and C_{1n}^q are determined to be

$$A_{1n}^v = A_{1n}^{TM} = \frac{j^{-n}}{\Delta}\Psi_{11}E_o^{TM} \quad , \quad A_{ln}^h = A_{1n}^{TE} = \frac{j^{-n}}{\Delta}\Psi_{12}H_o^{TE}$$
$$C_{1n}^v = C_{1n}^{TM} = \frac{j^{-n}}{\Delta}\Psi_{21}E_o^{TM} \quad , \quad C_{1n}^h = C_{1n}^{TE} = \frac{j^{-n}}{\Delta}\Psi_{22}H_o^{TE}$$

with

$$[\Psi] = \begin{bmatrix} \gamma_{33} - j\gamma_{43} & -\gamma_{13} + j\gamma_{23} \\ -\gamma_{31} + j\gamma_{41} & \gamma_{11} - j\gamma_{21} \end{bmatrix}$$

$$\Delta = (\gamma_{11} - j\gamma_{21})(\gamma_{33} - j\gamma_{43}) - (\gamma_{13} - j\gamma_{23})(\gamma_{31} - j\gamma_{41}) \tag{6b}$$

$$[\gamma] = \prod_{k=1}^{M}[\beta_{kn}]$$

The primes over the Bessel and the Neumann functions in (5) mean derivative with respect to the argument. The total electric field, within the multilayered two dimensional cylinder with oblique angle of incidence, in dyadic form can be expressed as

$$\begin{aligned}
&\underline{\underline{\mathbf{E}}}_{int}(\rho'_{m-1} < \rho' < \rho'_m) = \\
&\sum_{n=-\infty}^{+\infty} \Big\{ \underline{\underline{\mathbf{a}}}_{mn} J_n(\lambda_m \rho') + \underline{\underline{\mathbf{b}}}_{mn} N_n(\lambda_m \rho') \\
&\quad + \left[\underline{\underline{\mathbf{a}}}^c_{mn} \cos\phi' + \underline{\underline{\mathbf{a}}}^s_{mn} \sin\phi' \right] \frac{1}{\rho'} J_n(\lambda_m \rho') \\
&\quad + \left[\underline{\underline{\mathbf{b}}}^c_{mn} \cos\phi' + \underline{\underline{\mathbf{b}}}^s_{mn} \sin\phi' \right] \frac{1}{\rho'} N_n(\lambda_m \rho') \\
&\quad + \left[\underline{\underline{\mathbf{c}}}^c_{mn} \cos\phi' + \underline{\underline{\mathbf{c}}}^s_{mn} \sin\phi' \right] J'_n(\lambda_m \rho') \\
&\quad + \left[\underline{\underline{\mathbf{d}}}^c_{mn} \cos\phi' + \underline{\underline{\mathbf{d}}}^s_{mn} \sin\phi' \right] N'_n(\lambda_m \rho') \Big\} e^{jn\phi'} e^{jk_z z'}
\end{aligned} \tag{7}$$

The expressions for the dyadics $\underline{\underline{\mathbf{a}}}$, $\underline{\underline{\mathbf{b}}}$, $\underline{\underline{\mathbf{a}}}^k$, $\underline{\underline{\mathbf{b}}}^k$, $\underline{\underline{\mathbf{c}}}^k$, $\underline{\underline{\mathbf{d}}}^k$, $k \in \{c, s\}$ are given in Appendix B.

3. SCATTERING AMPLITUDES

Evaluation of Eq. (3) requires the integration of the internal dyadic electric field $\underline{\underline{\mathbf{E}}}_{int}$ within the volume which defines the cylinder. Substitution of Eq. (7) for $\underline{\underline{\mathbf{E}}}_{int}$ in Eq. (3) and following the procedure given in [4] yields the dydadic scattering amplitude as

$$\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = (\underline{\underline{\mathbf{I}}} - \underline{\mathbf{o}}\underline{\mathbf{o}}) \pi I_1 \sum_{n=-\infty}^{+\infty} \underline{\underline{\mathbf{I}}}_n j^n e^{jn(\phi_i - \psi)}$$

where

$$\begin{aligned}
I_1 &= \ell \sin c(jk_o \alpha \ell / 2) \\
\underline{\underline{\mathbf{I}}}_n &= \sum_{m=1}^{M} (\varepsilon_{rm} - 1) \int_{\rho_{m-1}}^{\rho_m} \underline{\underline{\mathbf{I}}}_{mn}(\rho') d\rho' \quad , \quad \rho'_o = 0
\end{aligned}$$

$$
\begin{aligned}
I_{mn}(\rho') = {} & 2\left[\underline{\underline{\mathbf{a}}}_{mn} J_n(\lambda_m\rho') + \underline{\underline{\mathbf{b}}}_{mn} N_n(\lambda_m\rho')\right]\rho' J_n(\lambda_o\rho') \\
& + \left[(\underline{\underline{\mathbf{a}}}^s_{mn} + j\underline{\underline{\mathbf{a}}}^c_{mn}) J_n(\lambda_m\rho') + (\underline{\underline{\mathbf{b}}}^s_{mn} + j\underline{\underline{\mathbf{b}}}^c_{mn}) N_n(\lambda_m\rho')\right] J_{n+1}(\lambda_o\rho') e^{j\psi} \\
& + \left[(\underline{\underline{\mathbf{a}}}^s_{mn} - j\underline{\underline{\mathbf{a}}}^c_{mn}) J_n(\lambda_m\rho') + (\underline{\underline{\mathbf{b}}}^s_{mn} - j\underline{\underline{\mathbf{b}}}^c_{mn}) N_n(\lambda_m\rho')\right] J_{n-1}(\lambda_o\rho') e^{-j\psi} \\
& + \left[(\underline{\underline{\mathbf{c}}}^s_{mn} + j\underline{\underline{\mathbf{c}}}^c_{mn}) J'_n(\lambda_m\rho') + (\underline{\underline{\mathbf{d}}}^s_{mn} + j\underline{\underline{\mathbf{d}}}^c_{mn}) N'_n(\lambda_m\rho')\right] J_{n+1}(\lambda_o\rho')\rho' e^{j\psi} \\
& + \left[(\underline{\underline{\mathbf{c}}}^s_{mn} - j\underline{\underline{\mathbf{c}}}^c_{mn}) J'_n(\lambda_m\rho') + (\underline{\underline{\mathbf{d}}}^s_{mn} - j\underline{\underline{\mathbf{d}}}^c_{mn}) N'_n(\lambda_m\rho')\right] J_{n-1}(\lambda_o\rho')\rho' e^{-j\psi}
\end{aligned} \tag{8}
$$

or by naming the integral expressions in Eq. (8), the dyadic scattering amplitude can be expressed as

$$
\begin{aligned}
& \underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = A(\underline{\underline{\mathbf{I}}} - \underline{\mathbf{oo}})\pi I_1 \times \\
& \sum_{n=-\infty}^{+\infty} j^n e^{jn(\phi_i-\psi)} \sum_{m=1}^{M} (\varepsilon_{rm} - 1)\Bigg\{ 2\left[\underline{\underline{\mathbf{a}}}_{mn} I_{1mn} + \underline{\underline{\mathbf{b}}}_{mn} I_{2mn}\right] \\
& \quad + \left[\left(\underline{\underline{\mathbf{a}}}^s_{mn} + j\underline{\underline{\mathbf{a}}}^c_{mn}\right) I_{3mn} + \left(\underline{\underline{\mathbf{b}}}^s_{mn} + j\underline{\underline{\mathbf{b}}}^c_{mn}\right) I_{4mn}\right] e^{j\psi} \\
& \quad + \left[\left(\underline{\underline{\mathbf{a}}}^s_{mn} - j\underline{\underline{\mathbf{a}}}^c_{mn}\right) I_{5mn} + \left(\underline{\underline{\mathbf{b}}}^s_{mn} - j\underline{\underline{\mathbf{b}}}^c_{mn}\right) I_{6mn}\right] e^{-j\psi} \\
& \quad + \left[\left(\underline{\underline{\mathbf{c}}}^s_{mn} + j\underline{\underline{\mathbf{c}}}^c_{mn}\right) I_{7mn} + \left(\underline{\underline{\mathbf{d}}}^s_{mn} + j\underline{\underline{\mathbf{d}}}^c_{mn}\right) I_{8mn}\right] e^{j\psi} \\
& \quad + \left[\left(\underline{\underline{\mathbf{c}}}^s_{mn} - j\underline{\underline{\mathbf{c}}}^c_{mn}\right) I_{9mn} + \left(\underline{\underline{\mathbf{d}}}^s_{mn} - j\underline{\underline{\mathbf{d}}}^c_{mn}\right) I_{10mn}\right] e^{-j\psi} \Bigg\}
\end{aligned} \tag{9}
$$

where,

$$
\begin{aligned}
\alpha &= \beta_3 + \cos\theta'_i \\
\lambda_s &= k_o\beta \\
\psi &= \arctan(\beta_2/\beta_1)
\end{aligned}
$$

and

$$
\begin{aligned}
\beta_1 &= \cos\overline{\phi_s}\sin\theta_s \\
\beta_2 &= \sin\theta_s\cos\theta\sin\overline{\phi_s} - \cos\theta_s\sin\theta \\
\beta_3 &= \sin\theta_s\sin\theta\sin\overline{\phi_s} + \cos\theta_s\cos\theta \\
\beta &= \sqrt{\beta_1^2 + \beta_2^2} \\
\overline{\phi_s} &= \phi - \phi_s
\end{aligned}
$$

The integrals $I_{kmn}(k = 1, 2, \ldots, 10)$ in Eq. (9) are as follows,

$$
\begin{aligned}
I_{1mn} &= \int_{\rho_{m-1}}^{\rho_m} J_n(\lambda_m\rho')J_n(\lambda_o\rho')\rho' d\rho' \\
I_{2mn} &= \int_{\rho_{m-1}}^{\rho_m} N_n(\lambda_m\rho')J_n(\lambda_o\rho')\rho' d\rho' \\
I_{3mn} &= \int_{\rho_{m-1}}^{\rho_m} J_n(\lambda_m\rho')J_{n+1}(\lambda_o\rho') d\rho' \\
I_{4mn} &= \int_{\rho_{m-1}}^{\rho_m} N_n(\lambda_m\rho')J_{n+1}(\lambda_o\rho') d\rho' \\
I_{5mn} &= \int_{\rho_{m-1}}^{\rho_m} J_n(\lambda_m\rho')J_{n-1}(\lambda_o\rho') d\rho' \\
I_{6mn} &= \int_{\rho_{m-1}}^{\rho_m} N_n(\lambda_m\rho')J_{n-1}(\lambda_o\rho') d\rho' \\
I_{7mn} &= \int_{\rho_{m-1}}^{\rho_m} J'_n(\lambda_m\rho')J_{n+1}(\lambda_o\rho')\rho' d\rho' \\
I_{8mn} &= \int_{\rho_{m-1}}^{\rho_m} N'_n(\lambda_m\rho')J_{n+1}(\lambda_o\rho')\rho' d\rho' \\
I_{9mn} &= \int_{\rho_{m-1}}^{\rho_m} J'_n(\lambda_m\rho')J_{n-1}(\lambda_o\rho')\rho' d\rho' \\
I_{10mn} &= \int_{\rho_{m-1}}^{\rho_m} N'_n(\lambda_m\rho')J_{n-1}(\lambda_o\rho')\rho' d\rho'
\end{aligned}
\tag{10}
$$

Although these integrals cannot be evaluated in closed analytic form, the following recursion relations should prove helpful in their evaluation:

$$\begin{aligned} I_{3mn} &= \lambda_m \left[\frac{1}{2n} \left(I_{1m(n+1)} - I_{1m(n-1)} \right) + \frac{1}{\lambda_s} I_{3m(n-1)} \right] \\ I_{4mn} &= \lambda_m \left[\frac{1}{2n} \left(I_{2m(n+1)} - I_{2m(n-1)} \right) + \frac{1}{\lambda_s} I_{4m(n-1)} \right] \\ I_{5mn} &= \lambda_s \left[\frac{1}{2(n-1)} \left(I_{1mn} - I_{1m(n-2)} \right) + \frac{1}{\lambda_m} I_{5m(n-1)} \right] \\ I_{6mn} &= \lambda_s \left[\frac{1}{2(n-1)} \left(I_{2mn} - I_{2m(n-2)} \right) + \frac{1}{\lambda_m} I_{6m(n-1)} \right] \end{aligned} \tag{11}$$

The integrals I_{1mn} and I_{2mn} are analytic.

Special case of a single layer cylinder:

In order to check the correctness of the scattering amplitude, the single layer cylinder is considered here as a special case. For the single layer case $B_{1n} \equiv D_{1n} \equiv 0$ then one can easily see that

$$\underline{\underline{\mathbf{b}}}_{1n} \equiv \underline{\underline{\mathbf{b}}}_{1n}^c \equiv \underline{\underline{\mathbf{b}}}_{1n}^s \equiv \underline{\underline{\mathbf{d}}}_{1n}^c \equiv \underline{\underline{\mathbf{d}}}_{1n}^s \equiv 0$$

thus the scattering amplitude is reduced to:

$$\begin{aligned} \underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = A(\underline{\underline{\mathbf{I}}} - \underline{\mathbf{oo}})\pi I_1 \sum_{n=-\infty}^{+\infty} j^n e^{jn(\phi_i - \psi)} (\varepsilon_{r1} - 1) \\ \left\{ 2\underline{\underline{\mathbf{a}}}_{1n} I_{1,1n} + \left(\underline{\underline{\mathbf{a}}}_{1n}^s + j\underline{\underline{\mathbf{a}}}_{1n}^c \right) I_{3,1n} e^{j\psi} + \left(\underline{\underline{\mathbf{a}}}_{1n}^s - j\underline{\underline{\mathbf{a}}}_{1n}^c \right) I_{5,1n} e^{-j\psi} \right. \\ \left. + (\underline{\underline{\mathbf{c}}}_{1n}^s + j\underline{\underline{\mathbf{c}}}_{1n}^c) I_{7,1n} e^{j\psi} + (\underline{\underline{\mathbf{c}}}_{1n}^s - j\underline{\underline{\mathbf{c}}}_{1n}^c) I_{9,1n} e^{-j\psi} \right\} \end{aligned}$$

Taking into consideration the following conversions in notations, the above expression is exactly the same as that given by ref. [4] where the experimental validation is also shown.

$$I_{1,1n} \rightarrow I_3^{(n)} \ , \ I_{7,1n} \rightarrow I_4^{(n)} \ , \ I_{9,1n} \rightarrow I_5^{(n)} \ , \ I_{3,1n} \rightarrow I_6^{(n)} \ , \ I_{5,1n} \rightarrow I_7^{(n)}$$

$$\underline{\underline{\mathbf{a}}}_{1n} \rightarrow \overline{\mathbf{a}}_o \ , \ \underline{\underline{\mathbf{a}}}_{1n}^s \rightarrow \overline{\mathbf{b}}_o \ , \ \underline{\underline{\mathbf{a}}}_{1n}^c \rightarrow \overline{\mathbf{a}}_2 \ , \ \underline{\underline{\mathbf{c}}}_{1n}^s \rightarrow \overline{\mathbf{b}}_1 \ , \ \underline{\underline{\mathbf{c}}}_{1n}^c \rightarrow \overline{\mathbf{a}}_1$$

The solution given by Eq. (9) applies for arbitrary dielectric constants, sizes and orientations. However, the solution should be applied with caution near edge-on incidence.

The behavior of the dyadic scattering amplitude $\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}})$ is perhaps best understood by considering those components associated with horizontal $(\underline{\mathbf{h}}_i, \underline{\mathbf{h}}_s)$ and vertical $(\underline{\mathbf{v}}_i, \underline{\mathbf{v}}_s)$ polarization of the incident and scattered waves respectively. They are given in terms of the incidence and scattering angles as

$$\begin{aligned} \underline{\mathbf{i}} &= -\sin\theta_i[\cos\phi_i\underline{\mathbf{x}} + \sin\phi_i\underline{\mathbf{y}}] - \cos\theta_i\underline{\mathbf{z}} \\ \underline{\mathbf{o}} &= \sin\theta_s[\cos\phi_s\underline{\mathbf{x}} + \sin\phi_s\underline{\mathbf{y}}] + \cos\theta_s\underline{\mathbf{z}} \end{aligned} \tag{12}$$

As $(\underline{\mathbf{h}}_s, \underline{\mathbf{v}}_s, \underline{\mathbf{o}})$ and $(\underline{\mathbf{h}}_i, \underline{\mathbf{v}}_i, \underline{\mathbf{i}})$ define mutually orthogonal unit vectors, then the scattering amplitude can be written in the form

$$\underline{\underline{\mathbf{f}}} = f_{hh}\underline{\mathbf{h}}_s\underline{\mathbf{h}}_i + f_{hv}\underline{\mathbf{h}}_s\underline{\mathbf{v}}_i + f_{vh}\underline{\mathbf{v}}_s\underline{\mathbf{h}}_i + f_{vv}\underline{\mathbf{v}}_s\underline{\mathbf{v}}_i \tag{13}$$

in as much as both the incident and the scattered radio waves are plane waves. The polarization vectors are defined as,

$$\begin{aligned} \underline{\mathbf{h}}_i &= \frac{\underline{\mathbf{i}} \times \underline{\mathbf{z}}}{|\underline{\mathbf{i}} \times \underline{\mathbf{z}}|} \quad , \quad \underline{\mathbf{v}}_i = \underline{\mathbf{h}}_i \times \underline{\mathbf{i}} \\ \underline{\mathbf{h}}_s &= \frac{\underline{\mathbf{o}} \times \underline{\mathbf{z}}}{|\underline{\mathbf{o}} \times \underline{\mathbf{z}}|} \quad , \quad \underline{\mathbf{v}}_s = \underline{\mathbf{h}}_s \times \underline{\mathbf{o}} \end{aligned} \tag{14}$$

The dyadic coefficients of $\underline{\underline{\mathbf{f}}}$ are given by

$$f_{pq} = \underline{\mathbf{p}} \cdot \underline{\underline{\mathbf{f}}} \cdot \underline{\mathbf{q}} \quad , \quad \underline{\mathbf{p}} \in \{\underline{\mathbf{h}}_s, \underline{\mathbf{v}}_s\} \quad , \quad \underline{\mathbf{q}} \in \{\underline{\mathbf{h}}_i, \underline{\mathbf{v}}_i\} \tag{15}$$

The resulting components f_{pq} are obtained below as

$$\begin{aligned} f_{pq} = A\pi I_1 \sum_{n=-\infty}^{+\infty} j^n e^{jn(\phi_i-\psi)} \sum_{m=1}^{M} (\varepsilon_{r_m} - 1) \\ \Bigg\{ 2\left[\underline{\mathbf{p}} \cdot \underline{\underline{\mathbf{a}}}_{mn} \cdot \underline{\mathbf{q}} I_{1mn} + \underline{\mathbf{p}} \cdot \underline{\underline{\mathbf{b}}}_{mn} \cdot \underline{\mathbf{q}} I_{2mn}\right] \\ + \left[\underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{a}}}^s_{mn} + j\underline{\underline{\mathbf{a}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{3mn} + \underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{b}}}^s_{mn} + j\underline{\underline{\mathbf{b}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{4mn}\right] e^{j\psi} \\ + \left[\underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{a}}}^s_{mn} - j\underline{\underline{\mathbf{a}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{5mn} + \underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{b}}}^s_{mn} - j\underline{\underline{\mathbf{b}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{6mn}\right] e^{-j\psi} \\ + \left[\underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{c}}}^s_{mn} + j\underline{\underline{\mathbf{c}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{7mn} + \underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{d}}}^s_{mn} + j\underline{\underline{\mathbf{d}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{8mn}\right] e^{j\psi} \\ + \left[\underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{c}}}^s_{mn} - j\underline{\underline{\mathbf{c}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{9mn} + \underline{\mathbf{p}} \cdot \left(\underline{\underline{\mathbf{d}}}^s_{mn} - j\underline{\underline{\mathbf{d}}}^c_{mn}\right) \cdot \underline{\mathbf{q}} I_{10mn}\right] e^{j\psi} \Bigg\} \end{aligned} \tag{16}$$

Simple No.	No. of layers	$k_0\rho_m$	$\varepsilon_{rm} = \varepsilon'_{rm} - j\varepsilon''_{rm}$
1	2	$0.3\pi,\ 0.4\pi$	$67.0 - j43.0,\ 6.0 - j0.5$
2	2	$0.02\pi,\ 0.052\pi$	$7.0 - j3.5,\ 70.0 - j125.0$
3	3	$0.2\pi,\ 0.3\pi,\ 0.4\pi$	$20.0 - j10.0,\ 10.0 - j20.0,\ 5.0 - j$
4	5	$0.2\pi,\ 0.4\pi,\ 0.6\pi,\ 0.8\pi,\ \pi$	$6.0,\ 5.0,\ 4.0,\ 3.0,\ 2.0$

Table 1. Parameters for sample cylinders used by Richmond [3].

4. CYLINDERS MADE OF MAGNETIC MATERIALS

In case of magnetic materials $(\mu_m \neq \mu_o, m = 1, 2, \ldots, M)$ the scattering amplitude, when $\varepsilon_m = \varepsilon_o$, can be obtained by

$$\underline{\underline{\mathbf{f}}}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = -\frac{k^2\eta_o}{4\pi} \int\limits_v (\mu_{rm} - 1)\underline{\mathbf{o}} \times \underline{\underline{\mathbf{H}}}_{int}(\underline{\mathbf{r}}')e^{jk_o(\underline{\mathbf{o}}\cdot\underline{\mathbf{r}}')} d\underline{\mathbf{r}'} \qquad (17)$$

where μ_{rm} is the relative magnetic permeability of the m th cylinder layer, and η_o is the characteristic impedance of free space. The internal magnetic field $\underline{\underline{\mathbf{H}}}_{int}$ can be obtained by using Eq. (7) by changing A^q_{mn} with C^q_{mn} , $q \in \{v, h\}$ and vise versa and by changing B^q_{mn} with D^q_{mn} and vise versa and μ_m with $-\varepsilon_m$ in the dyadic coefficients $\underline{\underline{\mathbf{a}}}$, $\underline{\underline{\mathbf{b}}}$, $\underline{\underline{\mathbf{a}}}^k$, $\underline{\underline{\mathbf{b}}}^k$, $\underline{\underline{\mathbf{c}}}^k$, $\underline{\underline{\mathbf{d}}}^k$, $k \in \{c, s\}$ which are given in Appendix B.

5. NUMERICAL RESULTS AND DISCUSSIONS

In order to see that the results obtained are in agreement with those published results; Richmond's work [3], where $\theta_i = \pi/2$, $\theta = \phi = 0$, are used to obtain the two dimensional scattering amplitude for infinite length, multi-layered cylinders. Then Richmond's work is used to find the three dimensional differential scattering cross section (DCS) by using the relation between the three dimensional and the two dimensional radar cross sections given in [7], finally the following relation for the DCS is obtained,

$$\sigma_d^{qq}(\underline{\mathbf{o}}, \underline{\mathbf{i}}) = |f_{qq}|^2 = (\ell/\pi)^2 |T(\phi)|^2 \quad , \quad q \in \{v,\ h\} \qquad (18)$$

where $T(\phi)$ is the two dimensional scattering amplitude, in direction ϕ, calculated using the values in Richmond' s results [3]. The cylinder parameters used are given in Table (1) and the DCS obtained from

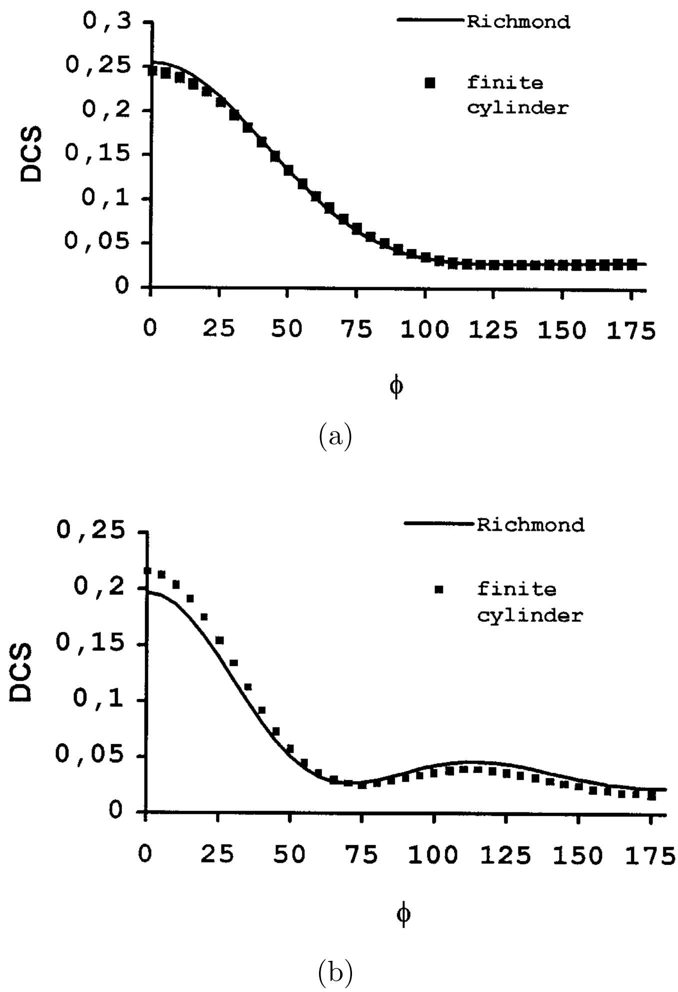


Figure 3. DCS for sample no. 1 of Table (1), obtained from results of this paper (referred to as finite cylinder) and from Richmond's work, for both vertical (TM) polarization (a) and horizontal (TE) polarization (b).

the above relation and those obtained from Eq. (16) for finite length cylinders, are plotted and compared in figures (3)–(6). As seen, the results are in excellent agreement for both TM and TE polarizations when $\theta_i = \pi/2$. In Table (1) the sample cylinders are non-magnetic.

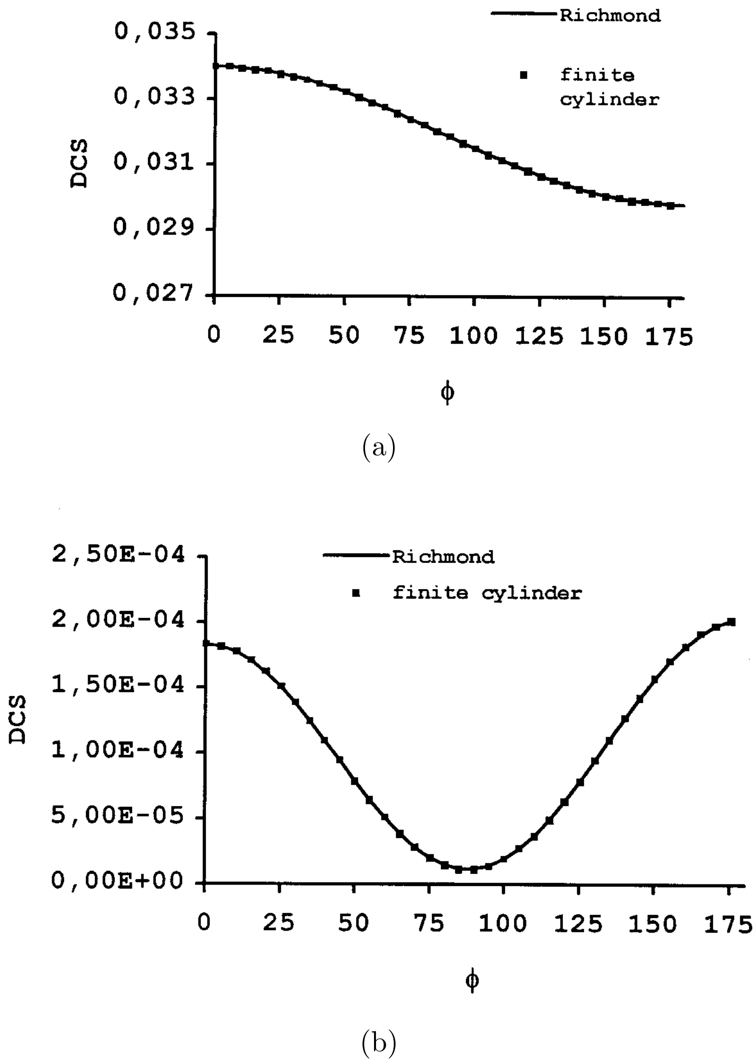


Figure 4. DCS for sample no. 2 of Table (1), for both vertical (TM) polarization (a) and horizontal (TE) polarization (b).

In figure (7) the relative error with respect to changing conductivity of a single layer cylinder is plotted for fixed frequency for both TM and TE polarizations. The cylinder parameters are: length $\ell = 0.01m$, radius $a = 0.002m$, complex relative dielectric constant $\varepsilon_r = 3.13 - j\varepsilon_r''$. The working frequency is 300 MHz. The relative error is defined

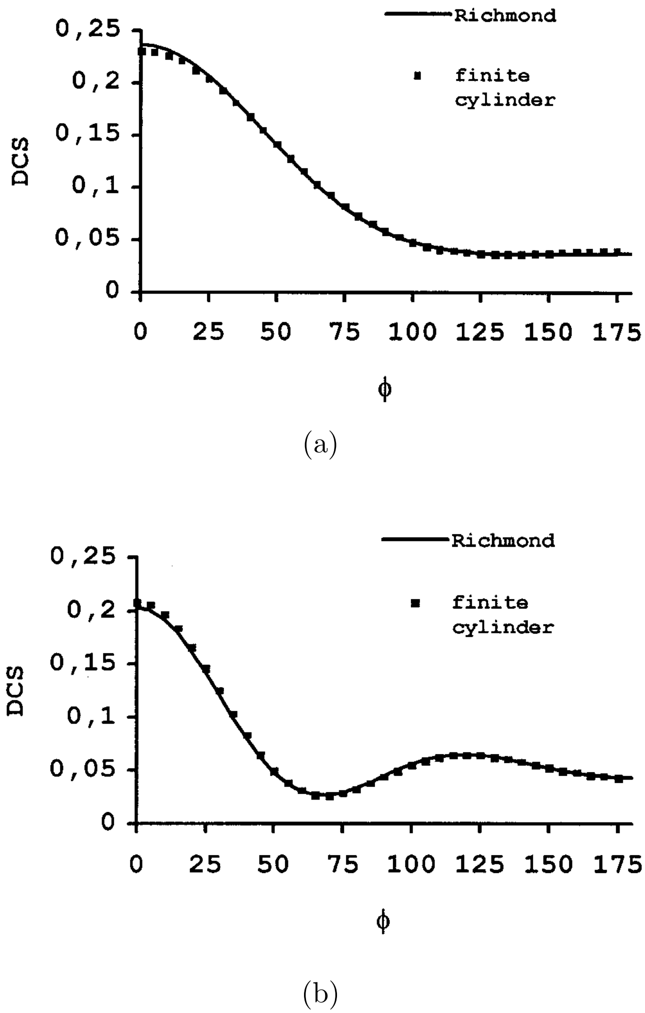
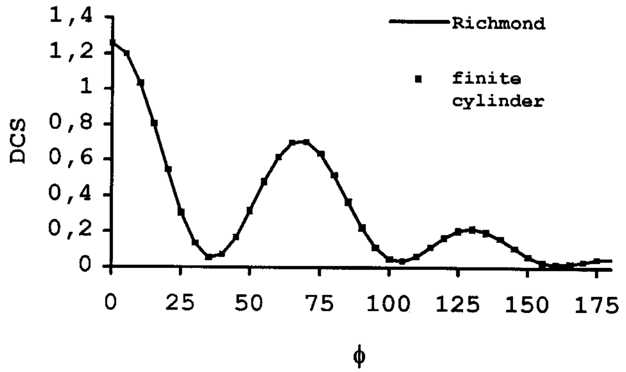


Figure 5. DCS for sample no. 3 of Table (1), for both vertical (TM) polarization (a) and horizontal (TE) polarization (b).

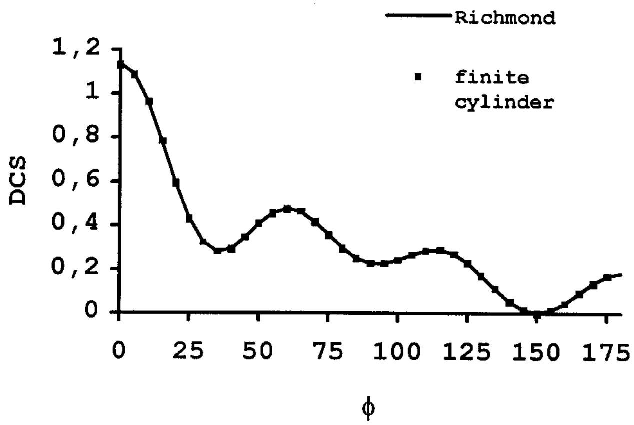
as,

$$\text{error} = \frac{|\sigma_t - \sigma_a - \sigma_s|}{\sigma_t}$$

where σ_t, σ_a and σ_s are the total, absorption and the scattering cross sections respectively defined in [8]. It is observed that as the conductivity, thus the imaginary part of the relative dielectric con-



(a)



(b)

Figure 6. DCS for sample no. 4 of Table (1), for both vertical (TM) polarization (a) and horizontal (TE) polarization (b).

stant, is increased the relative error decreases. This decrease is due to the diminishing contribution of the reflected waves at the tips of the cylinder.

In conclusion the results obtained in this work can be suitable for use in a variety of applications such as in remote sensing of vegetation as a model for trunks and branches or in remote sensing of clouds as a model for ice needles.

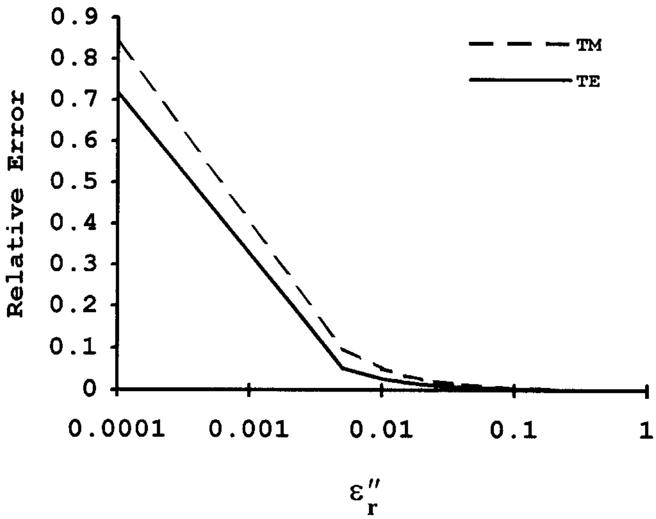


Figure 7. Relative error versus imaginary part of dielectric constant for a single layer cylinder at fixed frequency.

APPENDIX A

To determine the fields inside a multi-layer cylinder of M layers, boundary conditions must be solved on each of the M boundaries. In each layer the fields are represented by the complete set. The z and ϕ components of the TM and TE fields, in the mth layer are expressed respectively as

$$E_z^m = \sum_{n=-\infty}^{+\infty} \{A_{mn} J_n(\lambda_m \rho) + B_{mn} N_n(\lambda_m \rho)\} e^{jn\phi} e^{jk_z z}$$

$$H_z^m = \sum_{n=-\infty}^{+\infty} \{C_{mn} J_n(\lambda_m \rho) + D_{mn} N_n(\lambda_m \rho)\} e^{jn\phi} e^{jk_z z}$$

where

$$\lambda_m^2 = k_o^2 \varepsilon_{rm} \mu_{rm} - k_z^2 \ , \quad k_z = k_o \cos\theta_i$$

and from Maxwell's equations.

$$E_\phi^m = \sum_{n=-\infty}^{+\infty} \left\{ \begin{array}{l} \dfrac{-nk_z}{\lambda_m^2 \rho} \left[A_{mn} J_n(\lambda_m \rho) + B_{mn} N_n(\lambda_m \rho) \right] \\ + j \dfrac{\omega \mu_m}{\lambda_m} \left[C_{mn} J_n'(\lambda_m \rho) + D_{mn} N_n'(\lambda_m \rho) \right] \end{array} \right\} e^{jn\phi} e^{jk_z z}$$

$$H_\phi^m = \sum_{n=-\infty}^{+\infty} \left\{ \begin{array}{l} \dfrac{-nk_z}{\lambda_m^2 \rho} \left[C_{mn} J_n(\lambda_m \rho) + D_{mn} N_n(\lambda_m \rho) \right] \\ - j \dfrac{\omega \epsilon_m}{\lambda_m} \left[A_{mn} J_n'(\lambda_m \rho) + B_{mn} N_n'(\lambda_m \rho) \right] \end{array} \right\} e^{jn\phi} e^{jk_z z}$$

are obtained. At the boundary $\rho = \rho_m$ the following boundary conditions must be satisfied for each m.

$$\begin{aligned} E_z^{m+1} &= E_z^m \\ E_\phi^{m+1} &= E_\phi^m \\ H_z^{m+1} &= H_z^m \\ H_\phi^{m+1} &= H_\phi^m \end{aligned}$$

Imposing these conditions on the field expressions we have the following equality in matrix form.

$$\begin{bmatrix} J_n(\chi_{m+1}) & N_n(\chi_{m+1}) & 0 & 0 \\ \frac{-nk_z}{\lambda_{m+1}^2 \rho_m} J_n(\chi_{m+1}) & \frac{-nk_z}{\lambda_{m+1}^2 \rho_m} N_n(\chi_{m+1}) & j\frac{\omega\mu_{m+1}}{\lambda_{m+1}} J_n'(\chi_{m+1}) & j\frac{\omega\mu_{m+1}}{\lambda_{m+1}} N_n'(\chi_{m+1}) \\ 0 & 0 & J_n(\chi_{m+1}) & N_n(\chi_{m+1}) \\ -j\frac{\omega\varepsilon_{m+1}}{\lambda_{m+1}} J_n'(\chi_{m+1}) & -j\frac{\omega\varepsilon_{m+1}}{\lambda_{m+1}} N_n'(\chi_{m+1}) & \frac{-nk_z}{\lambda_{m+1}^2 \rho_m} J_n(\chi_{m+1}) & \frac{-nk_z}{\lambda_{m+1}^2 \rho_m} N_n(\chi_{m+1}) \end{bmatrix} \begin{bmatrix} A_{m+1} \\ B_{m+1} \\ C_{m+1} \\ D_{m+1} \end{bmatrix}$$

$$= \begin{bmatrix} J_n(\chi_m) & N_n(\chi_m) & 0 & 0 \\ \frac{-nk_z}{\lambda_m^2 \rho_m} J_n(\chi_m) & \frac{-nk_z}{\lambda_m^2 \rho_m} N_n(\chi_m) & j\frac{\omega\mu_m}{\lambda_m} J_n'(\chi_m) & j\frac{\omega\mu_m}{\lambda_m} N_n'(\chi_m) \\ 0 & 0 & J_n(\chi_m) & N_n(\chi_m) \\ -j\frac{\omega\varepsilon_m}{\lambda_m} J_n'(\chi_1) & -j\frac{\omega\varepsilon_m}{\lambda_m} N_n'(\chi_m) & \frac{-nk_z}{\lambda_m^2 \rho_m} J_n(\chi_m) & \frac{-nk_z}{\lambda_m^2 \rho_m} N_n(\chi_m) \end{bmatrix} \times$$

$$\begin{bmatrix} A_m \\ B_m \\ C_m \\ D_m \end{bmatrix}$$

The solution to the above matrix equation will give the coefficients of the fields in one layer in terms of the coefficients of the field in one layer inner. To solve this equation one has to determine the elements of the matrix $[\beta]$ below.

$$\begin{bmatrix} A_{m+1} \\ B_{m+1} \\ C_{m+1} \\ D_{m+1} \end{bmatrix} = [\beta] \begin{bmatrix} A_m \\ B_m \\ C_m \\ D_m \end{bmatrix}$$

At the innermost layer for a physical solution, the coefficients of the Neumann functions, seen in the field expressions, must be zero. Thus one has $B_{1n} = D_{1n} = 0$.

A_{1n} and C_{1n} are obtained by Eq. $(6a)$ and $(6b)$. The elements of the matrix $[\beta]$ are obtained as

$$\beta_{11} = \frac{\pi}{2} X_{m+1} \left[N_n'(X_{m+1}) J_n(X_m) - \frac{\lambda_{m+1}}{\lambda_m} \frac{\varepsilon_m}{\varepsilon_{m+1}} N_n(X_{m+1}) J_n'(X_m) \right]$$

$$\beta_{12} = \frac{\pi}{2} X_{m+1} \left[N_n'(X_{m+1}) N_n(X_m) - \frac{\lambda_{m+1}}{\lambda_m} \frac{\varepsilon_m}{\varepsilon_{m+1}} N_n(X_{m+1}) N_n'(X_m) \right]$$

$$\beta_{13} = j \frac{\pi}{2} \frac{n k_z \lambda_{m+1}^2}{\omega \varepsilon_{m+1}} \left(\frac{1}{\lambda_m^2} - \frac{1}{\lambda_{m+1}^2} \right) N_n(X_{m+1}) J_n(X_m)$$

$$\beta_{14} = j \frac{\pi}{2} \frac{n k_z \lambda_{m+1}^2}{\omega \varepsilon_{m+1}} \left(\frac{1}{\lambda_m^2} - \frac{1}{\lambda_{m+1}^2} \right) N_n(X_{m+1}) N_n(X_m)$$

$$\beta_{21} = \frac{\pi}{2} X_{m+1} \left[\frac{\lambda_{m+1}}{\lambda_m} \frac{\varepsilon_m}{\varepsilon_{m+1}} J_n(X_{m+1}) J_n'(X_m) - J_n'(X_{m+1}) J_n(X_m) \right]$$

$$\beta_{22} = \frac{\pi}{2} X_{m+1} \left[\frac{\lambda_{m+1}}{\lambda_m} \frac{\varepsilon_m}{\varepsilon_{m+1}} J_n(X_{m+1}) N_n'(X_m) - J_n'(X_{m+1}) N_n(X_m) \right]$$

$$\beta_{23} = j \frac{\pi}{2} \frac{n k_z \lambda_{m+1}^2}{\omega \varepsilon_{m+1}} \left(\frac{1}{\lambda_{m+1}^2} - \frac{1}{\lambda_m^2} \right) J_n(X_{m+1}) J_n(X_m)$$

$$\beta_{24} = j \frac{\pi}{2} \frac{n k_z \lambda_{m+1}^2}{\omega \varepsilon_{m+1}} \left(\frac{1}{\lambda_{m+1}^2} - \frac{1}{\lambda_m^2} \right) J_n(X_{m+1}) N_n(X_m)$$

$$\beta_{31} = j\frac{\pi}{2}\frac{nk_z\lambda_{m+1}^2}{\omega\mu_{m+1}}\left(\frac{1}{\lambda_{m+1}^2} - \frac{1}{\lambda_m^2}\right)N_n(X_{m+1})J_n(X_m)$$
$$\beta_{32} = j\frac{\pi}{2}\frac{nk_z\lambda_{m+1}^2}{\omega\mu_{m+1}}\left(\frac{1}{\lambda_{m+1}^2} - \frac{1}{\lambda_m^2}\right)N_n(X_{m+1})N_n(X_m)$$
$$\beta_{33} = \frac{\pi}{2}X_{m+1}\left[N_n'(X_{m+1})J_n(X_m) - \frac{\lambda_{m+1}}{\lambda_m}\frac{\mu_m}{\mu_{m+1}}N_n(X_{m+1})J_n'(X_m)\right]$$
$$\beta_{34} = \frac{\pi}{2}X_{m+1}\left[N_n'(X_{m+1})N_n(X_m) - \frac{\lambda_{m+1}}{\lambda_m}\frac{\mu_m}{\mu_{m+1}}N_n(X_{m+1})N_n'(X_m)\right]$$
$$\beta_{41} = j\frac{\pi}{2}\frac{nk_z\lambda_{m+1}^2}{\omega\mu_{m+1}}\left(\frac{1}{\lambda_m^2} - \frac{1}{\lambda_{m+1}^2}\right)J_n(X_{m+1})J_n(X_m)$$
$$\beta_{42} = j\frac{\pi}{2}\frac{nk_z\lambda_{m+1}^2}{\omega\mu_{m+1}}\left(\frac{1}{\lambda_m^2} - \frac{1}{\lambda_{m+1}^2}\right)J_n(X_{m+1})N_n(X_m)$$
$$\beta_{43} = \frac{\pi}{2}X_{m+1}\left[\frac{\lambda_{m+1}}{\lambda_m}\frac{\mu_m}{\mu_{m+1}}J_n(X_{m+1})J_n'(X_m) - J_n'(X_{m+1})J_n(X_m)\right]$$
$$\beta_{44} = \frac{\pi}{2}X_{m+1}\left[\frac{\lambda_{m+1}}{\lambda_m}\frac{\mu_m}{\mu_{m+1}}J_n(X_{m+1})N_n'(X_m) - J_n'(X_{m+1})N_n(X_m)\right]$$

where $X_m = \lambda_m\rho_m$ and $X_{m+1} = \lambda_{m+1}\rho_m$.

APPENDIX B

The dyadic coefficients seen in the field expressions of Eq. (7) are as follows,

$$\underline{\underline{\mathbf{a}}}_{mn} = A_{mn}^h\underline{\mathbf{z}}'\underline{\mathbf{h}}' + A_{mn}^v\underline{\mathbf{z}}'\underline{\mathbf{v}}'$$
$$\underline{\underline{\mathbf{b}}}_{mn} = B_{mn}^h\underline{\mathbf{z}}'\underline{\mathbf{h}}' + B_{mn}^v\underline{\mathbf{z}}'\underline{\mathbf{v}}'$$
$$\underline{\underline{\mathbf{a}}}_{mn}^c = \frac{n\omega\mu_m}{\lambda_m^2}\left(C_{mn}^h\underline{\mathbf{x}}'\underline{\mathbf{h}}' + C_{mn}^v\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right) - \frac{nk_z}{\lambda_m^2}\left(A_{mn}^h\underline{\mathbf{y}}'\underline{\mathbf{h}}' + A_{mn}^v\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right)$$
$$\underline{\underline{\mathbf{a}}}_{mn}^s = \frac{n\omega\mu_m}{\lambda_m^2}\left(C_{mn}^h\underline{\mathbf{y}}'\underline{\mathbf{h}}' + C_{mn}^v\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right) - \frac{nk_z}{\lambda_m^2}\left(A_{mn}^h\underline{\mathbf{x}}'\underline{\mathbf{h}}' + A_{mn}^v\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right)$$
$$\underline{\underline{\mathbf{b}}}_{mn}^c = \frac{n\omega\mu_m}{\lambda_m^2}\left(D_{mn}^h\underline{\mathbf{x}}'\underline{\mathbf{h}}' + D_{mn}^v\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right) - \frac{nk_z}{\lambda_m^2}\left(B_{mn}^h\underline{\mathbf{y}}'\underline{\mathbf{h}}' + B_{mn}^v\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right)$$
$$\underline{\underline{\mathbf{b}}}_{mn}^s = \frac{n\omega\mu_m}{\lambda_m^2}\left(D_{mn}^h\underline{\mathbf{y}}'\underline{\mathbf{h}}' + D_{mn}^v\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right) - \frac{nk_z}{\lambda_m^2}\left(B_{mn}^h\underline{\mathbf{x}}'\underline{\mathbf{h}}' + B_{mn}^v\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right)$$
$$\underline{\underline{\mathbf{c}}}_{mn}^c = j\frac{k_z}{\lambda_m}\left(A_{mn}^h\underline{\mathbf{x}}'\underline{\mathbf{h}}' + A_{mn}^v\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right) + j\frac{\omega\mu_m}{\lambda_m}\left(C_{mn}^h\underline{\mathbf{y}}'\underline{\mathbf{h}}' + C_{mn}^v\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right)$$

$$\underline{\underline{\mathbf{c}}}^s_{mn} = j\frac{k_z}{\lambda_m}\left(A^h_{mn}\underline{\mathbf{y}}'\underline{\mathbf{h}}' + A^v_{mn}\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right) - j\frac{n\omega\mu_m}{\lambda_m}\left(C^h_{mn}\underline{\mathbf{x}}'\underline{\mathbf{h}}' + C^v_{mn}\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right)$$

$$\underline{\underline{\mathbf{d}}}^c_{mn} = j\frac{k_z}{\lambda_m}\left(B^h_{mn}\underline{\mathbf{x}}'\underline{\mathbf{h}}' + B^v_{mn}\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right) + j\frac{\omega\mu_m}{\lambda_m}\left(D^h_{mn}\underline{\mathbf{y}}'\underline{\mathbf{h}}' + D^v_{mn}\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right)$$

$$\underline{\underline{\mathbf{d}}}^s_{mn} = j\frac{k_z}{\lambda_m}\left(B^h_{mn}\underline{\mathbf{y}}'\underline{\mathbf{h}}' + B^v_{mn}\underline{\mathbf{y}}'\underline{\mathbf{v}}'\right) - j\frac{n\omega\mu_m}{\lambda_m}\left(D^h_{mn}\underline{\mathbf{x}}'\underline{\mathbf{h}}' + D^v_{mn}\underline{\mathbf{x}}'\underline{\mathbf{v}}'\right)$$

The polarization vectors $\underline{\mathbf{v}}'$ and $\underline{\mathbf{h}}'$ can be derived from Eq. (14) for the local coordinate system.

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