Progress In Electromagnetics Research, *PIER 19, 223–240, 1998*

HIGH PRESSURE ELECTROMAGNETIC FRACTAL BEHAVIOUR OF SEDIMENTARY ROCKS

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1. INTRODUCTION

The transport properties of porous media are dependent mainly on pore structure. The changes in stress conditions that affect the pore structure and pore wall/fluid interactions are also expected to have measurable effects on electrical and hydraulic properties, since these are affected by reduction of the cross section available for conduction, and any changes in length of the flow passage. In the case of porous sedimentary rocks, electrical conduction is mainly electrolytic at low frequencies. The conducting medium is an aqueous solution of salts, distributed in complicated manner within the pore structure of the rock. Having accepted this description of the flow of fluid and electrical current in saturated rock, it follows that ions flowing through the rock under a potential gradient follow almost the same path as fluids flowing under a pressure gradient. Unfortunately, although electrical properties are extremely useful in being highly sensitive to subtle changes within the earth, they are also complicated to interpret due to a great number of parameters which can influence equivalent changes. One key question in well log analysis concerns the distribution of the conductive clay minerals in porous reservoir sandstones and the problem of resulting correlating hydrocarbon saturation of reservoir rocks to their bulk electrical conductivity. The complex conductivity parameter is related to the microgeometry of the pore space in addition to the petrophysical parameters. However, some electrical properties are related to the distribution of charges or to the accumulation of charges at discontinuities during charge transport. Although these properties are almost independent of frequency for water and most clay minerals, the composite properties vary appreciably with frequency. The effective conductivity as a slowly increasing function of frequency has been interpreted as being caused by geometric or textural heterogeneties in the rock system. These characteristics have also been related to electrochemical processes developed at the interfaces between clay minerals and electrolyte solutions. Interactions between charged clays and aqueous electrolytes give rise to an ionic double layer around these particles. Polarisation of such a layer by an applied electric field has been highlighted as the main mechanism for the anomalous behaviour observed in porous rocks with an appreciable clay content. In addition to the double layer, electrode-sample interface polarisation is also observed during these measurements.

1.1 Experimental Technique

In case of electrical measurements, the input parameter is the current and the output parameter is the voltage. The transfer impedance is measured directly using sinusoidal current excitation and monitoring the phase and gains of the resulting voltage response. This has been done at several different frequencies, resulting in an impedance spectrum displaying the frequency characteristics of each rock type. These frequency signatures are then used to describe the electrical behaviour of the rock. The approach here is to synthesise the transfer impedances of different sedimentary rock types with equivalent circuits composed of discrete and distributed linear, passive elements. The complex electrical conductivities for cylindrical rock samples 54mm in diameter and 61mm in length have been measured in stages initially to a hydrostatic effective stress of 20MPa over the frequency range 100Hz–30kHz. The consequence of narrow frequency band and few observations for gain and phase resulted in an inadequate explanation of the fractal parameters for the initial measured complex resistivities. This led to the realisation that a broad frequency band is needed for robust estimates of the model parameters. Rocha [1995] has mentioned that in her model, low frequencies are related to the fractal geometry of the rough grain surfaces. Scott et al. [1967], using a two-electrode probe for measuring electrical conductivity, showed that electrode-sample interface polarisation can give an effect of the same order of magnitude as the intrinsic electrode and double layer polarisation. This effect has been overcome to a great extent, here by introducing a stepwise approach [Malik, 1997]. Subsequently electrical conductivities were measured for increasing hydrostatic stresses to 40MPa over a lower and wider frequency range (10Hz–100kHz). The frequencies below 100Hz helped in modelling the effects of the rough surface of blocking grains. Three odd harmonics are provided for each frequency decade to give a discrete approximation to the actual decade response for the physical processes within the rocks. The phase measurements were observed to be very sensitive to effective stress changes and were found to fluctuate for each step increase in stress. Measurements were therefore made after their stabilisation over a period of time. This sensitivity of phase found at all stress levels, has been attributed to fluid movement with stress variation which stabilises over a period of time.

It was observed that all fractal parameters respond differently to low, intermediate and elevated stress levels. These changes in fractal

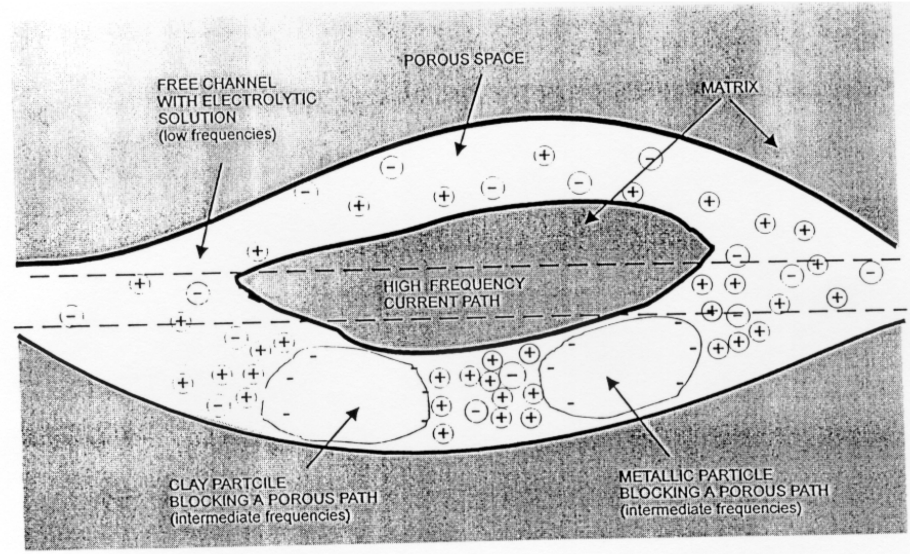

Figure 1a. Basic blocked and un-blocked paths for electrical conduction in rocks.

parameters are attributed to changes in the fractal pore surfaces which are sensitive to fractional changes in pore geometry, in contrast to the electrical conductivity curves. These fractal parameters vary from sample to sample, which in turn show their response to the mineralogical constituents of the rocks, the spatial distribution of the conductive clay-minerals, grain sizes and their distribution and pore geometry.

1.2 Fractal Model

Phenomenological models represented by a single conduit for fluid conduction are very basic and are not appropriate for describing the complicated behaviour of porous sedimentary rocks. A fractal model for the complex resistivity of rocks was first introduced by Rocha and Habashy [1995], using an analogue circuit including the diffusivity (K) of ions in the vicinity of the electrode/electrolyte interface and a fractal parameter (η). Later, Rocha [1995] proposed the adoption of a fractal time to substitute for the diffusivity (K) of ions. The fractal model proposed describes the electrical polarisation in porous sedimentary rocks under in situ conditions. The electrical current propagation in the fractal model is characterised by two paths, one through a free

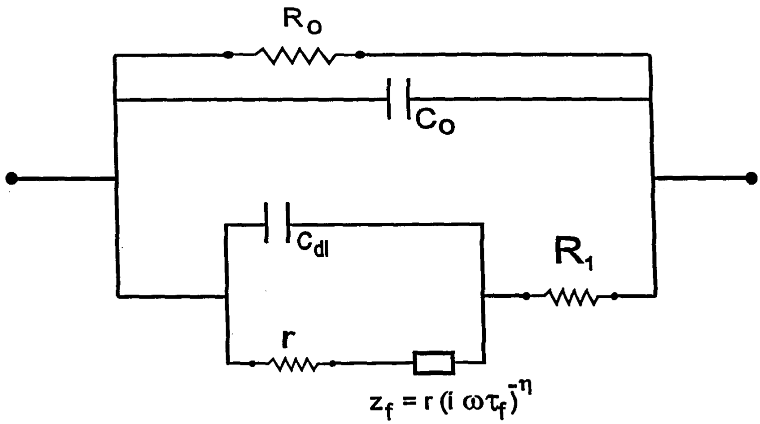

Figure 1b. Equivalent analog circuit of the mean behaviour of the medium for low-intermediate frequencies.

porous path channel free of clays and the second through a blocked pore with the presence of clay minerals as shown in Fig. 1(a).

The parameters of this model include DC resistivity (ρ_0), the chargeability (m), three relaxation times (τ , τ_f , τ_0), a grain ratio resistivity factor (δ_r), and the frequency exponent (η). The proposed analogue circuit, Fig. 1(b), represents the different conducting and polarising mechanisms involved in the electric behaviour of rocks. The fractal frequency exponent (η), describes the fractal nature of the roughness of pore surfaces. The fractal relaxation time (τ_f) and the frequency exponent are related to the fractal geometry of the rough pore interfaces between the conductive clay minerals which are blocking the pore paths and the electrolyte. The fractal roughness factor permits the investigation of the texture, an important factor explaining the electrical properties. The relaxation time (τ) is a result of low frequency relaxation of the electrical double layers formed between the electrolyte and the clay particles, whereas τ_0 is the macroscopic relaxation time of the bulk sample. The modelled fractal parameter τ_0 is the relaxation time constant related to grain size and type of disseminated blocking clay minerals. The grain ratio resistivity factor (δ_r) relates the resistivity of the conductive grains with the DC resistivity value of the rock. The DC resistivity of the rock and δ_t are related to the porosity, the electrolyte conductivity and the volumetric ratios between the matrix and the conductive grains.

The proposed circuit analogue, shown in Fig. 1(b), includes a non-linear impedance $r(i\omega\tau_f)^\eta$ which simulates the effects of the roughness of the interfaces between the blocking clay minerals and the electrolyte for the very low frequency response of the rocks. In this model the fractal time, τ_r, is involved in the transfer of charge and energy through the fractal interfaces. It is an independent parameter that can be used as a chronometer to observe the time that an individual ion will take to cross the interface. This generalised Warburg impedance is in series with the resistance of the blocking grains and both are shunted by the double layer capacitance. The charged double layer within the pores is represented by a resistive term (loss of energy due to collision of the free ions during their motion across the double layer) and a capacitive term (caused by the oscillation of bound charges in the double layer). As shown in Fig. 1(b) this combination is in series with the resistance of the electrolyte in the blocked pore passages. The un-blocked pore paths are represented by a resistance which corresponds to the normal DC resistivity of the rock. The parallel combination of this resistance with the bulk sample capacitance is finally connected in parallel to the rest of the above mentioned circuit.

Representing the time dependence of the electric field as $e^{i\omega t}$, the modified expression for the complex electrical resistivity ρ is defined as:

$$\rho(\omega) = \rho_0 \left[1 - m\left(1 - \frac{1}{1 + \frac{1+u}{\delta_r(1+v)}}\right)\right]\gamma h \tag{1}$$

$$m = \frac{R_0}{R_1 + R_0} \quad \text{chargeability, Siegel[1959]} \tag{2}$$

$$\delta_r = \frac{r}{R_1 + R_0} \quad \text{grain percent resistivity} \tag{3}$$

$$\gamma_h = \frac{1}{1 + i\omega\tau_0} \tag{4}$$

$$u = i\omega\tau(1 + v) \tag{5}$$

$$v = (i\omega\tau_f)^{-\eta} \tag{6}$$

$$\tau = rC_{dl} \quad \text{double layer relaxation time} \tag{7}$$

$$\tau_0 = R_0C_0, \quad \text{sample relaxation time} \tag{8}$$

$$\tau_f \quad \text{is the fractal relaxation time} \tag{9}$$

$\tau_0 = R_0 C_0$, where; $R_0 = g_0 \rho_0$ and $C_0 = \varepsilon_0 / g_0$. The factor g_0 represents a geometric factor given as a function of S/d, where S is the cross-section area and d is the corresponding electric length associated with the current path.

2. RESULTS AND DISCUSSION

It has been observed that all the fractal parameters respond differently to low, intermediate and elevated stress levels. These changes in fractal parameters are attributed to changes in the fractal pore surfaces which are sensitive to fractional changes in pore geometry, in contrast to the electrical conductivity curves. The fractal parameters vary from sample to sample, in response to their mineralogical constituents of the rocks, the spatial distribution of the conductive clay-minerals, grain sizes and their distribution and pore geometry. The fractal parameters obtained are discussed for each respective rock type: Berea, Clashach, and Doddington sandstone. The porosity, permeability, grain diameters and clay mineral contents for the respective sandstones are given in tables 1 and 2.

Sandstones	Porosity (%)	Permeability (md)	Bulk density (g/cm^3)	Grain-diameter (mm)
Berea 1	23.70	782.00	2.04	0.15
Berea 2	18.36	39.00	2.22	0.09
Doddington	19.90	1325.00	2.13	0.17
Clashach	16.80	1150.00	2.16	0.20

Table.1 Properties of Sandstones

Table 1 Properties of Sandstones.

Sandstones	Quartz	Kfeldspar	Clays	Ankerite	Albite	Rutile	Others
Berea 1	84.00	7.00	3.00	2.00	1.00	2.00	<1.00
Berea 2	89.00	2.00	3.00	< 0.50	2.00	2.00	<2.00
Doddington	97.00	2.00	< 0.5				<1.00
Clashach	95.00	3.00	< 0.50	< 0.50			<1.00

Table.2 Mineral composition (%) of Sandstones

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3. BEREA SANDSTONE NO.1

The σ_0 fractal parameter represents the bulk conductivity of the rock sample [Figure 2(f)] and responds differently for the low, intermediate and elevated stress levels. These different responses are attributed to separate mechanisms or physical processes taking place at the respective stress levels affecting the electrical conductivity. Initially on increasing the confining stress, cracks close. This results in a pressure build-up within the saturated rock sample. Arulanandan [1968] mentioned that the application of an alternating current sets the ions in an oscillatory motion. In clays and other ion exchangers the positive counter-ions required to balance the negative fixed charges on the solid particles are in the majority and hence they impart more momentum to the water. Thus there is net transfer in the direction of counter ion movement. Use can be made of this principle to examine and studying during increasing pressure by considering the coupling between electro-osmotic water flow and current flow. This pressure build-up under steady state conditions results in an electro-osmotic counter pressure for the double layer which prevents fluid flows resulting in a decrease in conductivity.

At intermediate stress levels, the clay edges are broken. Consequently the exposed surfaces created have unbalanced groups of charges on broken edge surfaces. These charges vary according to the hydrogen concentration within the electrolyte solution [Lockhart; 1979]. In the case of Berea 1, due to low hydrogen concentration values, the broken edges result in more positive charges. At these overburden conditions, the broken edges for the clays result in an increased electromagnetic interaction of ions. At the elevated stress levels, there is zero pressure gradient due to the earlier closure of cracks and also a decrease in pore volume. Thus there is no electro-osmotic counter pressure and the bulk conductivity decreases sharply with increasing pressure.

The τ_0 parameter is the time constant with the resistivity of the material as a whole and responds inversely to the conductivity parameter. For Berea 1 sandstone the double layer oscillation for ions is higher at lower confining stress conditions, resulting in higher τ_0 values. At elevated stress conditions the double layer oscillation activity is intensified because of the clay-electrolyte interface concentration and ion adsorption is comparatively higher, resulting in a higher values of (τ_0).

The τ_f parameter is the fractal relaxation time. For Berea 1 sandstone this parameter at first fluctuates with increasing confining stress but later shows a constant increase at elevated stress levels. Initially on increasing stress there is fluid pressure build up within the rock sample due to closure of cracks. Thus there is a net water transfer in the direction of the counter ion movement. Consequently the fractal relaxation time decreases initially due to the comparatively higher number of negative ions at the clay-mineral surfaces. At intermediate stress levels, however the broken edges and rough surfaces result in higher fractal times. At elevated stress conditions, the electrolyte saturated pores containing disseminated clay minerals tend to reduce in pore volume. The electrolyte is displaced from the saturated pores and a concentrated clay-electrolyte interface results an increase in ion adsorption, which lowers the hydrogen concentration and results in higher fractal times.

The value of δ_r which is the ratio of the conductive grain resistivity to the rock macroscopic resistivity decreases under the lower confining stress conditions and then rises steadily except at the highest confining pressure. The initial decrease of this parameter is due to an abrupt macroscopic increase in resistivity for the rock which in turn is due to closure of cracks at the lower confining pressures. At intermediate stress levels, the bulk resistivity decreases due to variation in pore geometry and as a result of broken clay edges. The broken clay edges

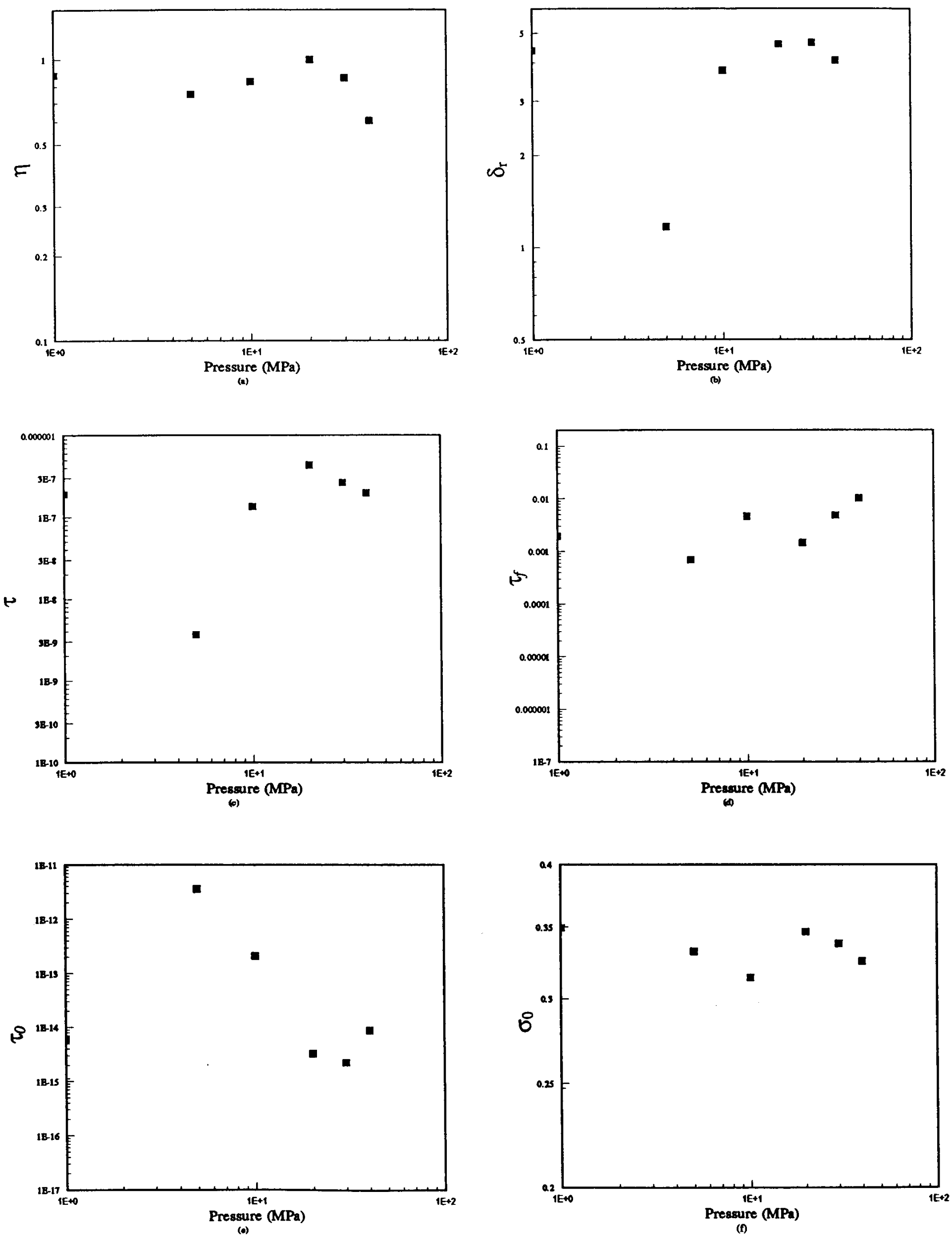

Figure 2. Modelled fractal parameters for double layer polarisation: Berea 1 Sandstone.

result in an increase and imbalance of the cation cloud which in turn results in a higher conductivity and an increase in δ_r. At the highest confining stress the parameter shows a slight decrease which is not well understood at this stage but may be due to closure of the pores at these elevated stress levels. Such an analysis does not provide an insight into physical mechanism causing its existence but is expected to be a function of particle size and type.

The modelled fractal parameter (η) is the fractal frequency exponent obtained at comparatively lower frequencies. This responds differently at low, intermediate and elevated confining stresses. The clay-mineral surfaces at the lower confining pressures are undisturbed and have an excess of bound ions. This is why (η) has a lower value. At intermediate confining stresses, the broken edges result in a higher fractal frequency exponent value. At elevated stress levels, the clay-electrolyte interface is more concentrated, which results in more adsorption of ions and consequently lower fractal characteristics of the rough surface interface.

The (τ) parameter for Berea 1 fluctuates in a manner similar to that of the resistivity of the rock. The cracks close at the lower confining stresses which changes the tortuosity path and results in an increase in the conductivity. Initially under the lower overburden condition (τ) decreases. This is because the bound ions are higher in number as compared to free ions. The double layer capacitance depends only on free ions. Since the free ions are less in number, the (τ) parameter therefore decreases. At intermediate stress levels, the clay edges are broken thus resulting in higher number of free electrons due to the broken edges. The double layer intensity tends therefore to be higher. In contrast, at the elevated stress levels the clay mineral surface behaves in manner similar to the lower confining pressure, this parameter decreases as the product of the resistivity of the blocking grains and capacitance associated to the double layer increases.

4. BEREA SANDSTONE NO. 2

The modelled fractal parameters for Berea 2 sandstone are shown in Figure 3. These fractal parameters respond differently to varying confining stresses but are not as pronounced as for Berea 1. This behaviour is attributed to the smaller grain size, lower porosity and permeability of the rock.

Variation in the bulk conductivity is not as pronounced as that of Berea 1 sample and the expected intermediate stress level responses are observed at comparatively higher stresses. At these stress levels the broken edges result in low hydrogen concentration values and consequently higher conductivity. At elevated stresses there is a pore volume decrease and furthermore, there is no electro-osmotic counter pressure. As a result the electrical conductivity decreases drastically. There is sufficient similarity of this behaviour between the Berea 1 and Berea 2 to justify the same general interpretations for the cases discussed earlier for Berea 1 sandstone. The modelled fractal parameter τ_0 (the macroscopic relaxation time of the sample) responds inversely to σ_0 (the bulk conductivity of the rock sample) is associated with the resistivity of the material. The response for τ_0 is not pronounced and is attributed to the smaller grain size, lower porosity and permeability as compared to the earlier Berea sample.

As shown in Figure 3, the fractal relaxation time τ_f for rough interfaces shows a decrease. This initial decrease is due to a higher number of negative charges on the clay mineral surfaces. At intermediate and elevated confining stress conditions, the fractal relaxation time adopts a parabolic shape for increasing confining pressure. At the elevated stress levels the pores with disseminated clay minerals reduce in volume. As a result, the clay-electrolyte interface roughness decreases and consequently the fractal relaxation time decreases accordingly.

The values obtained for (δ_r) are greater than unity as for Berea 1 sample, except at the highest stress level. This behaviour is not well understood at this point. The initial abrupt decrease of this parameter is the result of the macroscopic increase in resistivity for the rock, due to closure of cracks at the lower confining pressures. Later with increasing confining stress the bulk resistivity decreases due to broken edges. The increase and imbalance of cation cloud results in a higher conductivity and an increase in δ_r.

The fractal frequency exponent (η) defines the fractal nature of rough pore surfaces, comparatively at lower frequencies, responds differently to low, intermediate and elevated confining stresses. Initially the clay-mineral surfaces at the lower confining stresses are undisturbed. The smooth clay mineral surfaces have an excess of bound ions, so η has low values. With increasing overburden conditions, on reaching the intermediate stress levels the clays have broken edges, resulting in a higher fractal frequency exponent value. On the contrary,

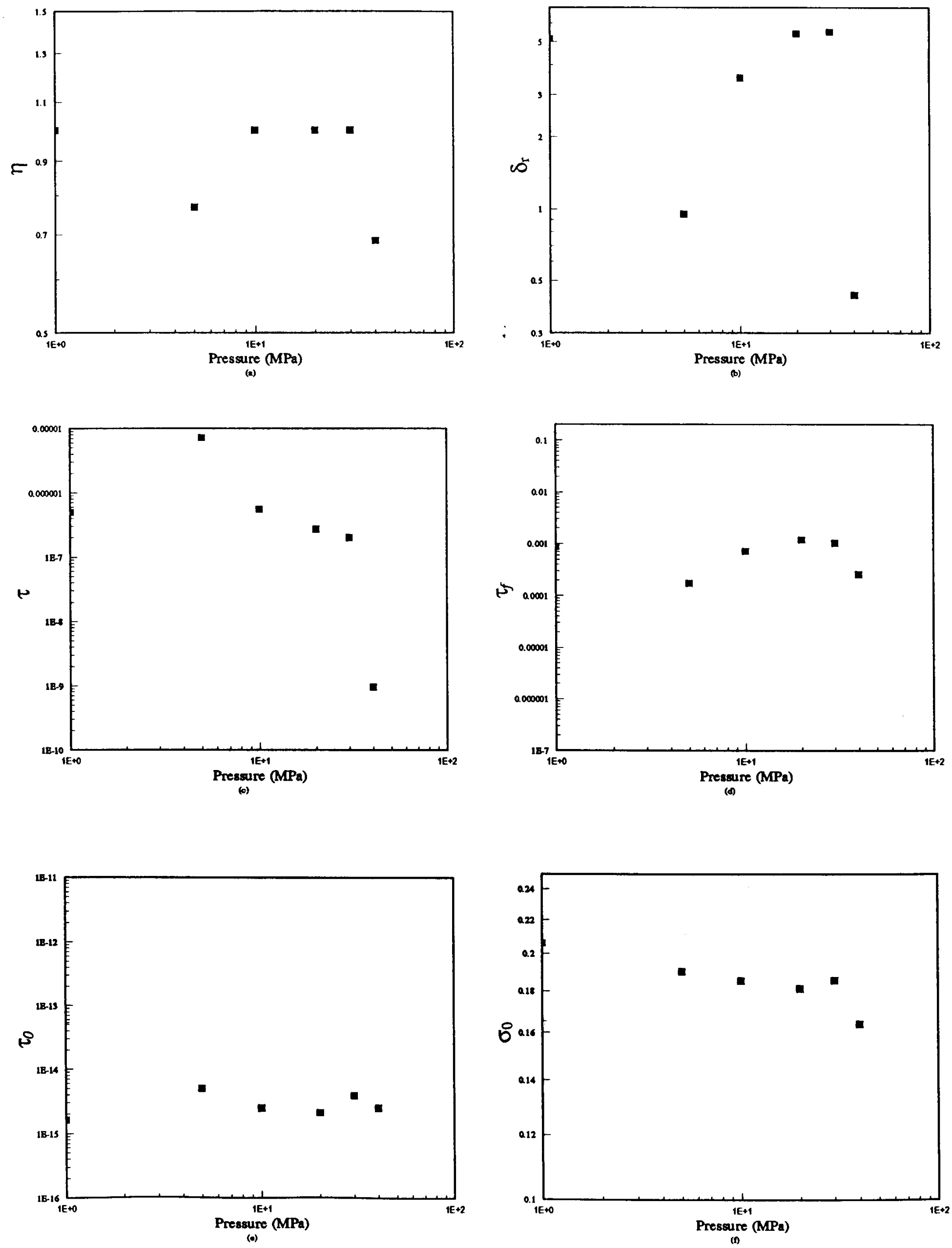

Figure 3. Modelled fractal parameters for double layer polarisation: Berea 2 Sandstone.

at elevated confining stresses, the pore volume decreases and the clay-electrolyte interface concentration increases, resulting in a lower fractal frequency exponent value.

The relaxation time constant related to the double layer, (τ) increases initially and then decreases with the increasing confining stress. Initially at lower confining stresses the double layer ion oscillation is higher than that at elevated stress levels.

5. CLASHACH SANDSTONE

The sample has fairly larger grain size (table 1) as compared to Berea 2 and is not significantly different to Berea 1. Because of the low clay mineral content, the Clashach sample is classified as fairly clean sandstone and thus there is no pronounced double layer polarisation effect. In spite of the significant clay and mineral grain variation the bulk conductivity of the Clashach sandstone behaves in a manner similar to Berea 2 (Figure 3).

The behaviour of the fractal parameters such as (τ_0) is as expected for fairly clean sandstones without the double layer polarisation effect.

The fractal relaxation time (τ_f), initially has a lower value due to the fractal nature of the rough interface. This initial abrupt decrease is due to low clay-mineral contents. The behaviour observed is in contrast to sandstone samples with higher clay-mineral contents. An increase in the time involved for the transfer of charge and energy through the rough interfaces at the higher stress levels is due to comparatively fractal nature of the rough interface.

The grain ratio resistivity (τ_r) decreases with increasing stresses. The decrease is not pronounced for other samples with higher clay contents, except at the highest stress levels. The grain ratio resistivity is less than unity, suggesting that grains are not conductive and is probably due to the blocked passage at these stress levels.

The fractal frequency exponent (η) for the pore surfaces, shows no significant variation, except at the highest stress levels. The insignificant variation is attributed to the low clay-mineral content.

6. DODDINGTON SANDSTONE

The fractal parameters (σ_0) as seen in Fig. 5(f), decreases steadily with increasing confining pressure. The decrease in conductivity is attributed to closure of cracks. Later, the decrease at elevated stress

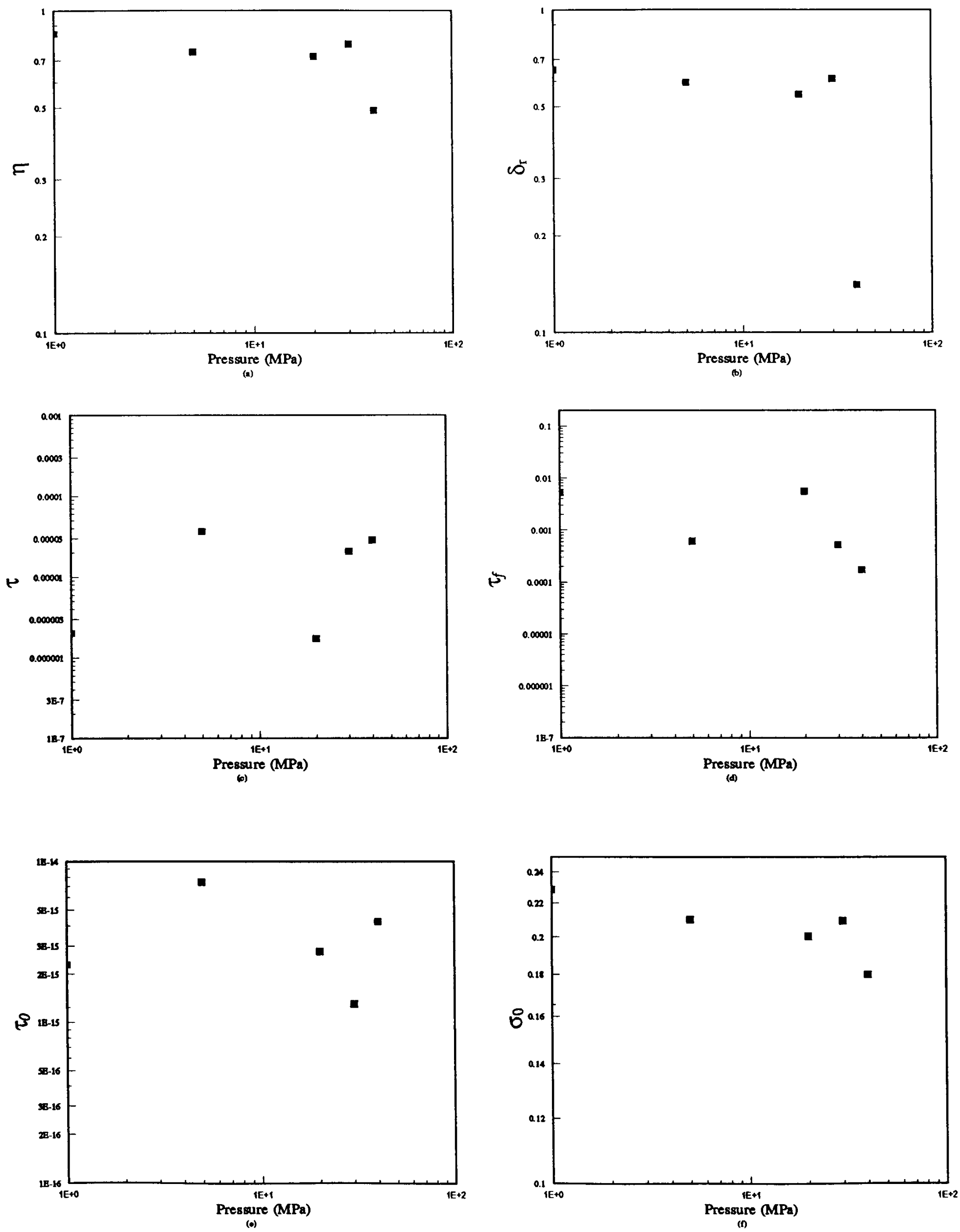

Figure 4. Modelled fractal parameters for double layer polarisation: Clashach Sandstone.

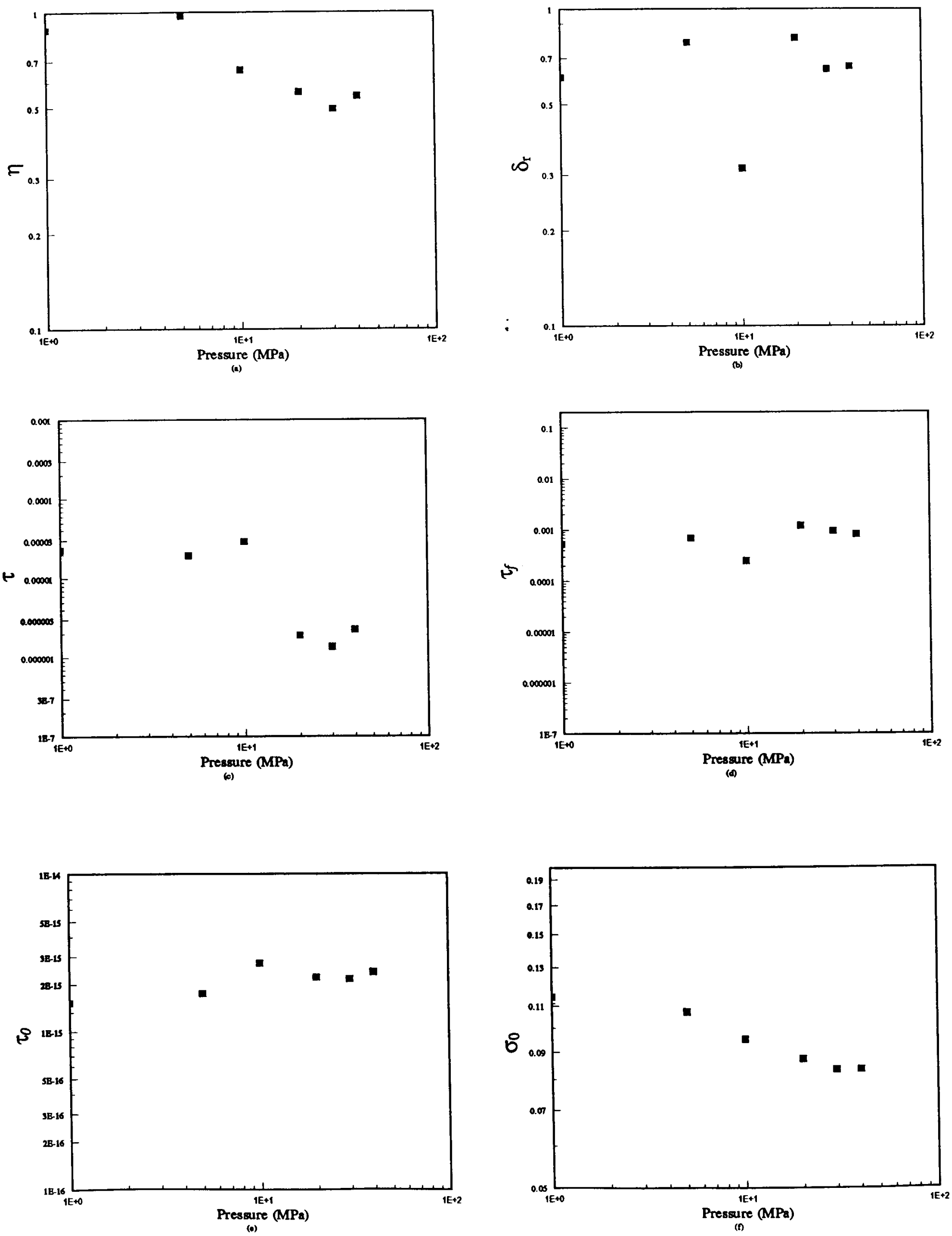

Figure 5. Modelled fractal parameters for double layer polarisation: Doddington Sandstone.

conditions is due to the reduction in pore volume. The time constant associated with the resistivity of the material as a whole responds inversely to σ_0. The fractal relaxation time (τ_f), behaves in a manner similar to Clashach sandstone. (τ_f) decreases with increasing confining pressure due to fine changes in pore geometry. The grain fraction resistivity shows considerable variation as compared to the other samples tested. The initial increase of conductivity is attributed to the surface conduction at these stress levels. Later the abrupt decrease of this parameter is due, to the macroscopic increase in resistivity for the rock due to closure of fine interstices. The (σ_r) values are less than unity showing that grains are highly resistive or the pores are blocked at the respective stress levels. The frequency exponent fractal parameter (η) decreases with increasing confining pressure except at the highest stress level. The relaxation time (τ) has higher values at lower overburden conditions as the double layer oscillation is higher as compared to elevated stress levels.

7. CONCLUSIONS

The fractal parameters under overburden condition, obtained by employing the fractal model is associated with the roughness of the pore spaces. The fractal parameters obtained respond differently to changes in stress, thus providing an opportunity to understand the physical processes occurring within the rocks. These fractal parameters need further investigations in further establishing the theory and can later play a vital role as an integral part of geophysical well logging.

ACKNOWLEDGEMENTS

Authors wish to thank Dr. Tarek Habashy and Dr. Reaz Taherian, Doll Schlumberger Research CT. USA for their acceptance to review the paper, authors appreciate their critical comments on the manuscript. This work was supported by Department of Earth Resources Engineering, Imperial College London and King Faisal Foundation, Saudi Arabia, their financial help is acknowledged.

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