Progress In Electromagnetics Research, **PIER 20, 25–44, 1998**

A SYSTEMATIC STUDY OF SYMMETRICAL ELECTROMAGNETIC THREE-PORTS WITH ISOTROPIC AND GYROTROPIC MEDIA

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1. INTRODUCTION

Electromagnetic reciprocal three-ports are used as components of circuits in a wide range of frequencies. They fulfill different functions such as dividing and combining. They serve also as elements of filters and matched circuits. The parameter matrices of three-ports (the scattering matrix $[S]$, the matrix of impedances $[Z]$ and admittances $[Y]$) have been extensively discussed in the literature [1–8].

Three-ports with gyrotropic media fulfill nonreciprocal and control functions at microwaves and in optics. They are circulators (isocirculators) based on different physical effects, filters, switches, commutators, control power dividers, mode convertors and so on [9–20].

The development of nonreciprocal and control devices in recent years has been directed to the accomplishment of more and more complicated functions. In particular, devices combining several functions have been developed, for example: dividers/combiners-control phase shifters, which provide beam-forming with the number of phase shifters reduced to half the number of elements [18], nonreciprocal impedance transformers [15] etc. This leads to a greater complexity in the structure of the devices and makes difficult their calculations.

A matrix analysis of the nonreciprocal three-ports is difficult in general case because of the large number of matrix parameters. Among the papers devoted to this problem one should mention [21], where the restrictions imposed by the unitary condition on the elements of the scattering matrix of a three-port were investigated. This paper shows the difficulties arising in treating 3×3 nonsymmetric scattering matrix even in the unitary case. In [9], the conditions for an arbitrary nondissipative three-port under which it can become an ideal circulator, were considered.

A classification of the three-ports may be accomplished in different ways, namely according to the functions, physical effects, types of waveguides or transmission lines being used. In [22] for example, a classification of lossless three-ports is based on comparing the $[S]$-matrix elements and equality of some of them to zero.

Two important cases of symmetrical three-ports have been considered in detail in literature. One of them is a three-port with three-fold axis symmetry C_3 (and closely related group C_{3V})[10]. The second one is the case of a mirror-symmetry. Two variants of such symmetry, namely nonreciprocal three-ports with gyrotropic symmetry and antisymmetry have been investigated in [14]. Their scattering matrices and the number of independent parameters of these matrices have been determined. Nondissipative three-ports have been analyzed in the paper [14] using the unitarity condition. Some problems concerning description of the devices with deviations from the ideal symmetry are discussed in [8].

The application of group theory to the electromagnetic N-ports in the case of gyrotropic media has been considered in [23, 28]. Symmetry leads in particularly to reducing the number of independent parameters of the matrices $[S]$, $[Z]$ and $[Y]$ of the multiports. A complete nomenclature of symmetrical two-ports with gyrotropic media which consists of eight 2×2 scattering matrices has been obtained in [25]. Many examples of the group theory application to electromagnetic problems are given in [30].

In this work, a systematical study of symmetrical three-ports with isotropic and gyrotropic media is fulfilled. A new classification of the three-ports based on geometrical symmetry with regard to the time reversal operator is suggested. Using the magnetic group approach and Curie principle, all the possible parameter matrices of symmetrical three-ports and their correspondence to magnetic groups are determined.

2. PROBLEM DESCRIPTION

Three-ports with linear, time invariable, passive and in general lossy isotropic and gyrotropic media will be considered in this paper. The dc magnetic field (and the dc magnetization of the media) may be nonuniform. Three uniform one-mode waveguides (transmission lines) are connected to a symmetrical volume. The mode, which propagates in the waveguide is the lowest one.

The waveguides and/or the volume may be filled with gyrotropic media. In the case of gyrotropic waveguides, the diagrams of the waveguide cross-section field structures for incident and reflected waves may be different, and it is a problem of bidirectionality.

It is known [26], that the symmetry of the properties of the multiport (i.e., the group of symmetry of the $[S]$-matrix) must include the operation of the point group of physical symmetry of this multiport. Therefore, we are to investigate the symmetry of the system: waveguides + interconnecting volume + dc magnetic field. In order to find all the possible scattering matrices of such a system and the correspondence of these matrices to magnetic groups of symmetry, i.e., to find the complete solution, we shall come from the highest possible symmetry of the system under consideration.

3. DETERMINATION OF THE HIGHEST POSSIBLE GROUP OF SYMMETRY OF THE THREE-PORTS

Let us apply first to the symmetries of the three-ports with nongyrotropic media. The symmetry of the interwaveguide volume and the waveguides themselves may be different. In this situation, one uses the Curie principle in order to find the resulting symmetry. In general, the

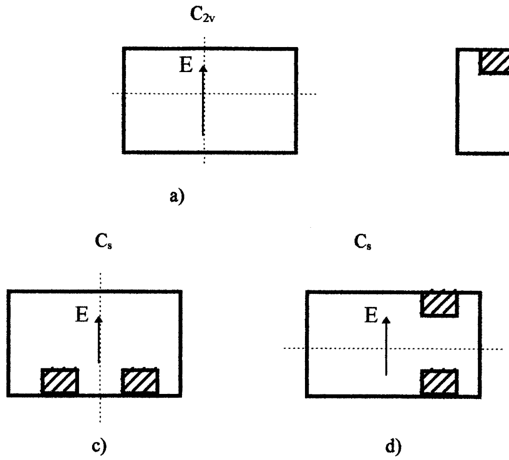

Figure 1. Cross-sections of rectangular waveguides with different symmetries.

geometrical symmetry of the complex system differs from the symmetry of its components. In order to find the highest possible symmetry of the system "interconnecting volume + waveguides", we may consider without loss of generality, the interconnecting volume in the form of a sphere. In this case, the resulting symmetry will be determined only by the symmetries of the waveguides and their mutual orientation.

As models of different uniform one-mode waveguides (transmission lines), there will be further considered rectangular waveguides with H_{10} modes (or quasi- H_{10} modes). The highest symmetry of the rectangular waveguide cross section is C_{2V} (in Schoenflies notation, the systems of group notation will be explained in Section 4). The other possible symmetries of the waveguide section are C_2, C_s (two variants of the symmetry plane orientation are possible) and C_1. The examples of such waveguides are given in Fig. 1 where additional insertions define the symmetry of rectangular waveguide.

It should be emphasized that the given figures are only spatial models of the symmetries. The real em structures with such symmetries may be infinitely diverse.

The highest symmetry of the section of a one-mode uniform rectangular waveguide is D_{2h}. The highest possible geometrical symmetry of the system interconnecting volume + waveguides of the three-ports is D_{3h}. It corresponds to the symmetry of Y-junction of three rectangular waveguides. All the other possible symmetries of the three-ports are subgroups of the group D_{3h} . Notice, that in particular case, the interconnecting volume may be absent.

The consideration of the coaxial line with lowest T-mode which has the symmetry $C_{\infty V}$ does not give new symmetries.

In order to find the symmetry of the physical system: geometrical structure + dc magnetic field, the Curie principle will also be used [23].

4. NOTATIONS OF MAGNETIC POINT GROUPS AND THEIR ELEMENTS

Two different systems of notation for the operations, elements and the groups of symmetry themselves will be used in the paper: the Schoenflies and the Shubnikov systems. The Schoenflies notation is as follows:

E -identity,
C_n -rotation about an axis through an angle $2\pi/n$,
σ -reflection in a plane passing through the axis C_n (σ is the plane passing through the axis C_n and i, σ_{ij} is the plane passing through C_n and midway between the axes i and j),
σ_h -reflection in a plane perpendicular to the axis C_n,
S_n -improper rotation, i.e. a rotation C_n and then a reflexion σ_h,
U_2 -two-fold rotation about an axis lying in the plane perpendicular to the axis C_n.

In the Shubnikov notation, an n-fold rotation axis is indicated by the symbol n. A plane of symmetry is denoted by the letter m. A single dot is used to indicate that two symmetry elements are parallel, for example the group C_{2V} in the Schoenflies notation would be $2 \bullet m$ Shubnikov one. A double dot is used to indicate that two symmetry elements are perpendicular. For example, the symbol $3 : m$ indicates that there is a plane of symmetry perpendicular to the three fold axis, it is C_{3h} in Schoenflies notation. The bar below a symbol means that one should take the product of the corresponding element with the time reversal operator T.

The Shubnikov notation is convenient in order to obtain generators of the group. Using generators, one can find all the elements of the group. From the Schoenflies notation, on the other hand, one may see the division of the group on the elements with T and without T.

The crystallographic magnetic point groups are divided into three categories [29]:

1. The 32 point groups including the time reversal operator T itself. These groups describe nonmagnetic media.
2. The 32 point groups without any form of the time reversal operator. These groups describe magnetic media, such as ferro-, ferri- and antiferromagnetics.
3. The 58 magnetic point groups of the third category include the time reversal operator only in combination with rotation and reflexion operators. These groups describe also magnetic media.

The Schoenflies (as well as Shubnikov) notations of the groups of the first and second categories coincide.

The groups of the third category have a particular Schoenflies notation. If G is a group of unitary and antiunitary operators and H is the invariant subgroup of unitary operators then the Schoenflies notation of this magnetic group is $G(H)$.

The first step in finding all the possible magnetic groups for three-ports is determining the highest group of symmetry for the case of nonmagnetic media. Beginning with the number of the ports and analyzing the subgroup decomposition of the 32 point groups of the first category, we have found that for the symmetrical three-ports with three different waveguides, the highest group is D_{3h}. Y-junction of three rectangular waveguides (as well as of the coaxial lines) has such a group of symmetry. All the other variants are the subgroups of the group D_{3h}.

The subgroup decomposition of the group D_{3h} is shown in Fig. 2 [29]. The group tree consists of 9 groups of the first category and therefore, of 9 groups of the second category. Notice that the parameter matrices which are defined by the groups of the first category are symmetrical about the main diagonal while the parameter matrices of the groups of the second category in general are not.

Every group is connected by a line to each of its subgroups. Heavy line indicates that the subgroup is not invariant. If the subgroup is not of index 2, the line is dotted.

The number of groups of the third category is equal to the number of lines connecting the groups in Fig. 2, excluding heavy and dashed lines. Therefore, the number of groups of the third category is 11 and they are given in Table 1. In this Table, the number of elements of the groups, the elements themselves and generators of the groups are written.

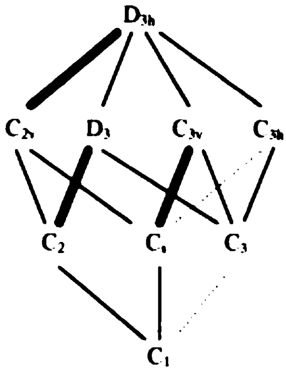

Figure 2. Group tree for the group D_{3h}. A heavy line indicates that the subgroup is not invariant, and a dotted line indicates that the subgroup is not of index 2 under the above group.

Shubnikov notation	Schoenflies notation	Number of group elements	Elements of the group	Generators of the group
$\underline{2}$	$C_2(C_1)$	2	E, TC_2	TC_2
$\underline{m}$	$C_s(C_1)$	2	$E, T\sigma$	$T\sigma$
$2\bullet\underline{m}$	$C_{2v}(C_2)$	4	$E, C_2, T\sigma_1, T\sigma_2$	$C_2, T\sigma_1$
$\underline{2}\bullet m$	$C_{2v}(C_s)$	4	$E, TC_2, \sigma_1, T\sigma_2$	TC_2, σ_1
$3{:}\underline{2}$	$D_3(C_3)$	6	$E, C_3, C_3^2,$ TU_2', TU_2'', TU_3'''	C_3, TU_2'
$3\bullet\underline{m}$	$C_{3v}(C_3)$	6	$E, C_3, C_3^2, T\sigma_1, T\sigma_2, T\sigma_3$	$C_3, T\sigma_1$
$3{:}\underline{m}$	$C_{3h}(C_3)$	6	$E, C_3, C_3^2, TS_3, TS_3^2,$ $T\sigma_h$	$C_3, T\sigma_h$
$\underline{m}\bullet 3{:}\underline{m}$	$D_{3h}(D_3)$	12	$E, C_3, C_3^2, U_2', U_2'',$ $U_2''',$ $T\sigma_1, T\sigma_2, T\sigma_3,$ $TS_3, TS_3^2, T\sigma_h$	$C_3, T\sigma_1, T\sigma_h$
$m\bullet 3{:}\underline{m}$	$D_{3h}(C_{3v})$	12	$E, C_3, C_3^2, \sigma_1, \sigma_2, \sigma_3,$ $TU_2', TU_2'', TU_3''',$ $TS_3, TS_3^2, T\sigma_h$	$C_3, \sigma_1, T\sigma_h$
$\underline{m}\bullet 3{:}m$	$D_{3h}(C_{3h})$	12	$E, C_3, C_3^2, S_3, S_3^2, \sigma_h,$ $TU_2', TU_2'', TU_3''',$ $T\sigma_1, T\sigma_2, T\sigma_3$	$C_3, T\sigma_1, \sigma_h$

Table 1. Magnetic Group of the Third Category Describing Symmetrical Three-ports:

5. COMMUTATION RELATIONS AND SYMMETRY MATRICES OF THE THREE-PORTS

Symmetry of the $[S]$-matrices will be understood in a broad meaning, namely in four-dimensional space, which includes the three spatial coordinates and time [26].

In order to find correlations between some elements of the scattering matrix, two types of the commutation conditions may be used:

for the case of gyrotropic symmetry (GS)

$$[R][S] = [S][R], \tag{1}$$

and for the case of gyrotropic antisymmetry (GA)

$$[R][S] = [S]^t[R], \tag{2}$$

where $[R]$ are generators of the symmetry group [23].

Let us apply to formal determination of symmetry matrices $[R]$ of the three-ports. We consider the matrices of geometrical symmetry of the three-ports therefore the dimension of these matrices must be 3×3. Each row and each column of the $[R]$-matrix contains only one element equal to +1 or -1 and the other elements are zeros. These matrices are 3×3 representations of the corresponding elements of the symmetry groups. The matrices are orthogonal, i.e., they satisfy the relation

$$[R]_i[R]_i^t = [E],$$

where $[E]$ is the unit matrix.

Our aim in this section is to find formally those $[R]$-matrices which will be used for symmetrical three-ports. First of all, it is the unit matrix

$$[R]_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{3}$$

This matrix, obviously corresponds to the identity operator, which is in every group. It will be shown later that the use of this matrix can give some information about the properties of the three-port because the matrix can describe a covering operation which causes some parts of the junction to interchange positions, but does not change the positions of the ports themselves. It is for example a reflection in a plane Cs

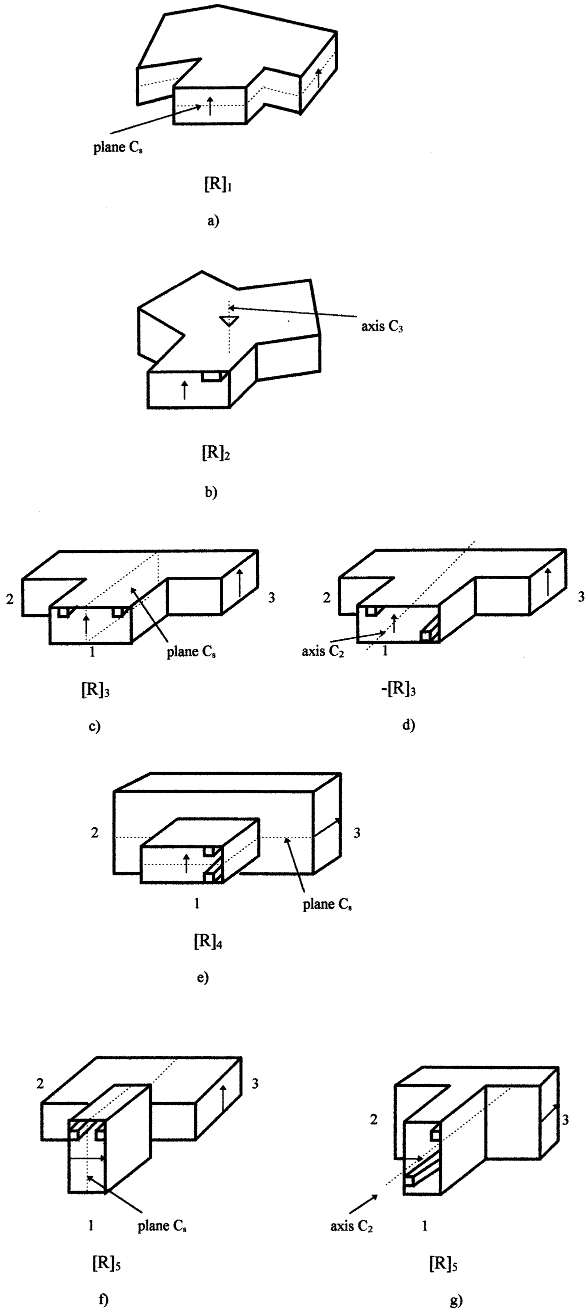

Figure 3. Examples of three-ports with different symmetries.

passing through all three ports (Fig. 3a, $[R] = -[R]_1$). Notice that the arrows in Fig. 3 correspond to the electric field orientation.

The second $[R]-$ matrix corresponds to a three-fold axis C_3 , i.e. to a rotation operation with a certain sense of rotation (Fig. 3b):

$$[R]_2 = \begin{vmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} \tag{4}$$

It is possible not to consider the rotation operation with the opposite sense of rotation because in all of the groups containing the three-fold axis C_3 , only one generator can be used for the set of elements E , C_3 , C_2^3 .

The next matrix describes the reflexion operation C_s (Fig. 3c), or the rotation C_2 about an axis passing through one of the ports between the two others (Fig. 3d). Without loss of generality one can assign the number 1 to the port which lies in the plane of symmetry or on the axis of the second order. The matrix has the following form :

$$[R]_3 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \tag{5}$$

Apply now to the question of the signs of the units in the matrices $[R]_1, [R]_2$ and $[R]_3$. Formally the signs plus or minus can be in the matrices $[R]_i$ at every unit, because it is not forbidden by the definition of these matrices given above. One of the units in the matrix $[R]_1$ can be with the negative sign:

$$[R]_4 = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} \tag{6}$$

An example of such a covering operation is a plane of symmetry in the Fig. 3e.

The matrix $[R]_2$ can not exist with units having different signs, because it contradicts to the meaning of this symmetry operation.

For a series junction, the voltage at port 1 will undergo a phase reversal under reflection operation (Fig. 3g). It means, that the symmetry operator $[R]_3$ in this case is transformed in

$$[R]_5 = \begin{vmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} \tag{7}$$

N	Matrix [S]	Symmetry Matrix $[R]_i$	Number of Independent Parameters	Symmetry Groups Which Generate this Matrix [S] and Qrientation of their Axes and Planes
1	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{vmatrix}$	$[R]_1$	6	C_s, the plane passes through all three ports (Fig.3a)
2	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$	4	C_2 or C_s, the axis or the plane is between ports 2 and 3 (Fig.3c,d), C_{2v} (Fig.3c without insertions)
3	$\begin{vmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & S_{23} \\ 0 & S_{23} & S_{33} \end{vmatrix}$	$[R]_4$	4	C_s, the plane passes through all three ports (Fig.3e, Fig 3f with insertions of Fig.1c)
4	$\begin{vmatrix} S_{11} & S_{12} & -S_{12} \\ S_{12} & S_{22} & S_{23} \\ -S_{12} & S_{23} & S_{22} \end{vmatrix}$	$[R]_5$	4	C_s, C_2, the axis or plane is between ports 2 and 3 (Fig.3f,g), C_{2v}((Fig.3g without insertions)
5	$\begin{vmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & S_{23} \\ 0 & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$, $[R]_5$	3	C_{2v}, the axis C_2 is between ports 2 and 3 (Fig.3e,f without insertions)
6	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{vmatrix}$	$[R]_2$	2	C_3, D_3, C_{3v}, C_{3h}, D_{3h} (Fig.3b with corresponding insertions of Fig.1). To obtain the matrix [S], it is sufficient to use only $[R]_2$

Table 2. Description of Symmetrical Scattering Matrices of the First Category:

with $[R]_{11} = -1$. The matrix (7) describes also reflexion operation in Fig. 3f.

A consideration of the matrices $-[R]_i (i = 1, ...5)$ does not give any new information, because the matrices $[R]_i$ are multipliers on both sides of identities (1) and (2), i.e. the use of $[R]_i$ and $-[R]_i$ gives the same results. Hence, from the point of view of symmetry one should consider $[R]_i$ and $-[R]_i$ as equivalent.

Notice that a change in the numbering scheme of the ports leads outwardly to a different matrix $[R]$, and consequently to different $[S]$. Of course, the numerical values of the elements in this case are not changed, but their positions in the matrix are different. Such matrices with renumbered ports may be transformed one into another using the similarity transformation

$$[S]' = [G]^t [S][G] \tag{8}$$

where $[G]$ is so called renumbering matrix which is analogous to the matrix $[R]$. The matrices corresponding the same structure with different numbering schemes will be considered as equivalent.

6. SOLUTION OF THE PROBLEM

It would require too much space in order to give all the solution of the problem for three- ports. Let us describe a method of finding the scattering matrices considering several examples. The cases with only one (anti)plane or one (anti)axis are rather simple. It is only necessary to take into account a possibility of different orientations of these elements. The groups, which include the three-fold axis C_3 , do not cause difficulties as well because this axis may be oriented uniquely. It should be noted a peculiarity of the 3-ports which are described by the groups with symmetries $C_3, D_3, C_{3V}, C_{3h}, D_{3h}$ having 3-fold axis. In order to find the matrix $[S]$ in isotropic case, it is sufficient to use only one generator $[R]_2$ The matrix $[S]$ in this case has minimal number of elements, namely 2, and the use of other possible generators does not give new information. In the gyrotropic case, the use of the generator $[R]_2$ gives the matrix with 3 independent parameters, and application of other possible generators may reduce the number of parameters, for example in matrix $[S]_7$, Table 3.

The group C_3 is not represented in the Table 4 because this group can not be of the third category.

N	Matrix [S]	Symmetry Matrix $[R]_i$	Number of Independent Parameters	Symmetry Groups Which Generate this Matrix and Orientation of their Axes and Planes
1	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{vmatrix}$	$[R]_1$	9	C_s, the plane passes through all three ports
2	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{21} & S_{22} & S_{23} \\ S_{21} & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$	5	C_2 or C_s, the axis or the plane is between ports 2 and 3, C_{2v}
3	$\begin{vmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & S_{23} \\ 0 & S_{32} & S_{33} \end{vmatrix}$	$[R]_4$	5	C_s, the plane passes through all three ports
4	$\begin{vmatrix} S_{11} & S_{12} & -S_{12} \\ S_{21} & S_{22} & S_{23} \\ -S_{21} & S_{23} & S_{22} \end{vmatrix}$	$[R]_5$	5	C_s, C_2, the axis or plane is between ports 2 and 3
5	$\begin{vmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & S_{23} \\ 0 & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$, $[R]_5$	3	C_{2v}, the axis C_2 is between ports 2 and 3
6	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{13} & S_{11} & S_{12} \\ S_{12} & S_{13} & S_{11} \end{vmatrix}$	$[R]_2$	3	C_3, D_3, C_{3v}, C_{3h}, D_{3h}; to obtain the matrix [S], it is sufficient to use only $[R]_2$
7	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{vmatrix}$	$[R]_2$, $[R]_3$	2	D_3, C_{3v}, D_{3h}

Table 3. Description of Symmetrical Scattering Matrices of the Second Category:

Let us apply to the case of the group $C_{2V}(C_2)$ ($2 \bullet m$ in Shubnikov notation). In this group (see Table 1), there is one two-fold axis of symmetry and two antiplanes of symmetry. The antiplanes are perpendicular to each other and pass through the axis of symmetry. The only possible orientation of the axis C_2 in the 3-ports under consideration is passing through one port and between the two others (for example, Fig. 3d, g, without insertions). The orientation of the (anti)planes C_s may be different: passing between two ports or through all the three ports.

Generators of the group $2 \bullet m$ are for instance the matrix $[R]_3$ and $[R]_5$. One of them corresponds to the rotation, and the other to any of the antireflections. Therefore, 4 combinations of these generators are possible:

1. $[R]_3, \mathrm{GS}$ and $[R]_3, \mathrm{GA}$, (Fig. 3c);
2. $[R]_5, \mathrm{GS}$ and $[R]_5, \mathrm{GA}$, (Fig. 3g);
3. $[R]_3, \mathrm{GA}$ and $[R]_5, \mathrm{GS}$, (Fig. 3e);
4. $[R]_3, \mathrm{GS}$ and $[R]_5, \mathrm{GA}$, (Fig. 3f),

where Fig. 3 should be considered without insertions. Using the commutation relations (1) and (2), one came to matrices $[S]_5, [S]_6, [S]_7$ and $[S]_8$ of Table 4, respectively (in the following discussion, the subscript of a matrix $[S]_i$ corresponds to the number of its position in Tables 2, 3 or 4). Notice that in Table 2, there are indicated the figures which illustrate the corresponding symmetries.

Similarly, one can find the matrices $[S]$, corresponding to the group $C_{2V}(C_s)$ which are in Table 4 as well.

7. DISCUSSION OF THE RESULTS

There are 6 matrices $[S]$ describing nonmagnetic symmetrical structures in Table 2, and also 7 matrices in Table 3 and 11 matrices in Table 4 for magnetic 3-ports. Some of the matrices for magnetic 3-ports are symmetrical about the main diagonal ($[S]_5$ and $[S]_7$ in Table3, and $[S]_4$, $[S]_5$ and $[S]_{11}$ in Table 4). Therefore, they describe reciprocal devices. Because there is no other magnetic groups of symmetry and all the variants of orientations of (anti)planes and (anti)axis of symmetry have been considered, it seems that all the possible solutions for symmetrical three-ports with three one-mode waveguides filled with isotropic and gyrotropic media have been found. The correspondence of the $[S]$-matrices and the magnetic point groups of symmetry is

N	Matrix [S]	Symmetry Matrix $[R]_i$	Number of Independent Parameters	Symmetry Groups Which Generate this Matrix and Orientation of their (Anti)Planes
1	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{13} & S_{22} & S_{23} \\ S_{12} & S_{32} & S_{22} \end{vmatrix}$	$[R]_3$, GA	6	$C_2(C_1)$, $C_s(C_1)$-the antiaxis or antiplane is between ports 2 and 3, $C_{2v}(C_s)$- the plane passes through all three ports
2	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ -S_{12} & S_{22} & S_{23} \\ -S_{13} & S_{23} & S_{33} \end{vmatrix}$	$[R]_4$, GA	6	$C_s(C_1)$-the antiplane passes through all 3 ports
3	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ -S_{13} & S_{22} & S_{23} \\ -S_{12} & S_{32} & S_{22} \end{vmatrix}$	$[R]_5$, GA	6	$C_2(C_1)$, $C_s(C_1)$-the antiplane is between ports 2 and 3, $C_{2v}(C_s)$- the plane is between ports 2 and 3
4	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{12} & S_{22} & S_{23} \\ S_{13} & S_{23} & S_{33} \end{vmatrix}$	$[R]_1$, GA	6	$C_s(C_1)$- the antiplane passes through all 3 ports
5	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$, GS $[R]_3$, GA	4	$C_{2v}(C_2)$, $C_{2v}(C_s)$- the plane is between ports 2 and 3
6	$\begin{vmatrix} S_{11} & S_{12} & -S_{12} \\ S_{12} & S_{22} & S_{23} \\ -S_{12} & S_{32} & S_{22} \end{vmatrix}$	$[R]_5$, GS $[R]_5$, GA	4	$C_{2v}(C_2)$
7	$\begin{vmatrix} S_{11} & S_{12} & -S_{12} \\ -S_{12} & S_{22} & S_{23} \\ S_{12} & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$, GA $[R]_5$, GS	4	$C_{2v}(C_2)$, $C_{2v}(C_s)$- the plane is between ports 2 and 3
8	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ -S_{12} & S_{22} & S_{23} \\ -S_{12} & S_{23} & S_{22} \end{vmatrix}$	$[R]_3$, GS $[R]_5$, GA	4	$C_{2v}(C_2)$, $C_{2v}(C_s)$- the plane is between ports 2 and 3
9	$\begin{vmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & S_{23} \\ 0 & S_{32} & S_{22} \end{vmatrix}$	$[R]_3$, GA $[R]_4$, GS	4	$C_{2v}(C_s)$- the plane passes through all 3 ports
10	$\begin{vmatrix} S_{11} & S_{12} & S_{13} \\ S_{13} & S_{11} & S_{12} \\ S_{12} & S_{13} & S_{11} \end{vmatrix}$	$[R]_2$, GS	3	$D_3(C_3)$, $C_{3v}(C_3)$, $D_{3h}(C_{3h})$
11	$\begin{vmatrix} S_{11} & S_{12} & S_{12} \\ S_{12} & S_{11} & S_{12} \\ S_{12} & S_{12} & S_{11} \end{vmatrix}$	$[R]_1$, GA $[R]_2$, GS	2	$C_{3h}(C_3)$, $D_{3h}(D_3)$, $D_{3h}(C_{3v})$

Table 4. Description of Symmetrical Scattering Matrices of the Third Category:

given in Tables 2, 3 and 4. These Tables may be considered as a classification of three-ports based on magnetic symmetry of the structures in four-dimensional space. In this connection, one can speak about symmetry of a matrix [S] in space-time coordinates. For example, the matrix $[S]_{10}$ (Table 4) may have the following symmetries: $D_3(C_3), C_{3V}(C_3)$, and $D_{3h}(C_{3h})$). Besides, such a matrix (which is $[S]_6$ in Table 3) may have the symmetries C_3, D_3, C_{3V}, C_{3h} and D_{3h} of the second category.

Notice that there exists a coincidence of some matrices $[S]_i$ in Tables 2, 3 and 4. For instance, the matrix $[S]_6$ of Table 2, the matrix $[S]_7$ of Table 3 and the matrix $[S]_{11}$ of Table 4 are identical (but they are described by different symmetries). There exist all together 18 different [S]-matrices of symmetrical 3-ports.

We shall not consider the properties of the derived matrices in Tables 2, 3, 4 because these properties are clear from the structure of the matrices. Notice only, that these results are sufficient for reciprocal properties of the three-ports and not sufficient for their nonreciprocal.

Investigating the N-ports, one can use further the unitarity condition and the condition of ideal matching. However, it is necessary to remember that these two conditions may be incomputable. For example, for the matrix $[S]_9$ (Table 4) the unitarity condition gives $|S_{11}| = 1$. It means that it is impossible to match ideally such a lossless three-port.

The suggested in this paper classification of the three-ports based on symmetry in four- dimensional space has the following advantages:

1. It is applied to many of the existing devices because many of them exhibit one or another symmetry.
2. The symmetry defines some functional properties of the device because the symmetry defines the values of some of the $[S]$-elements.
3. The results of the symmetry theory do not depend on frequency, physical effects and the types of waveguides being used.

One should also emphasized that all the results obtained in this paper are valid also for the matrices [Z] and [Y] [23].

8. DEVELOPMENT OF THE THEORY

The above theory may be developed for the cases of three-ports with multimode waveguides:

a) The case of two waveguides which are connected to the inter-waveguide volume. In one of the waveguides, only one mode propagates. In the second waveguide, two modes propagate. Therefore, one can treat this structure also as a three-port. The highest group of symmetry of the cross-section for the two-mode waveguides is C_{4V} with two orthogonal polarizations of modes. Notice that the group C_{4V} is not in the group tree of D_{3V}. Besides, we must also consider two variants of the two-mode waveguides described by the group C_{2V} (with equal and different orientation of the two ports). If a square and a rectangular waveguide are connected to each other, the highest possible symmetry of their common cross-section is C_{2V} (the junction of two waveguides with different cross-sections can not raise the resulting symmetry). Here, we must also consider circular and coaxial waveguides with the symmetry $C_{\infty V}$, and their combination with rectangular and square waveguides. A new generator will appear here:

$$[R]_6 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix} \quad (9)$$

b) The case of one waveguide with three propagating modes. For a three-mode waveguide, we must consider the group C_{3V} (it is for example, the symmetry of a triangular waveguide), C_{2V} and also $C_{\infty V}$. In order to consider such waveguides as three-ports, they must be bounded from one side (if the waveguide is unbounded it must be treated as a six-port). The waveguide may be terminated for example, with a resonator or with a load. Notice that in this case, one can not consider the group C_{4V} with polarization degeneracy, because the two orthogonal in space ports are equivalent and it corresponds to the even number of propagating modes. In this analysis, several new $[S]$-matrices will appear including diagonal ones.

9. CONCLUSIONS

The theory of magnetic groups and the Curie principle allows one to find all the symmetrical solutions which are possible for the multiports with isotropic and gyrotropic media. In this paper it has been made for the three-ports. Using these results, one can define a priory some general properties of the three-ports. It must be stressed that all the properties of the three-ports are a consequence of physical symmetry alone, i e., of the geometrical symmetry of the three-port and in the case of gyrotropic media, of dc magnetic field symmetry. The results of this paper may help to narrow down the range of searching for an optimal solution for some problems.

ACKNOWLEDGMENT

This work was supported by the Brazilian Agency CAPES.

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