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THE NEAR AND FAR FIELD DISTRIBUTIONS OF A THIN CIRCULAR LOOP ANTENNA IN A RADIALLY MULTILAYERED BIISOTROPIC SPHERE

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1. INTRODUCTION

More recently, much theoretical effort has been spent on the dyadic Green's function (DGF) theory of biisotropic chiral media in conjunction with different geometries. These are the derivation of one- and two-dimensional DGF in an unbounded and stratified biisotropic chiral media in the spectral and space domains, respectively, the construction of DGF in planar, two-layer or radially arbitrary multilayered cylindrical and spherical biisotropic regions using eigenfunction expansion method combined with the scattering superposition principle, etc. [1–22]. Directly, DGF relates a current to its field, is a very efficient means of dealing with a large number of electromagnetic boundary value-problems. According to the closed form of DGF, the radiation and scattering characteristics of sources in biisotropic chiral medium have been investigated by some authors, and the relevant models include the

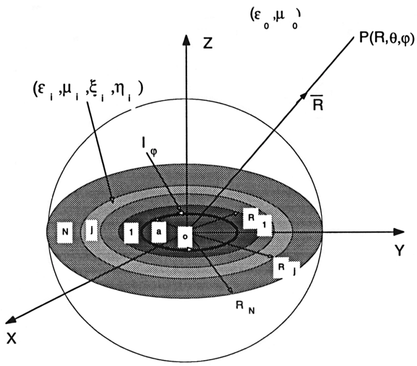


Figure 1. Geometry of a thin circular loop antenna in the N-layer spherical biisotropic media.

radiation and scattering from a thin wire antenna in unbounded chiral medium, dipole antenna radiation in the presence of a homogeneous or radially inhomogeneous chiral sphere, electromagnetic fields excited by a circular loop antenna in unbounded chiral medium and above a chiral half space, etc. [23–31].

Furthermore, this study aims at solving the problem of electromagnetic radiation from a thin circular loop antenna of arbitrary radius in a radially multilayered four-parameter biisotropic sphere, and the generalized, but analytical form of all field components in both near and far field zones for different current excitations are derived. Also, numerical examples are presented to show the unique effects of the cross coupling parameters on the near and far field characteristics. In the following sections, the mathematical treatment is based on the DGF in the form of an eigenfunction expansion using the normalized spherical vector wave function, and this procedure is very general, compact, and straightforward. It has also been used in Li's study for unbounded isotropic case more recently [32].

2. GEOMETRY OF THE PROBLEM

Fig. 1 shows the generalized geometry of a thin circular loop antenna in an arbitrary multilayered spherical biisotropic media, and the radii of N-layer regions are denoted by $R_1, \ldots, R_N$ respectively.

The constitutive characteristics for each layer of the four-parameter biisotropic media at an appropriate frequency range using the time dependence $e^{-j\omega t}$ can be described by

$$\overline{D}^{(i)} = \epsilon^{(i)}\overline{E}^{(i)} + \xi^{(i)}\overline{H}^{(i)} \tag{1a}$$

$$\overline{B}^{(i)} = \mu^{(i)}\overline{H}^{(i)} + \eta^{(i)}\overline{E}^{(i)} \qquad i = 1, \ldots, N \tag{1b}$$

where $\epsilon^{(i)}$, $\mu^{(i)}$, $\xi^{(i)}$, and $\eta^{(i)}$ are the permittivity, permeability and cross-coupling parameters, respectively. It is known that for the special case of a reciprocal chiral medium we have $\xi^{(i)} = \eta^{(i)^*} = j\kappa^{(i)}$, the superscript $*$ stands for the complex conjugation, and $\kappa^{(i)}$ is the chirality parameter. The thin circular loop is placed in the inner region and the volumetric electric current density is stated as [32]:

$$\overline{J}(R') = \frac{I(\varphi')\delta(R' - a)\delta(\theta' - \frac{\pi}{2})}{a}\overline{e}'_{\varphi} \tag{2}$$

where $I(\varphi')$ is an arbitrary function of φ', a is the radius of the circular loop $(a \leq R_1)$, and usually, the outer region $R > R_N$ is the free space (ϵ_0, μ_o).

3. MATHEMATICAL FORMULATION

Straightforwardly, the electromagnetic fields in each layer of the radially multilayered biisotropic sphere and its outer space excited by the thin circular loop antenna are determined as follows:

$$\overline{E}^{(i)} = j\omega\mu^{(1)} \iiint\limits_{v} \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') \cdot \overline{J}(\overline{R}')dV' \tag{3a}$$

$$\overline{H}^{(i)} = \iiint\limits_{v} \overline{\overline{G}}_{em}^{(i)}(\overline{R}|\overline{R}') \cdot \overline{J}(\overline{R}')dV' \tag{3b}$$

and

$$\overline{\overline{G}}_{em}^{(i)}(\overline{R}|\overline{R}') = \frac{\mu^{(1)}}{\mu^{(i)}}\nabla \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') - \frac{j\omega\eta^{(i)}\mu^{(1)}}{\mu^{(i)}}\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') \tag{3c}$$

where the subscript v denotes the volume occupied by the circular loop, $\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')$ is the electric DGF corresponding to each layer of the biisotropic sphere, and for $R > R_N$, it is noted by $\overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}')$ $(i = 0)$. Here $\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')$ takes the similar form as shown in [17] or [20] for the three-parameter reciprocal chiral medium, except that the source region term has been omitted. So it is further stated as

$$\overline{\overline{G}}_e^{(1)}(\overline{R}|\overline{R}') = \overline{\overline{G}}_{e0}^{(1)}(\overline{R}|\overline{R}') + \overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}') \tag{4a}$$

and

$$\begin{aligned}\overline{\overline{G}}_{e0}^{(1)}(\overline{R}|\overline{R}') = &- \frac{\overline{e}_R \overline{e}_R \delta(\overline{R} - \overline{R}')}{k^{(1)2}\left(1 - \frac{\xi^{(1)}\eta^{(1)}}{\epsilon^{(1)}\mu^{(1)}}\right)} + \frac{j}{2\pi(k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\ &\cdot \begin{cases} k_+^{(1)^2} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(1)}) \overline{V}'_{\substack{e\\o}mn}(k_+^{(1)}) + k_-^{(1)^2} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(1)}) \overline{W}'_{\substack{e\\o}mn}(k_-^{(1)}) \\ \qquad \text{if } a < R \leq R_1 \\ k_+^{(1)^2} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) \overline{V}'^{(1)}_{\substack{e\\o}mn}(k_+^{(1)}) + k_-^{(1)^2} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \overline{W}'^{(1)}_{\substack{e\\o}mn}(k_-^{(1)}) \\ \qquad \text{if } 0 < R \leq a \end{cases}\end{aligned} \tag{4b}$$

where $\delta(\overline{R} - \overline{R}')$ is the Dirac delta function and the factor $k^{(1)^2}(1 - \frac{\xi^{(1)}\eta^{(1)}}{\epsilon^{(1)}\mu^{(1)}})$ is to account for the cross coupling in the source region inside the inner biisotropic medium. $\overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}')$ represents the additional contributions of the multiple reflection and transmission waves from the spherical boundary $R = R_1$, expressed as

$$\begin{aligned}\overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}') = &\frac{j}{2\pi(k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\ &\cdot \left\{ \left[A^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) + B^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] \overline{V}'_{\substack{e\\o}mn}(k_+^{(1)}) \right. \\ &\left. + \left[C^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) + D^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] \overline{W}'_{\substack{e\\o}mn}(k_-^{(1)}) \right\} \\ &\quad 0 < R \leq R_1\end{aligned} \tag{4c}$$

In the region $R_{i-1} \leq R \leq R_i$ $(i = 2, \ldots, N)$, we have

$$\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') = \frac{j}{2\pi(k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn}$$

$$
\begin{aligned}
&\cdot \Big\{ \Big[A^{(i)} \overline{V}_{\substack{e\\o}mn}(k_+^{(i)}) + B^{(i)} \overline{W}_{\substack{e\\o}mn}(k_-^{(i)}) \\
&+ E^{(i)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(i)}) + F^{(i)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(i)}) \Big] \overline{V}'_{\substack{e\\o}mn}(k_+^{(1)}) \\
&+ \Big[C^{(i)} \overline{V}_{\substack{e\\o}mn}(k_+^{(i)}) + D^{(i)} \overline{W}_{\substack{e\\o}mn}(k_-^{(i)}) \\
&+ G^{(i)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(i)}) + H^{(i)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(i)}) \Big] \overline{W}'_{\substack{e\\o}mn}(k_-^{(1)}) \Big\} \quad (5)
\end{aligned}
$$

For $R \geq R_N$,

$$
\begin{aligned}
\overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}') = &\frac{j}{2\pi(k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\
&\cdot \Big\{ \Big[E^{(0)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) + F^{(0)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_0) \Big] \overline{V}'_{\substack{e\\o}mn}(k_+^{(1)}) \\
&+ \Big[G^{(0)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) + H^{(0)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_0) \Big] \overline{W}'_{\substack{e\\o}mn}(k_-^{(1)}) \Big\} \quad (6)
\end{aligned}
$$

and

$$
D_{mn} = (2 - \delta_{m0}) \frac{(2n+1)}{n(n+1)} \frac{(n-m)!}{(n+m)!} \quad (7)
$$

$$
k_\pm^{(i)} = \pm \frac{j\omega(\eta^{(i)} - \xi^{(i)})}{2} + \omega \sqrt{\epsilon^{(i)}\mu^{(i)} - \frac{(\xi^{(i)} + \eta^{(i)})^2}{4}} \quad (8\mathrm{a,b})
$$

where $k_\pm^{(i)}$ are the wavenumbers corresponding to two circularly polarized modes supported in the i th biisotropic layer, i.e., the right- and left-handed circularly polarized waves (RCP: $+$; LCP: $-$), and $k^{(i)^2} = \omega^2 \mu^{(i)} \epsilon^{(i)}$, $\delta_{m0} = \begin{cases} 1 & m = 0 \\ 0 & m \neq 0 \end{cases}$. The definitions and orthogonal properties of the normalized spherical vector wave functions $\overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(i)})$, $\overline{V}_{\substack{e\\o}mn}(k_+^{(i)})$, $\overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(i)})$ and $\overline{W}_{\substack{e\\o}mn}(k_-^{(i)})$ can be found in [5, 17, 21], and the prime in (4b)–(6) indicates the coordinates (R', θ', φ') of the source. $A^{(i)}, B^{(i)}, \ldots,$ and $H^{(i)}$ are the unknown coefficients determined by the boundary conditions at $R = R_i$ $(i = 1, \ldots, N-2)$, i.e.,

$$
\overline{e}_R \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') = \overline{e}_R \times \overline{\overline{G}}_e^{(i+1)}(\overline{R}|\overline{R}') \quad (9\mathrm{a})
$$

$$
\begin{aligned}
&\overline{e}_R \times \frac{1}{\mu^{(i)}} \Big[\nabla \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') - j\omega\eta^{(i)} \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') \Big] \\
&\quad = \overline{e}_R \times \frac{1}{\mu^{(i+1)}} \Big[\nabla \times \overline{\overline{G}}_e^{(i+1)}(\overline{R}|\overline{R}') - j\omega\eta^{(i+1)} \overline{\overline{G}}^{(i+1)}(\overline{R}|\overline{R}') \Big] \quad (9\mathrm{b})
\end{aligned}
$$

At $R = R^{(N)}$, (9b) becomes

$$\begin{aligned}&\bar{e}_R \times \frac{1}{\mu^{(N)}}\left[\nabla \times \overline{\overline{G}}_e^{(N)}(\overline{R}|\overline{R}') - j\omega\eta^{(N)}\overline{\overline{G}}_e^{(N)}(\overline{R}|\overline{R}')\right]\\ &\quad = \bar{e}_R \times \frac{1}{\mu_0}\nabla \times \overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}')\end{aligned} \tag{9c}$$

Then, inserting (4)–(6) into (9), and after some mathematical manipulations, it is found that the coefficients $A^{(1)}$, $B^{(1)}$, $C^{(1)}$, $D^{(1)}$ and $E^{(0)}$, $F^{(0)}$, $G^{(0)}$, $H^{(0)}$ are determined by

$$\left[M^{(1,1)}\right]\begin{bmatrix}A^{(1)}\\ B^{(1)}\end{bmatrix} - \left[M^{(1,2)}\right]\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\\ C_{31} & C_{32}\\ C_{41} & C_{42}\end{bmatrix}\begin{bmatrix}E^{(0)}\\ F^{(0)}\end{bmatrix} = -k_+^{(1)2}\begin{bmatrix}h_{1+}^{R_1}\\ \partial h_{1+}^{R_1}\\ h_{1+}^{R_1}/\eta_+^{(1)}\\ \partial h_{1+}^{R_1}/\eta_+^{(1)}\end{bmatrix} \tag{10a}$$

$$\left[M^{(1,1)}\right]\begin{bmatrix}C^{(1)}\\ D^{(1)}\end{bmatrix} - \left[M^{(1,2)}\right]\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\\ C_{31} & C_{32}\\ C_{41} & C_{42}\end{bmatrix}\begin{bmatrix}G^{(0)}\\ H^{(0)}\end{bmatrix} = -k_-^{(1)2}\begin{bmatrix}h_{1-}^{R_1}\\ -\partial h_{1-}^{R_1}\\ -h_{1-}^{R_1}/\eta_-^{(1)}\\ \partial h_{1-}^{R_1}/\eta_-^{(1)}\end{bmatrix} \tag{10b}$$

where

$$\eta_\pm^{(1)} = \frac{1}{\epsilon^{(1)}}\left[\pm\frac{j(\xi^{(1)}+\eta^{(1)})}{2} + \sqrt{\epsilon^{(1)}\mu^{(1)} - \frac{(\xi^{(1)}+\eta^{(1)})^2}{4}}\,\right] \tag{10c}$$

are the characteristic impedances of RCP $(+)$ and LCP $(-)$ modes in the inner region, respectively, and the relevant matrices in (10) are defined in Appendix 1.

Furthermore, substituting (2) together with (4), (5) and (6) into (3), respectively, the radiated electric fields excited by the thin circular loop antenna can be easily obtained, i.e.,

$$\overline{E}_<^{(1)} = -\frac{\omega\mu^{(1)}a}{2\sqrt{2}\pi(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$

$$\cdot \left\{ k_+^{(1)^2} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) P_{h\mp}^{(1+)} + k_-^{(1)^2} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) P_{h\pm}^{(1-)} \right.$$

$$+ \left[A^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) + B^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] P_{j\mp}^{(1+)}$$

$$\left. + \left[C^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) + D^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] P_{j\pm}^{(1-)} \right\}, \ 0 < R < a \quad (11)$$

$$\overline{E}_>^{(1)} = - \frac{\omega \mu^{(1)} a}{2\sqrt{2}\pi (k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn}$$

$$\cdot \left\{ \left[k_+^{(1)^2} \overline{V}_{\substack{e\\o}mn}^{(1)}(k_+^{(1)}) + A^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) + B^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] P_{j\mp}^{(1+)} \right.$$

$$+ \left[k_-^{(1)^2} \overline{W}_{\substack{e\\o}mn}^{(1)}(k_-^{(1)}) + C^{(1)} \overline{V}_{\substack{e\\o}mn}(k_+^{(1)}) \right.$$

$$\left. \left. + D^{(1)} \overline{W}_{\substack{e\\o}mn}(k_-^{(1)}) \right] P_{j\pm}^{(1-)} \right\}, \quad a < R \le R_1 \quad (12)$$

$$\overline{E}^{(i)} = - \frac{\omega \mu^{(1)} a}{2\sqrt{2}\pi (k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn}$$

$$\cdot \left\{ \left[A^{(i)} \overline{V}_{\substack{e\\o}mn}(k_+^{(i)}) + B^{(i)} \overline{W}_{\substack{e\\o}mn}(k_-^{(i)}) \right. \right.$$

$$\left. + E^{(i)} \overline{V}_{\substack{e\\o}mn}^{(1)}(k_+^{(i)}) + F^{(i)} \overline{W}_{\substack{e\\o}mn}^{(1)}(k_-^{(i)}) \right] P_{j\mp}^{(1+)}$$

$$+ \left[C^{(i)} \overline{V}_{\substack{e\\o}mn}(k_+^{(i)}) + D^{(i)} \overline{W}_{\substack{e\\o}mn}(k_-^{(i)}) \right.$$

$$\left. \left. + G^{(i)} \overline{V}_{\substack{e\\o}mn}^{(1)}(k_+^{(i)}) + H^{(i)} \overline{W}_{\substack{e\\o}mn}^{(1)}(k_-^{(i)}) \right] P_{j\pm}^{(1-)} \right\},$$

$$R_i \le R \le R_{i+1} \quad (13)$$

$$\overline{E}^{(0)} = - \frac{\omega \mu^{(1)} a}{2\sqrt{2}\pi (k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn}$$

$$\cdot \left\{ \left[E^{(0)} \overline{V}_{\substack{e\\o}mn}^{(1)}(k_0) + F^{(0)} \overline{W}_{\substack{e\\o}mn}^{(1)}(k_0) \right] P_{j\mp}^{(1+)} \right.$$

$$\left. + \left[G^{(0)} \overline{V}_{\substack{e\\o}mn}^{(1)}(k_0) + H^{(0)} \overline{W}_{\substack{e\\o}mn}^{(1)}(k_0) \right] P_{j\pm}^{(1-)} \right\}, \quad R \ge R_N \quad (14)$$

in which the shorthand notations in (11)–(14) are

$$P_{j\mp}^{(1+)} = \mp m P_n^m(0) \partial j_{1+}^a I_c^s - \frac{dP_n^m(0)}{d\theta'} j_{1+}^a I_s^c$$

$$\begin{aligned}
P_{j\pm}^{(1-)} &= \pm m P_n^m(0) \partial j_{1-}^a I_c^s - \frac{dP_n^m(0)}{d\theta'} j_{1-}^a I_s^c \\
P_{h\mp}^{(1+)} &= \mp m P_n^m(0) \partial h_{1+}^a I_c^s - \frac{dP_n^m(0)}{d\theta'} h_{1+}^a I_s^c \\
P_{h\pm}^{(1-)} &= \pm m P_n^m(0) \partial h_{1-}^a I_c^s - \frac{dP_n^m(0)}{d\theta'} h_{1-}^a I_s^c \\
I_s^c &= \int_0^{2\pi} \begin{matrix}\cos\\ \sin\end{matrix} (m\varphi') I(\varphi') d\varphi' \\
I_c^s &= \int_0^{2\pi} \begin{matrix}\sin\\ \cos\end{matrix} (m\varphi') I(\varphi') d\varphi' \\
j_{1\pm}^a &= j_n(k_\pm^{(1)} a) \\
\partial j_{1\pm}^a &= \frac{1}{k_\pm^{(1)} a} \frac{\partial}{\partial R'} \left[R' j_n(k_\pm^{(1)} R') \right]_{R'=a}
\end{aligned} \tag{15}$$

where $j_n(\cdot)$ and $h_n^{(1)}(\cdot)$ are the n-order spherical Bessel functions of the first and third kind, respectively. The signs $\mp$ above correspond to the even-mode $(+)$ and odd-mode $(-)$, while $\pm$ is just converse. $P_n^m(\cdot)$ is the associated Legendre function, and [32]

$$P_n^m(0) = \frac{2^m \cos\left[\frac{1}{2}(n+m)\pi\right] \Gamma\left(\frac{n+m+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{n-m}{2}+1\right)} \tag{16a}$$

$$\frac{dP_n^m(0)}{d\theta'} = -\frac{2^{m+1} \sin\left[\frac{1}{2}(n+m)\pi\right] \Gamma\left(\frac{n+m}{2}+1\right)}{\sqrt{\pi}\Gamma\left(\frac{n-m+1}{2}\right)} \tag{16b}$$

In addition, using the constitutive relations $(1a, b)$ or $(3b, c)$ the corresponding magnetic fields can be found, i.e.,

$$\begin{aligned}
\overline{H}_<^{(1)} =& \frac{j\omega\mu^{(1)} a}{2\sqrt{2}\pi(k_+^{(1)} + k_-^{(1)})} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\
& \cdot \left\{ \frac{k_+^{(1)^2}}{\eta_+^{(1)}} \overline{V}_{{}^e_o mn}(k_+^{(1)}) P_{h\mp}^{(1+)} - \frac{k_-^{(1)^2}}{\eta_-^{(1)}} \overline{W}_{{}^e_o mn}(k_-^{(1)}) P_{h\pm}^{(1-)} \right. \\
& + \left[\frac{A^{(1)}}{\eta_+^{(1)}} \overline{V}_{{}^e_o mn}(k_+^{(1)}) - \frac{B^{(1)}}{\eta_-^{(1)}} \overline{W}_{{}^e_o mn}(k_-^{(1)}) \right] P_{j\mp}^{(1+)} \\
& \left. + \left[\frac{C^{(1)}}{\eta_+^{(1)}} \overline{V}_{{}^e_o mn}(k_+^{(1)}) - \frac{D^{(1)}}{\eta_-^{(1)}} \overline{W}_{{}^e_o mn}(k_-^{(1)}) \right] P_{j\pm}^{(1-)} \right\} \quad 0 < R < a
\end{aligned} \tag{17}$$

$$\begin{aligned}\overline{H}^{(1)}_{>} =& \frac{j\omega\mu^{(1)}a}{2\sqrt{2}\pi(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}\\ &\cdot\left\{\left[\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(1)})+\frac{A^{(1)}}{\eta_+^{(1)}}\overline{V}_{\substack{e\\o}mn}(k_+^{(1)})-\frac{B^{(1)}}{\eta_-^{(1)}}\overline{W}_{\substack{e\\o}mn}(k_-^{(1)})\right]P_{j\mp}^{(1+)}\right.\\ &\left.+\left[-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(1)})+\frac{C^{(1)}}{\eta_+^{(1)}}\overline{V}_{\substack{e\\o}mn}(k_+^{(1)})-\frac{D^{(1)}}{\eta_-^{(1)}}\overline{W}_{\substack{e\\o}mn}(k_-^{(1)})\right]P_{j\pm}^{(1-)}\right\}\\ &a<R\le R_1 \end{aligned} \tag{18}$$

$$\begin{aligned}\overline{H}^{(i)} =& \frac{j\omega\mu^{(1)}a}{2\sqrt{2}\pi(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}\\ &\cdot\left\{\left[\frac{A^{(i)}}{\eta_+^{(i)}}\overline{V}_{\substack{e\\o}mn}(k_+^{(i)})-\frac{B^{(i)}}{\eta_-^{(i)}}\overline{W}_{\substack{e\\o}mn}(k_-^{(i)})\right.\right.\\ &\left.+\frac{E^{(i)}}{\eta_+^{(i)}}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(i)})-\frac{F^{(i)}}{\eta_-^{(i)}}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(i)})\right]P_{j\mp}^{(1+)}\\ &+\left[\frac{C^{(i)}}{\eta_+^{(i)}}\overline{V}_{\substack{e\\o}mn}(k_+^{(i)})-\frac{D^{(i)}}{\eta_-^{(i)}}\overline{W}_{\substack{e\\o}mn}(k_-^{(i)})\right.\\ &\left.\left.+\frac{G^{(i)}}{\eta_+^{(i)}}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_+^{(i)})-\frac{H^{(i)}}{\eta_-^{(i)}}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_-^{(i)})\right]P_{j\mp}^{(1+)}\right\}\ R_i\le R\le R_{i+1} \end{aligned} \tag{19}$$

$$\begin{aligned}\overline{H}^{(0)} =& \frac{j\omega\mu^{(1)}a}{2\sqrt{2}\pi(k_+^{(1)}+k_-^{(1)})\eta_0}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}\\ &\cdot\left\{\left[E^{(0)}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_0)-F^{(0)}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_0)\right]P_{j\mp}^{(+)}\right.\\ &\left.+\left[G^{(0)}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_0)-H^{(0)}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_0)\right]P_{j\pm}^{(-)}\right\}\quad R\ge R_N \end{aligned} \tag{20}$$

It should be noted that the field expressions (11)–(14) and (17)–(20) are valid for any current distribution on the thin circular-loop antenna and any observation point except that $R = a$. From above the scalar form of all field components with respect to the spherical coordinate system (R, θ, φ) can also be derived but are suppressed here. Now we pay our attention to two typical current distributions:

1. Fourier Series Form [31-32], i.e.,

$$I(\varphi') = \sum_{p=1}^{\infty} I_p \cos(p\varphi') \tag{21}$$

Correspondingly, the radiated fields (11)–(14) and (17)–(20) become

$$\begin{aligned}
\overline{E}_{<}^{(1)} =& -\frac{\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}I_m(1+\delta_{m0}) \\
&\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{k_+^{(1)^2}\overline{V}_{emn}(k_+^{(1)})h_{1+}^a + k_-^{(1)^2}\overline{W}_{emn}(k_-^{(1)})h_{1-}^a\right.\right. \\
&+\left[A^{(1)}\overline{V}_{emn}(k_+^{(1)}) + B^{(1)}\overline{W}_{emn}(k_-^{(1)})\right]j_{1+}^a \\
&\left.+\left[C^{(1)}\overline{V}_{emn}(k_+^{(1)}) + D^{(1)}\overline{W}_{emn}(k_-^{(1)})\right]j_{1-}^a\right\} \\
&+mP_n^m(0)\left\{k_+^{(1)^2}\overline{V}_{omn}(k_+^{(1)})\partial h_{1+}^a - k_-^{(1)^2}\overline{W}_{omn}(k_-^{(1)})\partial h_{1-}^a\right. \\
&+\left[A^{(1)}\overline{V}_{omn}(k_+^{(1)}) + B^{(1)}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1+}^a \\
&\left.\left.-\left[C^{(1)}\overline{V}_{omn}(k_+^{(1)}) + D^{(1)}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1-}^a\right\}\right\} \quad 0 < R < a
\end{aligned} \tag{22}$$

$$\begin{aligned}
\overline{E}_{>}^{(1)} =& -\frac{\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}I_m(1+\delta_{m0}) \\
&\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[k_+^{(1)^2}\overline{V}_{emn}(k_+^{(1)})\right.\right.\right. \\
&\left.+A^{(1)}\overline{V}_{emn}(k_+^{(1)}) + B^{(1)}\overline{W}_{emn}(k_-^{(1)})\right]j_{1+}^a \\
&\left.+\left[k_-^{(1)^2}\overline{W}_{emn}^{(1)}(k_-^{(1)}) + C^{(1)}\overline{V}_{emn}(k_+^{(1)}) + D^{(1)}\overline{W}_{emn}(k_-^{(1)})\right]j_{1-}^a\right\} \\
&+mP_n^m(0)\left\{\left[k_+^{(1)^2}\overline{V}_{omn}^{(1)}(k_+^{(1)})\right.\right. \\
&\left.+A^{(1)}\overline{V}_{omn}(k_+^{(1)}) + B^{(1)}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1+}^a \\
&\left.\left.-\left[k_-^{(1)^2}\overline{W}_{omn}^{(1)}(k_-^{(1)}) + C^{(1)}\overline{V}_{omn}(k_+^{(1)}) + D^{(1)}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1-}^a\right\}\right\} \\
&a < R \le R_1
\end{aligned} \tag{23}$$

$$\overline{E}^{(i)} = -\frac{\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}I_m(1+\delta_{m0})$$

$$\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[A^{(i)}\overline{V}_{emn}(k_+^{(i)})+B^{(i)}\overline{W}_{emn}(k_-^{(i)})\right.\right.\right.$$
$$\left.+E^{(i)}\overline{V}_{emn}^{(1)}(k_+^{(i)})+F^{(i)}\overline{W}_{emn}^{(1)}(k_-^{(i)})\right]j_{1+}^a$$
$$+\left[C^{(i)}\overline{V}_{emn}(k_+^{(i)})+D^{(i)}\overline{W}_{emn}(k_-^{(i)})\right.$$
$$\left.\left.+G^{(i)}\overline{V}_{emn}^{(1)}(k_+^{(i)})+H^{(i)}\overline{W}_{emn}^{(1)}(k_-^{(i)})\right]j_{1-}^a\right\}$$
$$+mP_n^m(0)\left\{\left[A^{(i)}\overline{V}_{omn}(k_+^{(i)})+B^{(i)}\overline{W}_{omn}(k_-^{(i)})\right.\right.$$
$$\left.+E^{(i)}\overline{V}_{omn}^{(1)}(k_+^{(i)})+F^{(i)}\overline{W}_{omn}^{(1)}(k_-^{(i)})\right]\partial j_{1+}^a$$
$$-\left[C^{(i)}\overline{V}_{omn}(k_+^{(i)})+D^{(i)}\overline{W}_{omn}(k_-^{(i)})\right.$$
$$\left.\left.\left.+G^{(i)}\overline{V}_{omn}^{(1)}(k_+^{(i)})+H^{(i)}\overline{W}_{omn}^{(1)}(k_-^{(i)})\right]\partial j_{1-}^a\right\}\right\}\ R_i\le R\le R_{i+1}\quad(24)$$

$$\overline{E}^{(0)}=-\frac{\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}I_m(1+\delta_{m0})$$
$$\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[E^{(0)}\overline{V}_{emn}^{(1)}(k_0)+F^{(0)}\overline{W}_{emn}^{(1)}(k_0)\right]j_{1+}^a\right.\right.$$
$$\left.+\left[G^{(0)}\overline{V}_{emn}^{(1)}(k_0)+H^{(0)}\overline{W}_{emn}^{(1)}(k_0)\right]j_{1-}^a\right\}$$
$$+mP_n^m(0)\left\{\left[E^{(0)}\overline{V}_{omn}^{(1)}(k_0)+F^{(0)}\overline{W}_{omn}^{(1)}(k_0)\right]\partial j_{1+}^a\right.$$
$$\left.\left.-\left[G^{(0)}\overline{V}_{omn}^{(1)}(k_0)+H^{(0)}\overline{W}_{omn}^{(1)}(k_0)\right]\partial j_{1-}^a\right\}\right\}\quad R\ge R_N\qquad(25)$$

and

$$\overline{H}_<^{(1)}=-\frac{j\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}I_m(1+\delta_{m0})$$
$$\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\overline{V}_{emn}(k_+^{(1)})h_{1+}^a-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}\overline{W}_{emn}(k_-^{(1)})h_{1-}^a\right.\right.$$
$$+\left[\frac{A^{(1)}}{\eta_+^{(1)}}\overline{V}_{emn}(k_+^{(1)})-\frac{B^{(1)}}{\eta_-^{(1)}}\overline{W}_{emn}(k_-^{(1)})\right]j_{1+}^a$$
$$\left.+\left[\frac{C^{(1)}}{\eta_+^{(1)}}\overline{V}_{emn}(k_+^{(1)})-\frac{D^{(1)}}{\eta_-^{(1)}}\overline{W}_{emn}(k_-^{(1)})\right]j_{1-}^a\right\}$$

$$+ mP_n^m(0)\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\overline{V}_{omn}(k_+^{(1)})\partial h_{1+}^a + \frac{k_-^{(1)^2}}{\eta_-^{(1)}}\overline{W}_{omn}(k_-^{(1)})\partial h_{1-}^a\right.$$
$$+ \left[\frac{A^{(1)}}{\eta_+^{(1)}}\overline{V}_{omn}(k_+^{(1)}) - \frac{B^{(1)}}{\eta_-^{(1)}}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1+}^a$$
$$\left.\left. - \left[\frac{C^{(1)}}{\eta_+^{(1)}}\overline{V}_{omn}(k_+^{(1)}) - \frac{D^{(1)}}{\eta_-^{(1)}}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1-}^a\right\}\right\}$$
$$0 < R < a \tag{26}$$

$$\overline{H}_>^{(1)} = \frac{j\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)} + k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}I_m(1+\delta_{m0})$$
$$\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\overline{V}_{emn}^{(1)}(k_+^{(1)})\right.\right.\right.$$
$$\left. + \frac{A^{(1)}}{\eta_+^{(1)}}\overline{V}_{emn}(k_+^{(1)}) - \frac{B^{(1)}}{\eta_-^{(1)}}\overline{W}_{emn}(k_-^{(1)})\right]j_{1+}^a$$
$$+ \left[-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}\overline{W}_{emn}^{(1)}(k_-^{(1)})\right.$$
$$\left.\left. + \frac{C^{(1)}}{\eta_+^{(1)}}\overline{V}_{emn}(k_+^{(1)}) - \frac{D^{(1)}}{\eta_-^{(1)}}\overline{W}_{emn}(k_-^{(1)})\right]j_{1-}^a\right\}$$
$$+ mP_n^m(0)\left\{\left[\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\overline{V}_{omn}^{(1)}(k_+^{(1)})\right.\right.$$
$$\left. + \frac{A^{(1)}}{\eta_+^{(1)}}\overline{V}_{omn}(k_+^{(1)}) - \frac{B^{(1)}}{\eta_-^{(1)}}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1+}^a$$
$$- \left[-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}\overline{W}_{omn}^{(1)}(k_-^{(1)})\right.$$
$$\left.\left.\left. + \frac{C^{(1)}}{\eta_+^{(1)}}\overline{V}_{omn}(k_+^{(1)}) - \frac{D^{(1)}}{\eta_-^{(1)}}\overline{W}_{omn}(k_-^{(1)})\right]\partial j_{1-}^a\right\}\right\}$$
$$a < R \leq R_1 \tag{27}$$

$$\overline{H}^{(i)} = \frac{j\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)} + k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}I_m(1+\delta_{m0})$$

$$
\begin{aligned}
&\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[\frac{A^{(i)}}{\eta_+^{(i)}}\overline{V}_{emn}(k_+^{(i)})-\frac{B^{(i)}}{\eta_-^{(i)}}\overline{W}_{emn}(k_-^{(i)})\right.\right.\right.\\
&\left.+\frac{E^{(i)}}{\eta_+^{(i)}}\overline{V}_{emn}^{(1)}(k_+^{(i)})-\frac{F^{(i)}}{\eta_-^{(i)}}\overline{W}_{emn}^{(1)}(k_-^{(i)})\right]j_{1+}^a\\
&+\left[\frac{C^{(i)}}{\eta_+^{(i)}}\overline{V}_{emn}(k_+^{(i)})-\frac{D^{(i)}}{\eta_-^{(i)}}\overline{W}_{emn}(k_-^{(i)})\right.\\
&\left.\left.+\frac{G^{(i)}}{\eta_+^{(i)}}\overline{V}_{emn}^{(1)}(k_+^{(i)})-\frac{H^{(i)}}{\eta_-^{(i)}}\overline{W}_{emn}^{(1)}(k_-^{(i)})\right]j_{1-}^a\right\}\\
&+mP_n^m(0)\left\{\left[\frac{A^{(i)}}{\eta_+^{(i)}}\overline{V}_{omn}(k_+^{(i)})-\frac{B^{(i)}}{\eta_-^{(i)}}\overline{W}_{omn}(k_-^{(i)})\right.\right.\\
&\left.+\frac{E^{(i)}}{\eta_+^{(i)}}\overline{V}_{omn}^{(1)}(k_+^{(i)})-\frac{F^{(i)}}{\eta_-^{(i)}}\overline{W}_{omn}^{(1)}(k_-^{(i)})\right]\partial j_{1+}^a\\
&+\left[\frac{C^{(i)}}{\eta_+^{(i)}}\overline{V}_{omn}(k_+^{(i)})-\frac{D^{(i)}}{\eta_-^{(i)}}\overline{W}_{omn}(k_-^{(i)})\right.\\
&\left.\left.\left.+\frac{G^{(i)}}{\eta_+^{(i)}}\overline{V}_{omn}^{(1)}(k_+^{(i)})-\frac{H^{(i)}}{\eta_-^{(i)}}\overline{W}_{omn}^{(1)}(k_-^{(i)})\right]\partial j_{1-}^a\right\}\right\}\\
&\quad R_i\leq R\leq R_{i+1}
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
\overline{H}^{(0)}=&\frac{j\omega\mu^{(1)}a}{2\sqrt{2}(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}I_m(1+\delta_{m0})\\
&\cdot\left\{-\frac{dP_n^m(0)}{d\theta'}\left\{\left[E^{(0)}\overline{V}_{emn}^{(1)}(k_0)-F^{(0)}\overline{W}_{emn}^{(1)}(k_0)\right]j_{1+}^a\right.\right.\\
&\left.+\left[G^{(0)}\overline{V}_{emn}^{(1)}(k_0)-H^{(0)}\overline{W}_{emn}^{(1)}(k_0)\right]j_{1-}^a\right\}\\
&+mP_n^m(0)\left\{\left[E^{(1)}\overline{V}_{omn}^{(1)}(k_0)-F^{(0)}\overline{W}_{omn}^{(1)}(k_0)\right]\partial j_{1+}^a\right.\\
&\left.\left.-\left[G^{(0)}\overline{V}_{omn}^{(1)}(k_0)-H^{(0)}\overline{W}_{omn}^{(1)}(k_0)\right]\partial j_{1-}^a\right\}\right\}\quad R\geq R_N.
\end{aligned}
\tag{29}
$$

As a special case, the electromagnetic fields in both near and far field zones can be obtained when the thin circular-loop antenna is excited by a sinusoidal current, i.e.,

$$I(\varphi')=I_p\cos(p\varphi') \tag{30}$$

In this case only the factor I_m in (22)–(29) should be replaced by $I_m = I_p\delta_{mp}$.

2. Uniform Current Distribution [32–34], i.e.,

$$I(\varphi') = I_0 \tag{31}$$

This represents the simplest case of the circular loop antenna radiation, and it can also be regarded as the special case of $m = 0$ shown above. Therefore, the electric fields in the near and far field zones are given in the scalar form as follows:

$$\begin{aligned}E^{(1)}_{R<} =&\frac{\omega\mu^{(1)}aI_0}{2(k^{(1)}_+ + k^{(1)}_-)R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP^{(0)'}_n\Bigg\{k^{(1)}_+ h^a_{1+}j^R_{1+} - k^{(1)}_- h^a_{1-}j^R_{1-}\\&+\left[\frac{A^{(1)}}{k^{(1)}_+}j^R_{1+} - \frac{B^{(1)}}{k^{(1)}_-}j^R_{1-}\right]j^a_{1+} + \left[\frac{C^{(1)}}{k^{(1)}_+}j^R_{1+} - \frac{D^{(1)}}{k^{(1)}_-}j^R_{1-}\right]j^a_{1-}\Bigg\}\end{aligned} \tag{32a}$$

$$\begin{aligned}E^{(1)}_{\theta<} =&\frac{\omega\mu^{(1)}aI_0}{2(k^{(1)}_+ + k^{(1)}_-)}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP^{(0)'}_n dP^{(\theta)}_n\\&\cdot\Big\{k^{(1)^2}_+ h^a_{1+}\partial j^R_{1+} - k^{(1)^2}_- h^a_{1-}\partial j^R_{1-} + \left[A^{(1)}\partial j^R_{1+} - B^{(1)}\partial j^R_{1-}\right]j^a_{1+}\\&+\left[C^{(1)}\partial j^R_{1+} - D^{(1)}\partial j^R_{1-}\right]j^a_{1-}\Big\}\end{aligned} \tag{32b}$$

$$\begin{aligned}E^{(1)}_{\varphi<} =& -\frac{\omega\mu^{(1)}aI_0}{2(k^{(1)}_+ + k^{(1)}_-)}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP^{(0)'}_n dP^{(\theta)}_n\Big\{k^{(1)^2}_+ h^a_{1+}j^R_{1+} - k^{(1)^2}_- h^a_{1-}j^R_{1-}\\&+\left[A^{(1)}j^R_{1+} - B^{(1)}j^R_{1-}\right]j^a_{1+} + \left[C^{(1)}j^R_{1+} - D^{(1)}j^R_{1-}\right]j^a_{1-}\Big\}\end{aligned} \tag{32c}$$

$$\begin{aligned}E^{(1)}_{R>} =&\frac{\omega\mu^{(1)}aI_0}{2(k^{(1)}_+ + k^{(1)}_-)R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP^{(0)'}_n\Bigg\{k^{(1)}_+ h^R_{1+}j^a_{1+} - k^{(1)}_- h^R_{1-}j^a_{1-}\\&+\left[\frac{A^{(1)}}{k^{(1)}_+}j^R_{1+} - \frac{B^{(1)}}{k^{(1)}_-}j^R_{1-}\right]j^a_{1+} + \left[\frac{C^{(1)}}{k^{(1)}_+}j^R_{1+} - \frac{D^{(1)}}{k^{(1)}_-}j^R_{1-}\right]j^a_{1-}\Bigg\}\end{aligned} \tag{33a}$$

$$\begin{aligned}E^{(1)}_{\theta>} =&\frac{\omega\mu^{(1)}aI_0}{2(k^{(1)}_+ + k^{(1)}_-)}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP^{(0)'}_n dP^{(\theta)}_n\\&\cdot\Big\{k^{(1)^2}_+ \partial h^R_{1+}j^a_{1+} - k^{(1)^2}_- \partial h^R_{1-}j^a_{1-} + \left[A^{(1)}\partial j^R_{1+} - B^{(1)}\partial j^R_{1-}\right]j^a_{1+}\end{aligned}$$

$$+\left[C^{(1)}\partial j_{1+}^{R}-D^{(1)}\partial j_{1-}^{R}\right]j_{1-}^{a}\bigg\} \tag{33b}$$

$$E_{\varphi>}^{(1)}=-\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}\bigg\{k_+^{(1)^2}h_{1+}^{R}j_{1+}^{a}+k_-^{(1)^2}h_{1-}^{R}j_{1-}^{a}$$

$$+\left[A^{(1)}j_{1+}^{R}+B^{(1)}j_{1-}^{R}\right]j_{1+}^{a}+\left[C^{(1)}j_{1+}^{R}+D^{(1)}j_{1-}^{R}\right]j_{1-}^{a}\bigg\} \tag{33c}$$

$$E_R^{(i)}=\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)'}$$

$$\cdot\left\{\left[\frac{A^{(i)}}{k_+^{(i)}}j_{i+}^{R}-\frac{B^{(i)}}{k_-^{(i)}}j_{i-}^{R}+\frac{E^{(i)}}{k_+^{(i)}}h_{i+}^{R}-\frac{F^{(i)}}{k_-^{(i)}}h_{i-}^{R}\right]j_{1+}^{a}\right.$$

$$\left.+\left[\frac{C^{(i)}}{k_+^{(i)}}j_{i+}^{R}-\frac{D^{(i)}}{k_-^{(i)}}j_{i-}^{R}+\frac{G^{(i)}}{k_+^{(i)}}h_{i+}^{R}-\frac{H^{(i)}}{k_-^{(i)}}h_{i-}^{R}\right]j_{1-}^{a}\right\} \tag{34a}$$

$$E_\theta^{(i)}=\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$

$$\cdot\left\{\left[A^{(i)}\partial j_{i+}^{R}-B^{(i)}\partial j_{i-}^{R}+E^{(i)}\partial h_{i+}^{R}-F^{(i)}\partial h_{i-}^{R}\right]j_{1+}^{a}\right.$$

$$\left.+\left[C^{(i)}\partial j_{i+}^{R}-D^{(i)}\partial j_{i-}^{R}+G^{(i)}\partial h_{i+}^{R}-H^{(i)}\partial h_{i-}^{R}\right]j_{1-}^{a}\right\} \tag{34b}$$

$$E_\varphi^{(i)}=-\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$

$$\cdot\left\{\left[A^{(i)}j_{i+}^{R}+B^{(i)}j_{i-}^{R}+E^{(i)}h_{i+}^{R}+F^{(i)}h_{i-}^{R}\right]j_{1+}^{a}\right.$$

$$\left.+\left[C^{(i)}j_{i+}^{R}+D^{(i)}j_{i-}^{R}+G^{(i)}h_{i+}^{R}+H^{(i)}h_{i-}^{R}\right]j_{1-}^{a}\right\} \tag{34c}$$

$$E_R^{(0)}=\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})k_0R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)'}h_0^R$$

$$\cdot\left[(E^{(0)}-F^{(0)})j_{1+}^{a}+(G^{(0)}-H^{(0)})j_{1-}^{a}\right] \tag{35a}$$

$$E_\theta^{(0)}=\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}\partial h_0^R$$

$$\cdot\left[(E^{(0)}-F^{(0)})j_{1+}^{a}+(G^{(0)}-H^{(0)})j_{1-}^{a}\right] \tag{35b}$$

$$E_{\varphi}^{(0)} = -\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}h_0^R \cdot\left[(E^{(0)}+F^{(0)})j_{1+}^a+(G^{(0)}+H^{(0)})j_{1-}^a\right] \tag{35c}$$

and the magnetic fields are

$$H_{R<}^{(1)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)'} \cdot\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}h_{1+}^a j_{1+}^R+\frac{k_-^{(1)}}{\eta_-^{(1)}}h_{1-}^a j_{1-}^R+\left[\frac{A^{(1)}}{k_+^{(1)}\eta_+^{(1)}}j_{1+}^R+\frac{B^{(1)}}{k_-^{(1)}\eta_-^{(1)}}j_{1-}^R\right]j_{1+}^a +\left[\frac{C^{(1)}}{k_+^{(1)}\eta_+^{(1)}}j_{1+}^R+\frac{D^{(1)}}{k_-^{(1)}\eta_-^{(1)}}j_{1-}^{\overline{R}}\right]j_{1-}^a\right\} \tag{36a}$$

$$H_{\theta<}^{(1)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)} \cdot\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}h_{1+}^a\partial j_{1+}^R+\frac{k_-^{(1)^2}}{\eta_-^{(1)}}h_{1-}^a\partial j_{1-}^R+\left[\frac{A^{(1)}}{\eta_+^{(1)}}\partial j_{1+}^R+\frac{B^{(1)}}{\eta_-^{(1)}}\partial j_{1-}^R\right]j_{1+}^a +\left[\frac{C^{(1)}}{\eta_+^{(1)}}\partial j_{1+}^R+\frac{D^{(1)}}{\eta_-^{(1)}}\partial j_{1-}^R\right]j_{1-}^a\right\} \tag{36b}$$

$$H_{\varphi<}^{(1)} = \frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)} \cdot\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}h_{1+}^a j_{1+}^R-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}h_{1-}^a j_{1-}^R+\left[\frac{A^{(1)}}{\eta_+^{(1)}}j_{1+}^R-\frac{B^{(1)}}{\eta_-^{(1)}}j_{1-}^R\right]j_{1+}^a +\left[\frac{C^{(1)}}{\eta_+^{(1)}}j_{1+}^R-\frac{D^{(1)}}{\eta_-^{(1)}}j_{1-}^R\right]j_{1-}^a\right\} \tag{36c}$$

$$H_{R>}^{(1)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)'} \cdot\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}h_{1+}^R j_{1+}^a+\frac{k_-^{(1)}}{\eta_-^{(1)}}h_{1-}^R j_{1-}^a+\left[\frac{A^{(1)}}{k_+^{(1)}\eta_+^{(1)}}j_{1+}^R+\frac{B^{(1)}}{k_-^{(1)}\eta_-^{(1)}}j_{1-}^R\right]j_{1+}^a +\left[\frac{C^{(1)}}{k_+^{(1)}\eta_+^{(1)}}j_{1+}^R+\frac{D^{(1)}}{k_-^{(1)}\eta_-^{(1)}}j_{1-}^R\right]j_{1-}^a\right\} \tag{37a}$$

$$H_{\theta>}^{(1)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$
$$\cdot\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}\partial h_{1+}^R j_{1+}^a+\frac{k_-^{(1)^2}}{\eta_-^{(1)}}\partial h_{1-}^R j_{1-}^a+\left[\frac{A^{(1)}}{\eta_+^{(1)}}\partial j_{1+}^R+\frac{B^{(1)}}{\eta_-^{(1)}}\partial j_{1-}^R\right]j_{1+}^a\right.$$
$$\left.+\left[\frac{C^{(1)}}{\eta_+^{(1)}}\partial j_{1+}^R+\frac{D^{(1)}}{\eta_-^{(1)}}\partial j_{1-}^R\right]j_{1-}^a\right\} \tag{37b}$$

$$H_{\varphi>}^{(1)} = \frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$
$$\cdot\left\{\frac{k_+^{(1)^2}}{\eta_+^{(1)}}h_{1+}^R j_{1+}^a-\frac{k_-^{(1)^2}}{\eta_-^{(1)}}h_{1-}^R j_{1-}^a+\left[\frac{A^{(1)}}{\eta_+^{(1)}}j_{1+}^R-\frac{B^{(1)}}{\eta_-^{(1)}}j_{1-}^R\right]j_{1+}^a\right.$$
$$\left.+\left[\frac{C^{(1)}}{\eta_+^{(1)}}j_{1+}^R-\frac{D^{(1)}}{\eta_-^{(1)}}j_{1-}^R\right]j_{1-}^a\right\} \tag{37c}$$

$$H_R^{(i)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)'}$$
$$\cdot\left\{\left[\frac{A^{(i)}}{k_+^{(i)}\eta_+^{(i)}}j_{i+}^R+\frac{B^{(i)}}{k_-^{(1)}\eta_-^{(i)}}j_{i-}^R+\frac{E^{(i)}}{k_+^{(i)}\eta_+^{(i)}}h_{i+}^R+\frac{F^{(i)}}{k_-^{(i)}\eta_-^{(i)}}h_{i-}^R\right]j_{1+}^a\right.$$
$$\left.+\left[\frac{C^{(i)}}{k_+^{(i)}\eta_+^{(i)}}j_{i+}^R+\frac{D^{(i)}}{k_-^{(i)}\eta_-^{(i)}}j_{i-}^R+\frac{G^{(i)}}{k_+^{(i)}\eta_+^{(i)}}h_{i+}^R+\frac{H^{(i)}}{k_-^{(i)}\eta_-^{(i)}}h_{i-}^R\right]j_{1-}^a\right\} \tag{38a}$$

$$H_{\theta}^{(i)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$
$$\cdot\left\{\left[\frac{A^{(i)}}{\eta_+^{(i)}}\partial j_{i+}^R+\frac{B^{(i)}}{\eta_-^{(i)}}\partial j_{i-}^R+\frac{E^{(i)}}{\eta_+^{(i)}}\partial h_{i+}^R+\frac{F^{(i)}}{\eta_-^{(i)}}\partial h_{i-}^R\right]j_{1+}^a\right.$$
$$\left.+\left[\frac{C^{(i)}}{\eta_+^{(i)}}\partial j_{i+}^R+\frac{D^{(i)}}{\eta_-^{(i)}}\partial j_{i-}^R+\frac{G^{(i)}}{\eta_+^{(i)}}\partial h_{i+}^R+\frac{H^{(i)}}{\eta_-^{(i)}}\partial h_{i-}^R\right]j_{1-}^a\right\} \tag{38b}$$

$$H_{\varphi}^{(i)} = \frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)'}dP_n^{(\theta)}$$

$$\cdot\left\{\left[\frac{A^{(i)}}{\eta_+^{(i)}}j_{i+}^R - \frac{B^{(i)}}{\eta_-^{(i)}}j_{i-}^R + \frac{E^{(i)}}{\eta_+^{(i)}}h_{i+}^R - \frac{F^{(i)}}{\eta_-^{(i)}}h_{i-}^R\right]j_{1+}^a\right.$$
$$\left.+\left[\frac{C^{(i)}}{\eta_+^{(i)}}j_{i+}^R - \frac{D^{(i)}}{\eta_-^{(i)}}j_{i-}^R + \frac{G^{(i)}}{\eta_+^{(i)}}h_{i+}^R - \frac{H^{(i)}}{\eta_-^{(i)}}h_{i-}^R\right]j_{1-}^a\right\} \tag{38c}$$

$$H_R^{(0)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})\eta_0 k_0 R}\sum_{n=1}^{\infty}(2n+1)P_n(\cos\theta)dP_n^{(0)\prime}h_0^R$$
$$\cdot\left[\left(E^{(0)}+F^{(0)})j_{1+}^a + (G^{(0)}+H^{(0)})j_{1-}^a\right)\right] \tag{39a}$$

$$H_\theta^{(0)} = -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})\eta_0}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)\prime}dP_n^{(\theta)}\partial h_0^R$$
$$\cdot\left[\left(E^{(0)}+F^{(0)})j_{1+}^a + (G^{(0)}+H^{(0)})j_{1-}^a\right)\right] \tag{39b}$$

$$H_\varphi^{(0)} = \frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})\eta_0}\sum_{n=1}^{\infty}\frac{(2n+1)}{n(n+1)}dP_n^{(0)\prime}dP_n^{(\theta)}h_0^R$$
$$\cdot\left[\left(E^{(0)}-F^{(0)})j_{1+}^a + (G^{(0)}-H^{(0)})j_{1-}^a\right)\right] \tag{39c}$$

where

$$dP_n^{(0)\prime} = \frac{dP_n(0)}{d\theta'}, \qquad dP_n^{(\theta)} = \frac{dP_n(\cos\theta)}{d\theta} \tag{40}$$

The above formulation show that, there exist six radiated field components for the thin circular loop antenna in a radially multilayered biisotropic sphere, while for isotropic case the field components $E_{R,\varphi}$ and H_θ are all zero. So cross coupling coefficients cause new componenets in the radiated fields in both the near and far field zones. In addition, from (32a)–(39a) it is obvious that as $R \to \infty$ the radial field component is faster approaching to zero than the other components, and only it contributes to the source region term shown in (4b). For uniform current distribution we further consider two cases:

1°. Electrically Large Size of Circular Loop Antenna $(k_\pm^{(1)}a \gg 1)$

Since the thin circular loop antenna is placed in the inner region $(a \leq R_1)$, the condition $k_\pm^{(1)}a \gg 1$ also implies that $x_\pm^{(i)} = k_\pm^{(i)}R_i \gg 1$. Under such circumstances, the spherical Bessel and Hankel functions together with their derivations can be stated as follows:

$$\begin{aligned} j_n(x_\pm^{(i)}) &\approx (-j)^{n+1}\frac{\cos x_\pm^{(i)}}{x_\pm^{(i)}} \\ \frac{d}{x_\pm^{(i)}dx_\pm^{(i)}}\left[x_\pm^{(i)}j_n(x_\pm^{(i)})\right] &\approx -(-j)^{n+1}\frac{\sin x_\pm^{(i)}}{x_\pm^{(i)}} \\ h_n^{(1)}(x_\pm^{(i)}) &\approx (-j)^{n+1}\frac{e^{jx_\pm^{(i)}}}{x_\pm^{(i)}} \\ \frac{d}{x_\pm^{(i)}dx_\pm^{(i)}}\left[x_\pm^{(i)}h_n^{(1)}(x_\pm^{(i)})\right] &\approx (-j)^{n}\frac{e^{jx_\pm^{(i)}}}{x_\pm^{(i)}} \\ x_\pm^{(i)} &\gg 1 \end{aligned} \tag{41}$$

These approximate expressions can be exploited for reducing the coefficient equations greatly for the unknown coefficients $A^{(i)}, B^{(i)}, \ldots, H^{(i)}$ in (10). In addition,

$$\sum_{n=0}^{\infty}(\pm j)^{n-m}(2n+1)\frac{(n-m)!}{(n+m)!}P_n^m(0)P_n^m(\cos\theta)j_n(k_\pm^{(i)}R) = J_{m\pm}^{(i,R)} \tag{42}$$

can be used to simplify the field components in the near or far field zone, here $J_{m\pm}^{(i,R)} = J_m(k_\pm^{(i)}R\sin\theta)$, $J_m(\cdot)$ is the cylindrical Bessel function of the first kind. For instance, in the near field zone ($0 < R < a$),

$$\begin{aligned} E_{R<}^{(1)} \approx &-\frac{\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})R_1}\sum_{n=1}^{\infty}(-j)^{n+1}(2n+1)P_n^1(0)P_n(\cos\theta) \\ &\cdot\left[e^{jk_+^{(1)}a}j_{1+}^R - e^{k_-^{(1)}a}j_{1-}^R + \frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(A^{(1)}\frac{j_{1+}^R}{k_+^{(1)}} - B^{(1)}\frac{j_{1-}^R}{k_-^{(1)}}\right)\right. \\ &\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(C^{(1)}\frac{j_{1-}^R}{k_-^{(1)}} - D^{(1)}\frac{j_{1-}^R}{k_-^{(1)}}\right)\right] \end{aligned} \tag{43a}$$

$$E_{\theta<}^{(1)} \approx -\frac{\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})}\sin\theta\left[k_+^{(1)}e^{jk_+^{(1)}a}J_{0+}^{(1,R)} - k_-^{(1)}e^{jk_-^{(1)}a}J_{0-}^{(1,R)}\right.$$

$$+\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(A^{(1)}J_{0+}^{(1,R)}-B^{(1)}J_{0-}^{(1,R)}\right)$$
$$\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(C^{(1)}J_{0+}^{(1,R)}-D^{(1)}J_{0-}^{(1,R)}\right)\right] \tag{43b}$$

$$E_{\varphi<}^{(1)}\approx\frac{\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})}\left[k_+^{(1)}e^{jk_+^{(1)}a}J_{1+}^{(1,R)}-k_-^{(1)}e^{jk_-^{(1)}a}J_{1-}^{(1,R)}\right.$$
$$+\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(A^{(1)}J_{1+}^{(1,R)}-B^{(1)}J_{1-}^{(1,R)}\right)$$
$$\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(C^{(1)}J_{1+}^{(1,R)}-D^{(1)}J_{1-}^{(1,R)}\right)\right] \tag{43c}$$

$$H_{R<}^{(1)}\approx\frac{j\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})R}\sum_{n=1}^{\infty}(-j)^{n+1}(2n+1)P_n^1(0)P_n(\cos\theta)$$
$$\cdot\left[\frac{e^{jk_+^{(1)}a}}{\eta_+^{(1)}}j_{1+}^R+\frac{e^{jk_-^{(1)}a}}{\eta_-^{(1)}}j_{1-}^R+\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(\frac{A^{(1)}}{\eta_+^{(1)}}\frac{j_{1+}^R}{k_+^{(1)}}+\frac{B^{(1)}}{\eta_-^{(1)}}\frac{j_{1-}^R}{k_-^{(1)}}\right)\right.$$
$$\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(\frac{C^{(1)}}{\eta_+^{(1)}}\frac{j_{1+}^R}{k_+^{(1)}}+\frac{D^{(1)}}{\eta_-^{(1)}}\frac{j_{1-}^R}{k_-^{(1)}}\right)\right] \tag{44a}$$

$$H_{\theta<}^{(1)}\approx\frac{j\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})}\sin\theta\left[\frac{k_+^{(1)}}{\eta_+^{(1)}}e^{jk_+^{(1)}a}J_{0+}^{(1,R)}+\frac{k_-^{(1)}}{\eta_-^{(1)}}e^{jk_-^{(1)}a}J_{0-}^{(1,R)}\right.$$
$$+\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(\frac{A^{(1)}}{\eta_+^{(1)}}J_{0+}^{(1,R)}+\frac{B^{(1)}}{\eta_-^{(1)}}J_{0-}^{(1,R)}\right)$$
$$\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(\frac{C^{(1)}}{\eta_+^{(1)}}J_{0+}^{(1,R)}+\frac{D^{(1)}}{\eta_-^{(1)}}J_{0-}^{(1,R)}\right)\right] \tag{44b}$$

$$H_{\varphi<}^{(1)}\approx\frac{j\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})}\left[-\frac{k_+^{(1)}}{\eta_+^{(1)}}e^{jk_+^{(1)}a}J_{1+}^{(1,R)}+\frac{k_-^{(1)}}{\eta_-^{(1)}}e^{jk_-^{(1)}a}J_{1-}^{(1,R)}\right.$$
$$+\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left(-\frac{A^{(1)}}{\eta_+^{(1)}}J_{1+}^{(1,R)}+\frac{B^{(1)}}{\eta_-^{(1)}}J_{1-}^{(1,R)}\right)$$
$$\left.+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left(-\frac{C^{(1)}}{\eta_+^{(1)}}J_{1+}^{(1,R)}+\frac{D^{(1)}}{\eta_-^{(1)}}J_{1-}^{(1,R)}\right)\right] \tag{44c}$$

In the near field zone $(a < R \leq R_1)$ $(k_{\pm}^{(1)}R, k_{\pm}^{(1)}R \gg 1)$,

$$\begin{aligned}
E_{R>}^{(1)} \approx & \frac{\omega\mu^{(1)} I_0}{2(k_+^{(1)} + k_-^{(1)})R^2} \sum_{n=1}^{\infty} (-1)^n (2n+1) P_n^1(0) P_n(\cos\theta) \\
& \cdot \left[\frac{\cos k_+^{(1)} a}{k_+^{(1)}} \left(e^{jk_+^{(1)}R} + \frac{A^{(1)}}{k_+^{(1)^2}} \cos k_+^{(1)} R - \frac{B^{(1)}}{k_-^{(1)^2}} \cos k_-^{(1)} R \right) \right. \\
& \left. + \frac{\cos k_-^{(1)} a}{k_-^{(1)}} \left(-e^{jk_-^{(1)}R} + \frac{C^{(1)}}{k_+^{(1)^2}} \cos k_+^{(1)} R - \frac{D^{(1)}}{k_-^{(1)^2}} \cos k_-^{(1)} R \right) \right]
\end{aligned} \tag{45a}$$

$$\begin{aligned}
E_{\theta>}^{(1)} \approx & -\frac{j\omega\mu^{(1)} a I_0}{2(k_+^{(1)} + k_-^{(1)})R} \left[k_+^{(1)} e^{jk_+^{(1)}R} J_{1+}^{(1,a)} - k_-^{(1)} e^{jk_-^{(1)}R} J_{1-}^{(1,a)} \right. \\
& + j \left(A^{(1)} \frac{\sin k_+^{(1)} R}{k_+^{(1)}} - B^{(1)} \frac{\sin k_-^{(1)} R}{k_-} \right) J_{1+}^{(1,a)} \\
& \left. + j \left(C^{(1)} \frac{\sin k_+^{(1)} R}{k_+^{(1)}} - D^{(1)} \frac{\sin k_-^{(1)} R}{k_-^{(1)}} \right) J_{1-}^{(1,a)} \right]
\end{aligned} \tag{45b}$$

$$\begin{aligned}
E_{\varphi>}^{(1)} \approx & \frac{\omega\mu^{(1)} a I_0}{2(k_+^{(1)} + k_-^{(1)})R} \left[k_+^{(1)} e^{jk_+^{(1)}R} J_{1+}^{(1,a)} + k_-^{(1)} e^{jk_-^{(1)}R} J_{1-}^{(1,a)} \right. \\
& + \left(A^{(1)} \frac{\cos k_+^{(1)} R}{k_+^{(1)}} + B^{(1)} \frac{\cos k_-^{(1)} R}{k_-^{(1)}} \right) J_{1+}^{(1,a)} \\
& \left. + \left(C^{(1)} \frac{\cos k_+^{(1)} R}{k_+^{(1)}} + D^{(1)} \frac{\cos k_-^{(1)} R}{k_-^{(1)}} \right) J_{1-}^{(1,a)} \right]
\end{aligned} \tag{45c}$$

$$\begin{aligned}
H_{R>}^{(1)} \approx & -\frac{j\omega\mu^{(1)} I_0}{2(k_+^{(1)} + k_-^{(1)})R} \sum_{n=1}^{\infty} (-j)^{n+1} (2n+1) P_n^1(0) P_n(\cos\theta) \\
& \cdot \left[\frac{\cos k_+^{(1)} a}{k_+^{(1)}} \left(\frac{e^{jk_+^{(1)}R}}{\eta_+^{(1)}} + \frac{A^{(1)}}{\eta_+^{(1)} k_+^{(1)^2}} \cos k_+^{(1)} R + \frac{B^{(1)}}{\eta_-^{(1)} k_-^{(1)^2}} \cos k_-^{(1)} R \right) \right. \\
& \left. + \frac{\cos k_-^{(1)} a}{k_-^{(1)}} \left(\frac{e^{jk_-^{(1)}R}}{\eta_-^{(1)}} + \frac{C^{(1)}}{\eta_+^{(1)} k_+^{(1)^2}} \cos k_+^{(1)} R + \frac{D^{(1)}}{\eta_-^{(1)} k_-^{(1)^2}} \cos k_-^{(1)} R \right) \right]
\end{aligned} \tag{46a}$$

$$H_{\theta>}^{(1)} \approx -\frac{\omega\mu^{(1)} a I_0}{2(k_+^{(1)} + k_-^{(1)})R} \left[\frac{k_+^{(1)}}{\eta_+^{(1)}} e^{jk_+^{(1)}R} J_{1+}^{(1,a)} + \frac{k_-^{(1)}}{\eta_-^{(1)}} e^{jk_-^{(1)}R} J_{1-}^{(1,a)} \right.$$

$$+ j\left(\frac{A^{(1)}}{\eta_+^{(1)}k_-^{(1)}}\sin k_+^{(1)}R + \frac{B^{(1)}}{\eta_-^{(1)}k_-^{(1)}}\sin k_-^{(1)}R\right)J_{1+}^{(1,a)}$$

$$+ j\left(\frac{C^{(1)}}{\eta_+^{(1)}k_+^{(1)}}\sin k_+^{(1)}R + \frac{D^{(1)}}{\eta_-^{(1)}k_-^{(1)}}\sin k_-^{(1)}R\right)J_{1-}^{(1,a)}\Bigg] \tag{46b}$$

$$H_{\varphi>}^{(1)} \approx -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\left[-\frac{k_+^{(1)}}{\eta_+^{(1)}}e^{jk_+^{(1)}R}J_{1+}^{(1,a)} + \frac{k_-^{(1)}}{\eta_-^{(1)}}e^{jk_-^{(1)}R}J_{1-}^{(1,a)}\right.$$

$$-\left(\frac{A^{(1)}}{\eta_+^{(1)}k_-^{(1)}}\cos k_+^{(1)}R - \frac{B^{(1)}}{\eta_-^{(1)}k_-^{(1)}}\cos k_-^{(1)}R\right)J_{1+}^{(1,a)}$$

$$\left.-\left(\frac{C^{(1)}}{\eta_+^{(1)}k_+^{(1)}}\cos k_+^{(1)}R - \frac{D^{(1)}}{\eta_-^{(1)}k_-^{(1)}}\cos k_-^{(1)}R\right)J_{1-}^{(1,a)}\right] \tag{46c}$$

In the region $R_i \le R \le R_{i+1}$,

$$E_R^{(i)} \approx \frac{\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})R^2}\sum_{n=1}^{\infty}(-1)^n(2n+1)P_n^1(0)P_n(\cos\theta)$$

$$\cdot\left\{\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left[A^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)^2}} - B^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)^2}}\right.\right.$$

$$\left. + E^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)^2}} - F^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)^2}}\right]$$

$$+ \frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left[C^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)^2}} - D^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)^2}}\right.$$

$$\left.\left. + G^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)^2}} - H^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)^2}}\right]\right\} \tag{47a}$$

$$E_\theta^{(i)} \approx -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}$$

$$\cdot\left\{\left[j\left(A^{(i)}\frac{\sin k_+^{(i)}R}{k_+^{(i)}} - B^{(i)}\frac{\sin k_-^{(i)}R}{k_-^{(i)}}\right)\right.\right.$$

$$\left. + E^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}} - F^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}}\right]J_{1+}^{(1,a)}$$

$$+\left[j\left(C^{(i)}\frac{\sin k_+^{(i)}R}{k_+^{(i)}}-D^{(i)}\frac{\sin k_-^{(i)}R}{k_-^{(i)}}\right)\right.$$

$$\left.\left.+G^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}}-H^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}}\right]J_{1-}^{(1,a)}\right\} \tag{47b}$$

$$E_\varphi^{(i)}\approx\frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}$$

$$\cdot\left\{\left[A^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)}}+B^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)}}\right.\right.$$

$$\left.+E^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}}+F^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}}\right]J_{1+}^{(1,a)}$$

$$+\left[C^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)}}+D^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)}}\right.$$

$$\left.\left.+G^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}}+H^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}}\right]J_{1-}^{(1,a)}\right\} \tag{47c}$$

$$H_R^{(i)}\approx-\frac{j\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})R^2}\sum_{n=1}^{\infty}(-1)^n(2n+1)P_n^1(0)P_n(\cos\theta)$$

$$\cdot\left\{\frac{\cos k_+^{(1)}a}{k_+^{(1)}}\left[A^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)^2}\eta_+^{(i)}}+B^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)^2}\eta_-^{(i)}}\right.\right.$$

$$\left.+E^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)^2}\eta_+^{(i)}}+F^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)^2}\eta_-^{(i)}}\right]$$

$$+\frac{\cos k_-^{(1)}a}{k_-^{(1)}}\left[C^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)^2}\eta_+^{(i)}}+D^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)^2}\eta_-^{(i)}}\right.$$

$$\left.\left.+G^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)^2}\eta_+^{(i)}}+H^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)^2}\eta_-^{(i)}}\right]\right\} \tag{48a}$$

$$H_\theta^{(i)}\approx\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}$$

$$\cdot\left\{\left[\left(-A^{(i)}\frac{\sin k_+^{(i)}R}{k_+^{(i)}\eta_+^{(i)}}-B^{(i)}\frac{\sin k_-^{(i)}R}{k_-^{(i)}\eta_-^{(i)}}\right)\right.\right.$$

$$
\begin{aligned}
&+ j\left(E^{(i)}\frac{je^{ik_+^{(i)}R}}{k_+^{(i)}\eta_+^{(i)}} + F^{(i)}\frac{je^{ik_-^{(i)}R}}{k_-^{(i)}\eta_-^{(i)}}\right)\Bigg]J_{1+}^{(1,a)}\\
&+\left[-C^{(i)}\frac{\sin k_+^{(i)}R}{k_+^{(i)}\eta_+^{(i)}} - D^{(i)}\frac{\sin k_-^{(i)}R}{k_-^{(i)}\eta_-^{(i)}}\right.\\
&\left.+ j\left(G^{(i)}\frac{je^{ik_+^{(i)}R}}{k_+^{(i)}\eta_+^{(i)}} + H^{(i)}\frac{je^{ik_-^{(i)}R}}{k_-^{(i)}\eta_-^{(i)}}\right)\right]J_{1-}^{(1,a)}\Bigg\} \qquad (48b)
\end{aligned}
$$

$$
\begin{aligned}
H_\varphi^{(i)} \approx& -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})R}\\
&\cdot\Bigg\{\left[A^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)}\eta_+^{(i)}} - B^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)}\eta_-^{(i)}}\right.\\
&\left.+E^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}\eta_+^{(i)}} - F^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}\eta_-^{(i)}}\right]J_{1+}^{(1,a)}\\
&+\left[C^{(i)}\frac{\cos k_+^{(i)}R}{k_+^{(i)}\eta_+^{(i)}} - D^{(i)}\frac{\cos k_-^{(i)}R}{k_-^{(i)}\eta_-^{(i)}}\right.\\
&\left.+G^{(i)}\frac{e^{ik_+^{(i)}R}}{k_+^{(i)}\eta_+^{(i)}} - H^{(i)}\frac{e^{ik_-^{(i)}R}}{k_-^{(i)}\eta_-^{(i)}}\right]J_{1-}^{(1,a)}\Bigg\} \qquad (48c)
\end{aligned}
$$

In the region $R \geq R_N$, the electric field components

$$
\begin{aligned}
E_R^{(0)} \approx& \frac{\omega\mu^{(1)}I_0}{2(k_+^{(1)}+k_-^{(1)})k_0R}\frac{e^{jk_0R}}{k_0R}\sum_{n=1}^{\infty}(-1)^n(2n+1)P_n(\cos\theta)P_n^1(0)\\
&\cdot\left[(E^{(0)}-F^{(0)})\frac{\cos k_{1-}a}{k_{1-}} + (G^{(0)}-H^{(0)})\frac{\cos k_{1-}a}{k_{1-}}\right] \qquad (49a)\\
E_\theta^{(0)} \approx& -\frac{j\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\frac{e^{jk_0R}}{k_0R}\\
&\cdot\left[(E^{(0)}-F^{(0)})J_{1+}^{(1,a)} + (G^{(0)}-H^{(0)})J_{1-}^{1,a)}\right] \qquad (49b)\\
E_\varphi^{(0)} \approx& \frac{\omega\mu^{(1)}aI_0}{2(k_+^{(1)}+k_-^{(1)})}\frac{e^{jk_0R}}{k_0R}\\
&\cdot\left[(E^{(0)}+F^{(0)})J_{1+}^{(1,a)} + (G^{(0)}+H^{(0)})J_{1-}^{(1,a)}\right] \qquad (49c)
\end{aligned}
$$

It is clear that each field component is just the combination of the RCP and LCP modes.

2°. Electrically Small Circular Loop $(x_{\pm}^{(1)} = k_{\pm}^{(1)} a \ll 1)$

For this extreme case, only the dominant term $n = 1$ must be retained in the spherical Bessel and Hankel functions, and the radiated fields in the far field zone from the electrically small circular loop antenna in a radially multilayered biisotropic sphere can be stated as:

$$\begin{aligned}\overline{E}^{(0)} \approx & -\frac{j\omega\mu^{(1)} a^2 I_0}{4(k_+^{(1)} + k_-^{(1)})k_0 R} e^{jk_0 R} \sin\theta \cdot \Big\{ (\overline{e}_\theta + j\overline{e}_\varphi)[k_+^{(1)} E^{(0)} + k_-^{(1)} G^{(0)}] \\ & - (\overline{e}_\theta - j\overline{e}_\varphi)[k_+^{(1)} F^{(0)} + k_-^{(1)} H^{(0)}] \Big\} \end{aligned} \tag{50}$$

$$\begin{aligned}\overline{H}^{(0)} \approx & -\frac{\mu^{(1)} a^2 I_0}{4(k_+^{(1)} + k_-^{(1)})\mu_0 R} e^{jk_0 R} \sin\theta \cdot \Big\{ (\overline{e}_\theta + j\overline{e}_\varphi)[k_+^{(1)} E^{(0)} + k_-^{(1)} G^{(0)}] \\ & + (\overline{e}_\theta - j\overline{e}_\varphi)[k_+^{(1)} F^{(0)} + k_-^{(1)} H^{(0)}] \Big\} \end{aligned} \tag{51}$$

This indicates that the far fields of circular loop antenna are also the sums of RCP and LCP modes. The total radiated power of the loop antenna is calculated by

$$P = \frac{1}{2} \mathrm{Re} \int \overline{e}_R \cdot (\overline{E}^{(0)} \times \overline{H}^{(0)}) ds \tag{52}$$

so inserting (49) and (50) into (51), we have

$$\begin{aligned} P = & \frac{\pi\omega {\mu^{(1)}}^2 a^4 I_0^2}{6(k_+^{(1)} + k_-^{(1)})^2 k_0 \mu_0} \left[\left| k_+^{(1)} E^{(0)} + k_-^{(1)} G^{(0)} \right|^2 \right. \\ & \left. + \left| k_+^{(1)} F^{(0)} + k_-^{(1)} H^{(0)} \right|^2 \right] \end{aligned} \tag{53}$$

Physically, the polarized state of radiated fields can be represented by the point on the Poincare's sphere with latitude 2χ as

$$\sin 2\chi = \frac{\left| k_+^{(1)} E^{(0)} + k_-^{(1)} G^{(0)} \right|^2 - \left| k_+^{(1)} F^{(0)} + k_-^{(1)} H^{(0)} \right|^2}{\left| k_+^{(1)} E^{(0)} + k_-^{(1)} G^{(0)} \right|^2 + \left| k_+^{(1)} F^{(0)} + k_-^{(1)} H^{(0)} \right|^2} \tag{54}$$

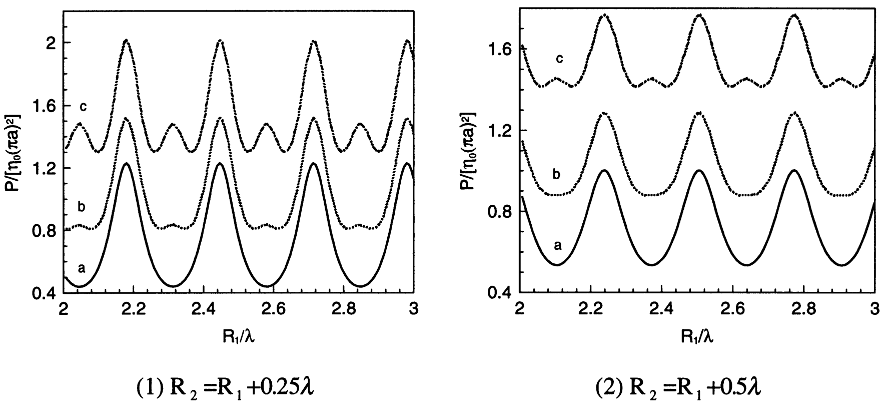


Figure 2. Normalized radiated power as a function of the radius of biisotro- pic sphere. $\epsilon^{(1)} = 3.5\epsilon_0$, $\mu^{(1)} = \mu_0$, $\epsilon^{(2)} = 2.5\epsilon_0$, $\mu^{(2)} = \mu_0$, $\xi^{(2)} = \eta^{(2)} = 0$, λ is the operating wavelength, $I_0 = 1A$, $\xi^{(1)} = \eta^{(1)^*} = j\kappa\sqrt{\mu_0\epsilon_0}$, $\kappa = 0.1(a)$, $0.5(b)$, $0.8(c)$.

where $\sin 2\chi \in [-1, 1]$ is the ellipticity of the polarization ellipse and it can indicate the right- and left-hand elliptically, circularly, and linear polarized waves of the far fields, respectively.

4. NUMERICAL EXAMPLES

Based on the developed mathematical formulation for the radiated fields in different regions, some numerical examples are presented to show the influences of cross coupling parameters on the radiated fields of thin circular loop antenna. The relevant constitutive parameters of the radially multilayered biisotropic sphere chosen for calculation are in a realizable range and related to some publications. Here the loss of biisotropic medium is neglected so that the attenuation can not mask effects produced by cross coupling parameters.

At first, Figs. 2(1) and (2) depict the radiated power characteristics of an electrically small thin circular loop antenna in a two-layer spherical structure $(k_{\pm}^{(1)} a \ll 1)$, and such geometry is assumed to be a biisotropic sphere covered by one-layer isotropic medium corresponding to different thickness, respectively.

In Figs. 2(1) and (2), since the cross coupling parameters are chosen to be $\xi^{(1)} = \eta^{(1)^*} = j\kappa^{(1)}\sqrt{\mu_0\epsilon_0}$, it just stands for the reciprocal chiral

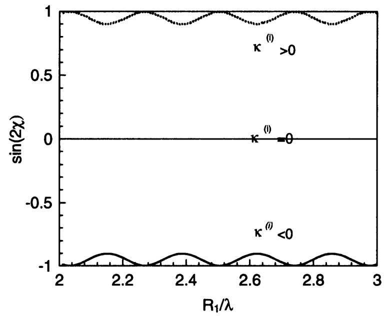


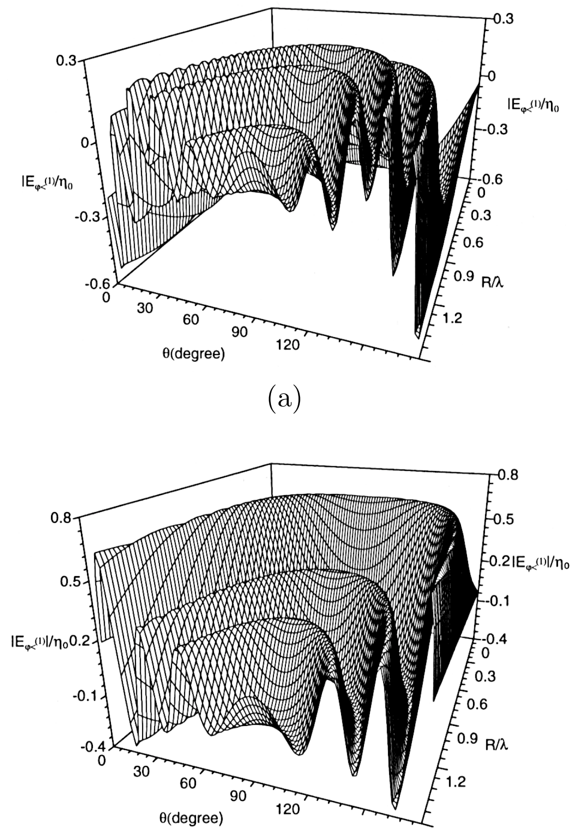
Figure 3. The ellipticity of the polarization ellipse as a function of the radii of four-layer biisotropic sphere. $I_0 = 1A$, $\epsilon^{(1)} = 4.5\epsilon_0$, $\epsilon^{(2)} = 3.5\epsilon_0$, $\epsilon^{(3)} = 2.5\epsilon_0$, $\epsilon^{(4)} = 1.5\epsilon_0$, $\mu^{(i)} = \mu_0$, $i = 1,2,3,4$, $\xi^{(i)} = \eta^{(i)*} = j\kappa^{(i)}\sqrt{\mu_0\epsilon_0}$, $\kappa^{(1)} = 1.0$, $\kappa^{(2)} = 0.8$, $\kappa^{(3)} = 0.6$, $\kappa^{(4)} = 0.4$, $R_2 = R_1 + 0.25\lambda$, $R_3 = R_1 + 0.5\lambda$, $R_4 = R_1 + 0.75\lambda$.

case $(R \leq R_1)$. In addition, this two-layer sphere is assumed to be non-magnetic $(\mu^{(i)} = \mu_0)$, and the thickness of the cover layer is equal to be $\frac{\lambda}{4}$ and $\frac{\lambda}{2,}$ respectively. It is interesting to note that, with an increasing of $\kappa^{(1)}$ the radiated power curve is enhanced greatly for both cover layers. When $\kappa^{(1)}$ reaches certain magnitude, sub-peak is developed between two main resonant peaks, while the locations of all main resonant peaks do not change for the given cover layer. By changing the thickness of cover layer the height of sub-peak can be adjusted effectively.

Fig. 3 shows the influence of $\kappa^{(i)}$ on the polarized state of radiated field for an electrically small circular loop antenna in a four-layer biisotropic sphere $(k_{\pm}^{(1)}a \ll 1, k_{\pm}^{(i)}R_i \gg 1)$.

It is evident that, the polarized state of the radiated fields is completely governed by the cross coupling or chirality parameter, and it is generally in the right- or left-handed elliptically polarized state. For a positive value of $\kappa^{(i)}$, it is $0 < \sin 2\chi < 1$, and conversely, for a negative value of $\kappa^{(i)}$, $-1 < \sin 2\chi < 0$. In an extreme case, the right- or left-handed circularly polarized wave could be achieved $(\sin 2\chi \approx \pm 1)$. When $\kappa^{(i)} = 0$, i.e., for isotropic structure, the radiated fields is in the pure linear polarized state.

As another illustrative example, the three-dimensional near field pattern excited by a thin circular loop antenna in a four-layer biisotropic sphere is demonstrated in Fig. 4(a), while Fig. 4(b) corre-



(a)

(b)

Figure 4. Three-dimensional pattern of the near zone $E^{(1)}_{\varphi<}$ as a function of θ for various R. $I_0 = 1A$, $\epsilon^{(1)} = 4.5\epsilon_0$, $\epsilon^{(2)} = 3.5\epsilon_0$, $\epsilon^{(3)} = 2.5\epsilon_0$, $\epsilon^{(4)} = 1.5\epsilon_0$, $\mu^{(i)} = \mu_0$, $i = 1, 2, 3, 4$, $R_1 = 2\lambda$, $R_2 = R_1 + 0.25\lambda$, $R_3 = R_1 + 0.5\lambda$, $R_4 = R_1 + 0.75\lambda$. (a) $\xi^{(i)} = \eta^{(i)*} = j\kappa^{(i)}\sqrt{\mu_0\epsilon_0}$, $\kappa^{(1)} = 1.0$, $\kappa^{(2)} = 0.8$, $\kappa^{(3)} = 0.6$, $\kappa^{(4)} = 0.4$. (b) $\xi^{(i)} = \eta^{(i)} = 0$.

sponds to the isotropic case. The constitutive parameters chosen for calculation are the same as in Fig. 3, and the radius of the circular loop is assumed to be $a = 1.5\lambda$, so we have $k^{(i)}_{\pm} a \gg 1$. Under such circumstances, the relevant radiated field components in the near zone $0 < R < a$ or $a < R \leq R_1$ are given by (43)–(46).

In Fig. 4, only the field component $E^{(1)}_{\varphi<}$ in the near zone $0 < R < a$

is shown. It is obvious that strong resonance occurs in the near zone, and the variation of such resonance for four-layered biisotropic sphere is the combination of RCP and LCP modes, so it is more complex than that of isotropic structure. In addition, similar conclusions can be drawn for another field components in the near zone.

5. CONCLUSIONS

Here, we have examined the radiation from a thin circular loop antenna in an arbitrary multilayered biisotropic sphere. The technique of dyadic Green's function in terms of the normalized spherical vector wave functions has been imposed in the mathematical analysis. Such method is very general as well as straightforward to apply. The formulation developed for calculating the radiated fields in both near and far zones is also suitable for radially multilayered reciprocal chiral and isotropic spheres. It is believed that the present work can provide us deep physical insight into the cross coupling effects on the radiation effects of sources in inhomogeneous biisotropic regions.

APPENDIX 1

In (10),

$$\left[M^{(1,1)}\right]=\begin{bmatrix} j_{1+}^{R_1} & j_{1-}^{R_1} \\ \partial j_{1+}^{R_1} & -\partial j_{1-}^{R_1} \\ \dfrac{j_{1+}^{R_1}}{\eta_+^{(1)}} & -\dfrac{j_{1-}^{R_1}}{\eta_-^{(1)}} \\ \dfrac{\partial j_{1+}^{R_1}}{\eta_+^{(1)}} & \dfrac{\partial j_{1-}^{R_1}}{\eta_-^{(1)}} \end{bmatrix}$$

$$\left[M^{(0,0)}\right]=\begin{bmatrix} h_0^{R_N} & h_0^{R_N} \\ \partial h_0^{R_N} & -\partial h_0^{R_N} \\ \dfrac{h_0^{R_N}}{\eta_0} & -\dfrac{h_0^{R_N}}{\eta_0} \\ \dfrac{\partial h^{R_N}}{\eta_0} & \dfrac{\partial h^{R_N}}{\eta_0} \end{bmatrix}$$

$$\begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \\ C_{31} & C_{32} \\ C_{41} & C_{42} \end{bmatrix}=\left[M^{(2,2)}\right]^{-1}\left[M^{(2,3)}\right]\cdots$$

$$\cdots\left[M^{(i,i)}\right]^{-1}\left[M^{(i,i+1)}\right]\cdots\left[M^{(N,N)}\right]^{-1}\left[M^{(0,0)}\right]$$

$$\left[M^{(i,i)}\right] = \begin{bmatrix} j_{i+}^{R_i} & j_{i-}^{R_i} & h_{i+}^{R_i} & h_{i-}^{R_i} \\ \partial j_{i+}^{R_i} & -\partial j_{i-}^{R_i} & \partial h_{i+}^{R_i} & -\partial h_{i-}^{R_i} \\ \frac{j_{i+}^{R_i}}{\eta_+^{(i)}} & -\frac{j_{i-}^{R_i}}{\eta_-^{(i)}} & \frac{h_{i+}^{R_i}}{\eta_+^{(i)}} & -\frac{h_{i-}^{R_i}}{\eta_-^{(i)}} \\ \frac{\partial j_{i+}^{R_i}}{\eta_+^{(i)}} & \frac{\partial j_{i-}^{R_i}}{\eta_-^{(i)}} & \frac{\partial h_{i+}^{R_i}}{\eta_+^{(i)}} & \frac{\partial h_{i-}^{R_i}}{\eta_-^{(i)}} \end{bmatrix}, \ i = 1, 2 \ldots, N$$

$$\left[M^{(i,i+1)}\right] = \begin{bmatrix} j_{(i+1)+}^{R_i} & j_{(i+1)-}^{R_i} & h_{(i+1)+}^{R_i} & h_{(i+1)-}^{R_i} \\ \partial j_{(i+1)+}^{R_i} & -\partial j_{(i+1)-}^{R_i} & \partial h_{(i+1)+}^{R_i} & -\partial h_{(i+1)-}^{R_i} \\ \frac{j_{(i+1)+}^{R_i}}{\eta_+^{(i+1)}} & -\frac{j_{(i+1)-}^{R_i}}{\eta_-^{(i+1)}} & \frac{h_{(i+1)+}^{R_i}}{\eta_+^{(i+1)}} & -\frac{h_{(i+1)-}^{R_i}}{\eta_-^{(i+1)}} \\ \frac{\partial j_{(i+1)+}^{R_i}}{\eta_+^{(i+1)}} & \frac{\partial j_{(i+1)-}^{R_i}}{\eta_-^{(i+1)}} & \frac{\partial h_{(i+1)+}^{R_i}}{\eta_+^{(i+1)}} & \frac{\partial h_{(i+1)-}^{R_i}}{\eta_-^{(i+1)}} \end{bmatrix},$$

$$\begin{aligned} i =& 1, 2 \ldots, N-1 \\ j_{i\pm}^{R_i} =& j_n(k_\pm^{(i)} R_i), \ \partial j_{i\pm}^{R_i} = \frac{1}{k_\pm^{(i)} R_i} \frac{\partial}{\partial R} \left[R j_n(k_\pm^{(i)} R)\right]_{R=R_i} \\ h_{i\pm}^{R_i} =& h_n^{(1)}(k_\pm^{(i)} R_i), \ \partial h_{i\pm}^{R_i} = \frac{1}{k_\pm^{(i)} R_i} \frac{\partial}{\partial R} \left[R h_n^{(1)}(k_\pm^{(i)} R)\right]_{R=R_i} \\ \eta_\pm^{(i)} =& \frac{1}{\epsilon^{(1)}} \left[\pm \frac{j(\xi^{(i)} + \eta^{(i)})}{2} + \sqrt{\epsilon^{(i)} \mu^{(i)} - \frac{(\xi^{(i)} + \eta^{(i)})^2}{4}}\right] \end{aligned} \quad \text{(A1)}$$

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