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ELECTROMAGNETIC FIELDS OF TWO THIN CIRCULAR LOOP ANTENNAS IN A RADIALLY MULTILAYERED BIISOTROPIC CYLINDER

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1. INTRODUCTION

It is well known that there exists a substantial body of literature devoted to the radiation characteristics of circular loop antennas in homogeneous and inhomogeneous isotropic media [1–3]. On the other hand, the radiation of canonical sources in unbounded and bounded biisotropic chiral regions has gained some attention in electromagnetic community more recently, and the studies carried out include the radiation from a thin wire antenna in an unbounded reciprocal chiral medium [4–6], dipole antenna radiation in the presence of homogeneous and radially inhomogeneous chiral spheres, cylinders, and grounded biisotropic chiral slabs, respectively [7–15], electromagnetic fields excited by a circular loop antenna in an unbounded chiral medium, above a chiral half space as well as in a radially multilayered biisotropic sphere [16–18].

In contrast to the previous work, this paper aims at solving the problem of electromagnetic radiation from two thin circular loop antennas of arbitrary radius in a radially multilayered biisotropic cylinder. The mathematical treatment here is based on the DGF in the form of an eigenfunction expansion using the normalized cylindrical vector wave functions, and such method is very general, compact, and straightforward. It has also been used in the authors' study for the geometry of a multilayered biisotropic sphere recently [18]. While for this biisotropic cylindrical structure, the analytical form of all field components in both near and far field zones for different current excitations is derived in the following sections, and numerical examples are presented to show the unique effects of the magnetoelectric cross coupling parameters on the radiated field characteristics of circular loop antennas.

2. GEOMETRY OF THE PROBLEM

Fig. 1 shows the generalized geometry of two thin circular loop antennas in a radially multilayered biisotropic cylinder, and the radii of N-layer regions are denoted by $r_1, \cdots, r_N$, respectively.

The constitutive characteristics for each layer of the biisotropic media at an appropriate frequency range using the time dependence $e^{-j\omega t}$ can be described by

$$\overline{D}^{(i)} = \varepsilon^{(i)}\overline{E}^{(i)} + \xi^{(i)}\overline{H}^{(i)} \tag{1a}$$

$$\overline{B}^{(i)} = \mu^{(i)}\overline{H}^{(i)} + \eta^{(i)}\overline{E}^{(i)} \qquad i = 1, \cdots, N \tag{1b}$$

where $\varepsilon^{(i)}$, $\mu^{(i)}$, $\xi^{(i)}$ and $\eta^{(i)}$ are the permittivity, permeability and cross-coupling parameters, respectively. It is known that for the special case of a reciprocal chiral medium we have $\xi^{(i)} = \eta^{(i)*} = j\kappa^{(i)}$, the superscript * stands for the complex conjugation, and $\kappa^{(i)}$ is the chirality parameter. Here, two thin circular loop antennas are placed in the inner region and the volumetric electric current density are stated as:

$$\overline{J}_q(\overline{R}') = \frac{I_q\left(\varphi_q'\right)\delta\left(r_q' - a_q\right)\delta\left(z_q' - z_q\right)}{a_q}\overline{e}_{q\varphi}' \qquad q = 1, 2 \tag{2}$$

where $\overline{e}_{q\varphi}'$ is the unit vector of the φ_q'-coordinate with respect to the source coordinates (r_q', φ_q', z_q'), $I(\varphi_q')$ is an arbitrary function of φ_q', a_q is the radius of the circular loop $(a_q \leq r_1)$, and usually, the outer region $r > r_N$ is the free space $(\varepsilon_0,\ \mu_0)$.

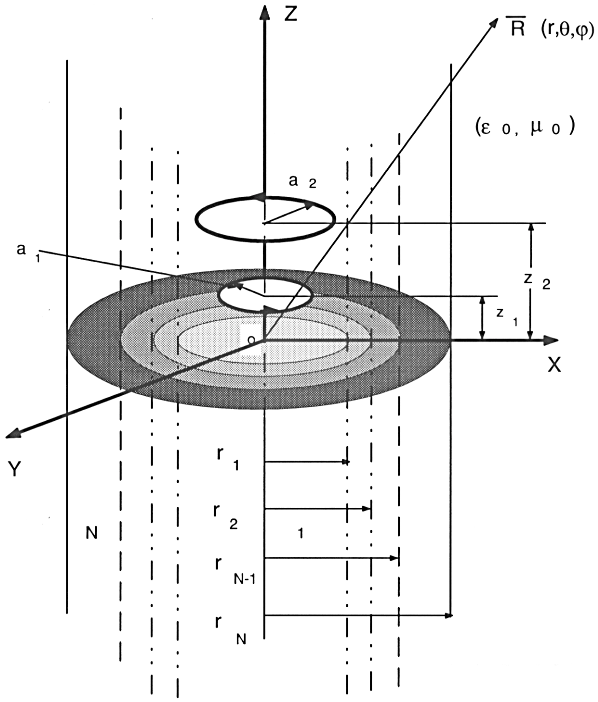


Figure 1. Geometry of two thin circular loop antennas in a N-layer biisotropic cylinder $(a_1 < a_2 \leq r_1)$.

3. MATHEMATICAL FORMULATION

The electromagnetic fields in each layer of the radially multilayered biisotropic cylinder and its outer space excited by two thin circular loop antennas are determined as follows:

$$\overline{E}^{(i)} = j\omega\mu^{(1)} \int_{\nu} \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') \cdot \left[\overline{J}_1(\overline{R}') + \overline{J}_2(\overline{R}')\right] dV' \tag{3a}$$

$$\overline{H}^{(i)} = \int_{\nu} \overline{\overline{G}}_{em}^{(i)}(\overline{R}|\overline{R}') \cdot \left[\overline{J}_1(\overline{R}') + \overline{J}_2(\overline{R}')\right] dV' \tag{3b}$$

and

$$\overline{\overline{G}}_{em}^{(i)}(\overline{R}|\overline{R}') = \frac{\mu^{(1)}}{\mu^{(i)}} \nabla \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') - \frac{j\omega\eta^{(i)}\mu^{(1)}}{\mu^{(i)}} \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') \tag{3c}$$

where the subscript ν denotes the volume occupied by the circular loops, $\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')$ is the electric DGF corresponding to each layer of the biisotropic cylinder, and for $r > r_N$, it is noted by $\overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}')(i = 0)$. Here $\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')$ takes the similar form as shown in [12] for the three-parameter reciprocal chiral medium, except that the source region term has been omitted. So it is further stated as

$$\overline{\overline{G}}_e^{(1)}(\overline{R}|\overline{R}') = \overline{\overline{G}}_{e0}^{(1)}(\overline{R}|\overline{R}') + \overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}') \tag{4a}$$

and

$$\begin{aligned}\overline{\overline{G}}_{e0}^{(1)}(\overline{R}|\overline{R}') = &-\frac{\overline{e}_r\overline{e}_r\delta(\overline{R}-\overline{R}')}{k^{(1)2}\left(1-\dfrac{\xi^{(1)}\eta^{(1)}}{\varepsilon^{(1)}\mu^{(1)}}\right)} \\ &+\frac{j}{4\pi\left(k_+^{(1)}+k_-^{(1)}\right)}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\delta_{n0}^{(2)} \\ &\cdot\begin{cases}\tilde{K}_+^{(1)}\overline{V}_{\substack{e\\0}n\eta_+^{(1)}}(h)\overline{V}'_{\substack{e\\0}n\eta_+^{(1)}}(-h)+\tilde{K}_-^{(1)}\overline{W}^{(1)}_{\substack{e\\0}n\eta_-^{(1)}}(h)\overline{W}'_{\substack{e\\0}n\eta_-^{(1)}}(-h) \\ \text{if } a_q < r \le r_1 \\ \tilde{K}_+^{(1)}\overline{V}_{\substack{e\\0}n\eta_+^{(1)}}(h)\overline{V}'^{(1)}_{\substack{e\\0}n\eta_+^{(1)}}(-h)+\tilde{K}_-^{(1)}\dfrac{k_-^{(1)}}{\eta_-^2}\overline{W}_{\substack{e\\0}n\eta_-^{(1)}}(h)\overline{W}'^{(1)}_{\substack{e\\0}n\eta_-^{(1)}}(-h) \\ \text{if } 0 < r < a_q\end{cases}\end{aligned} \tag{4b}$$

where $\overline{e}_r$, is the unit vector of the r-coordinate in the coordinate system (r, φ, z), $\delta(\overline{R}-\overline{R}')$ is the Dirac delta function and the factor $k^{(1)2}\left(1-\frac{\xi^{(1)}\eta^{(1)}}{\varepsilon^{(1)}\mu^{(1)}}\right)$ is to account for the cross coupling in the source region inside the inner biisotropic medium, $k^{(1)^2} = \omega^2\mu^{(1)}\varepsilon^{(1)}$, $\delta_{n0}^{(2)} = 2-\delta_{n0}$, $\delta_{n0} = \begin{cases}1 & n=0\\ 0 & n\neq 0\end{cases}$, $\tilde{K}_\pm^{(1)} = \frac{k_\pm^{(1)}}{\eta_\pm^{(1)2}}$. $\overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}')$ represents the additional contributions of the multiple reflection and transmission waves from the cylindrical boundary $r = r_1$, and expressed as [12]

$$\begin{aligned}\overline{\overline{G}}_{es}^{(1)}(\overline{R}|\overline{R}') =& \frac{j}{4\pi\left(k_+^{(1)}+k_-^{(1)}\right)}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\delta_{n0}^{(2)} \\ &\cdot\Big\{\tilde{K}_+^{(1)}\Big[A_1^{(1)}\overline{V}_{\substack{e\\o}n\eta_+^{(1)}}(h)+A_2^{(1)}\overline{W}_{\substack{e\\o}n\eta_-^{(1)}}(h) \\ &+A_3^{(1)}\overline{V}_{\substack{o\\e}n\eta_+^{(1)}}(h)+A_4^{(1)}\overline{W}_{\substack{o\\e}n\eta_-^{(1)}}(h)\Big]\overline{V}'_{\substack{e\\o}n\eta_+^{(1)}}(-h)\end{aligned}$$

$$+\tilde{K}_{-}^{(1)}\left[B_1^{(1)}\overline{V}_{\substack{e\\o}n\eta_+^{(1)}}(h)+B_2^{(1)}\overline{W}_{\substack{e\\o}n\eta_-^{(1)}}(h)\right.$$
$$\left.+B_3^{(1)}\overline{V}_{\substack{o\\e}n\eta_+^{(1)}}(h)+B_4^{(1)}\overline{W}_{\substack{o\\e}n\eta_-^{(1)}}(h)\right]\overline{W}'_{\substack{e\\o}n\eta_-^{(1)}}(-h)\Big\}$$
$$0<r\le r_1 \tag{4c}$$

In the region $r_{i-1}\le r\le r_i$ $(i=2,\ldots,N)$, we have

$$\begin{aligned}\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')=&\frac{j}{4\pi\left(k_+^{(1)}+k_-^{(1)}\right)}\int_{-\infty}^{+\infty}dh\sum_{n=0}^{\infty}\delta_{n0}^{(2)}\\
&\cdot\Big\{\tilde{K}_+^{(1)}\Big[A_1^{(i)}\overline{V}_{\substack{e\\o}n\eta_+^{(i)}}(h)+A_2^{(i)}\overline{W}_{\substack{e\\o}n\eta_-^{(i)}}(h)+A_3^{(i)}\overline{V}_{\substack{o\\e}n\eta_+^{(i)}}(h)\\
&+A_4^{(i)}\overline{W}_{\substack{o\\e}n\eta_-^{(i)}}(h)+A_5^{(i)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_+^{(i)}}(h)+A_6^{(i)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_-^{(i)}}(h)\\
&+A_7^{(i)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_+^{(i)}}(h)+A_8^{(i)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_-^{(i)}}(h)\Big]\overline{V}'_{\substack{e\\o}n\eta_+^{(1)}}(-h)\\
&+\tilde{K}_-^{(1)}\Big[B_1^{(i)}\overline{V}_{\substack{e\\o}n\eta_+^{(i)}}(h)+B_2^{(i)}\overline{W}_{\substack{e\\o}n\eta_-^{(i)}}(h)+B_3^{(i)}\overline{V}_{\substack{o\\e}n\eta_+^{(i)}}(h)\\
&+B_4^{(i)}\overline{W}_{\substack{o\\e}n\eta_-^{(i)}}(h)+B_5^{(i)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_+^{(i)}}(h)+B_6^{(i)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_-^{(i)}}(h)\\
&+B_7^{(i)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_+^{(i)}}(h)+B_8^{(i)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_-^{(i)}}(h)\Big]\overline{W}'_{\substack{e\\o}n\eta_+^{(1)}}(-h)\Big\}\end{aligned}\tag{5}$$

For $r\ge r_N$,

$$\begin{aligned}\overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}')=&\frac{j}{4\pi\left(k_+^{(1)}+k_-^{(1)}\right)}\int_{-\infty}^{+\infty}dh\sum_{n=0}^{\infty}\delta_{n0}^{(2)}\\
&\cdot\Big\{\tilde{K}_+^{(1)}\Big[A_1^{(0)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_0}(h)+A_2^{(0)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_0}(h)+A_3^{(0)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_0}(h)\\
&+A_4^{(0)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_0}(h)\Big]\overline{V}'_{\substack{e\\o}n\eta_+^{(1)}}(-h)\\
&+\tilde{K}_-^{(1)}\Big[B_1^{(0)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_0}(h)+B_2^{(0)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_0}(h)+B_3^{(0)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_0}(h)\\
&+B_4^{(0)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_0}(h)\Big]\overline{W}'_{\substack{e\\o}n\eta_-^{(1)}}(-h)\Big\}\end{aligned}\tag{6}$$

where

$$k_\pm^{(i)}=\pm\frac{j\omega\left(\eta^{(i)}-\xi^{(i)}\right)}{2}+\omega\sqrt{\varepsilon^{(i)}\mu^{(i)}-\frac{\left(\xi^{(i)}+\eta^{(i)}\right)^2}{4}}\tag{7a, b}$$

and $\eta_\pm^{(i)^2} = k_\pm^{(i)^2} - h^2$ $(i = 1, \ldots, N)$, $\eta_0^2 = k_0^2 - h^2$, $k_0^2 = \omega^2\mu_0\epsilon_0$. $k_\pm^{(i)}$ are the wavenumbers corresponding to two circularly polarized modes supported in the ith biisotropic layer, i.e., the right- and left-handed circularly polarized waves $(RCP : +; LCP : -)$. The definitions and orthogonal properties of the normalized cylindrical vector wave functions $\overline{V}^{(1)}_{{}^e_o n\eta_+^{(i)}}(h)$, $\overline{V}_{{}^e_o n\eta_+^{(i)}}(h)$, $\overline{W}^{(1)}_{{}^e_o n\eta_-^{(i)}}(h)$ and $\overline{W}_{{}^e_o n\eta_-^{(i)}}(h)$ can be found in [12], and the prime in (4b)–(6) indicates the coordinates (r', φ', z') of the source. The notations ${}^e_o n\eta_\pm^{(i)}$ and ${}^o_e n\eta_\pm^{(i)}$ of the normalized cylindrical vector wave functions mean that the summation form of both even and odd modes should be taken into account when the above integral is evaluated. $A_1^{(i)}$–$A_8^{(i)}$, $B_1^{(i)}$–$B_8^{(i)}, \ldots, A_1^{(0)}$–$A_4^{(0)}$ and $B_1^{(0)}$–$B_4^{(0)}$ are the unknown coefficients determined by the boundary conditions at $r = r_i$ $(i = 1, \ldots, r_{N-1})$, i.e.,

$$\overline{e}_r \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') = \overline{e}_r \times \overline{\overline{G}}_e^{(i+1)}(\overline{R}|\overline{R}') \tag{8a}$$

$$\begin{aligned}\overline{e}_r \times \frac{1}{\mu^{(i)}} &\left[\nabla \times \overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}') - j\omega\eta^{(i)}\overline{\overline{G}}_e^{(i)}(\overline{R}|\overline{R}')\right] \\ &= \overline{e}_r \times \frac{1}{\mu^{(i+1)}}\left[\nabla \times \overline{\overline{G}}_e^{(i+1)}(\overline{R}|\overline{R}') - j\omega\eta^{(i+1)}\overline{\overline{G}}_e^{(i+1)}(\overline{R}|\overline{R}')\right]\end{aligned} \tag{8b}$$

At $r = r_N$, (8b) becomes

$$\begin{aligned}\overline{e}_r \times \frac{1}{\mu^{(N)}} &\left[\nabla \times \overline{\overline{G}}_e^{(N)}(\overline{R}|\overline{R}') - j\omega\eta^{(N)}\overline{\overline{G}}_e^{(N)}(\overline{R}|\overline{R}')\right] \\ &= \overline{e}_r \times \frac{1}{\mu_0}\nabla \times \overline{\overline{G}}_e^{(0)}(\overline{R}|\overline{R}')\end{aligned} \tag{8c}$$

Then, inserting (4)–(6) into (8), and after tedious mathematical manipulations, four sets of linear equations independently can be derived for determining all the unknown coefficients.

Furthermore, substituting (2) together with (4)–(6) into (3), respectively, the radiated electric fields excited by two thin circular loop antennas in each layer can be found, i.e.,

$$\overline{E}_<^{(1)} = \overline{E}^{(10)} + \overline{E}^{(1s)} \qquad 0 < r < r_1 \tag{9a}$$

and

$$\overline{E}^{(10)} = -\frac{c}{\sqrt{2}\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \delta_{n0}^{(2)}$$

$$\cdot\begin{cases}\Big\{\tilde{K}_{+}^{(1)}\overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)\left[e_1P_{H\pm}^{(1+,1)}+e_2P_{H\pm}^{(1+,2)}\right] & \\ +\tilde{K}_{-}^{(1)}\overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)\left[e_1P_{H\mp}^{(1-,1)}+e_2P_{H\mp}^{(1-,2)}\right]\Big\} & \text{if } 0<r<a_1\\ \Big\{\tilde{K}_{+}^{(1)}\left[\overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)e_1P_{J\pm}^{(1+,1)}+\overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)e_2P_{H\pm}^{(1+,2)}\right] & \\ +\tilde{K}_{-}^{(1)}\left[\overline{W}^{(1)}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)e_1P_{J\mp}^{(1-,1)}+\overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)e_2P_{H\mp}^{(1-,2)}\right]\Big\} & \text{if } a_1<r<a_2\\ \Big\{\tilde{K}_{+}^{(1)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)\left[e_1P_{J\pm}^{(1+,1)}+e_2P_{J\pm}^{(1+,2)}\right] & \\ +\tilde{K}_{-}^{(1)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)\left[e_1P_{J\mp}^{(1-,1)}+e_2P_{J\mp}^{(1-,2)}\right]\Big\} & \text{if } a_2<r<r_1\end{cases}\tag{9b}$$

$$\begin{aligned}\overline{E}^{(1s)}=&-\frac{c}{\sqrt{2}\pi}\int_{-\infty}^{+\infty}dh\sum_{n=0}^{\infty}\delta_{n0}^{(2)}\\ &\cdot\Big\{\tilde{K}_{+}^{(1)}\Big[A_1^{(1)}\overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)+A_2^{(1)}\overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)+A_3^{(1)}\overline{V}_{\substack{o\\e}n\eta_{+}^{(1)}}(h)\\ &+A_4^{(1)}\overline{W}_{\substack{o\\e}n\eta_{-}^{(1)}}(h)\Big]\left[e_1P_{J\pm}^{(1+,1)}+e_2P_{J\pm}^{(1+,2)}\right]\\ &+\tilde{K}_{-}^{(1)}\Big[B_1^{(1)}\overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h)+B_2^{(1)}\overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h)+B_3^{(1)}\overline{V}_{\substack{o\\e}n\eta_{+}^{(1)}}(h)\\ &+B_4^{(1)}\overline{W}_{\substack{o\\e}n\eta_{-}^{(1)}}(h)\Big]\left[e_1P_{J\mp}^{(1-,1)}+e_2P_{J\mp}^{(1-,2)}\right]\Big\}\end{aligned}\tag{10}$$

$$\begin{aligned}\overline{E}^{(i)}=&-\frac{c}{\sqrt{2}\pi}\int_{-\infty}^{+\infty}dh\sum_{n=0}^{\infty}\delta_{n0}^{(2)}\\ &\cdot\Big\{\tilde{K}_{+}^{(1)}\Big[A_1^{(i)}\overline{V}_{\substack{e\\o}n\eta_{+}^{(i)}}(h)+A_2^{(i)}\overline{W}_{\substack{e\\o}n\eta_{-}^{(i)}}(h)+A_3^{(i)}\overline{V}_{\substack{o\\e}n\eta_{+}^{(i)}}(h)\\ &+A_4^{(i)}\overline{W}_{\substack{o\\e}n\eta_{-}^{(i)}}(h)+A_5^{(i)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(i)}}(h)+A_6^{(i)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_{-}^{(i)}}(h)\\ &+A_7^{(i)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_{+}^{(i)}}(h)+A_8^{(i)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_{-}^{(i)}}(h)\Big]\left[e_1P_{J\pm}^{(1+,1)}+e_2P_{J\pm}^{(1+,2)}\right]\\ &+\tilde{K}_{-}^{(1)}\Big[B_1^{(i)}\overline{V}_{\substack{e\\o}n\eta_{+}^{(i)}}(h)+B_2^{(i)}\overline{W}_{\substack{e\\o}n\eta_{-}^{(i)}}(h)+B_3^{(i)}\overline{V}_{\substack{o\\e}n\eta_{+}^{(i)}}(h)\\ &+B_4^{(i)}\overline{W}_{\substack{o\\e}n\eta_{-}^{(i)}}(h)+B_5^{(i)}\overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(i)}}(h)+B_6^{(i)}\overline{W}^{(1)}_{\substack{e\\o}n\eta_{-}^{(i)}}(h)\\ &+B_7^{(i)}\overline{V}^{(1)}_{\substack{o\\e}n\eta_{+}^{(i)}}(h)+B_8^{(i)}\overline{W}^{(1)}_{\substack{o\\e}n\eta_{-}^{(i)}}(h)\Big]\left[e_1P_{J\mp}^{(1-,1)}+e_2P_{J\mp}^{(1-,2)}\right]\Big\}\\ &\qquad r_{i-1}\le r\le r_i\ (i=2,\ldots,N)\end{aligned}\tag{11}$$

$$\begin{aligned}\overline{E}^{(0)} =& -\frac{c}{\sqrt{2}\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty}\delta_{n0}^{(2)} \\ &\cdot\left\{\tilde{K}_{+}^{(1)}\left[A_1^{(0)}\overline{V}_{\substack{e\\o}n\eta_0}^{(1)}(h) + A_2^{(0)}\overline{W}_{\substack{e\\o}n\eta_0}^{(1)}(h) + A_3^{(0)}\overline{V}_{\substack{o\\e}n\eta_0}^{(1)}(h)\right.\right. \\ &\left. +A_4^{(0)}\overline{W}_{\substack{o\\e}n\eta_0}^{(1)}(h)\right]\left[e_1 P_{J\pm}^{(1+,1)} + e_2 P_{J\pm}^{(1+,2)}\right] \\ &+\tilde{K}_{-}^{(1)}\left[B_1^{(0)}\overline{V}_{\substack{e\\o}n\eta_0}^{(1)}(h) + B_2^{(0)}\overline{W}_{\substack{e\\o}n\eta_0}^{(1)}(h) + B_3^{(0)}\overline{V}_{\substack{o\\e}n\eta_0}^{(1)}(h)\right. \\ &\left.\left. +B_4^{(0)}\overline{W}_{\substack{o\\e}n\eta_0}^{(1)}(h)\right]\left[e_1 P_{J\mp}^{(1-,1)} + e_2 P_{J\mp}^{(1-,2)}\right]\right\} \\ & \qquad r \geq r_N \end{aligned} \tag{12}$$

where the shorthand notations in (9)–(12) are

$$\begin{aligned} P_{J\pm}^{(1+,q)} &= \pm\frac{jhn}{k_{+}^{(1)}a_q} J_{n+}^{(1a_q)} I_{cq}^{s} - dJ_{n+}^{(1a_q)} I_{sq}^{c}, \\ P_{J\mp}^{(1-,q)} &= \mp\frac{jhn}{k_{-}^{(1)}a_q} J_{n-}^{(1a_q)} I_{cq}^{s} - dJ_{n-}^{(1a_q)} I_{sq}^{c}, \\ P_{H\pm}^{(1+,q)} &= \pm\frac{jhn}{k_{+}^{(1)}a_q} H_{n+}^{(1a_q)} I_{cq}^{s} - dH_{n+}^{(1a_q)} I_{sq}^{c}, \\ P_{H\mp}^{(1-,q)} &= \mp\frac{jhn}{k_{-}^{(1)}a_q} H_{n-}^{(1a_q)} I_{cq}^{s} - dH_{n-}^{(1a_q)} I_{sq}^{c}, \\ I_{sq}^{c} &= \int_0^{2\pi} \begin{matrix}\cos\\ \sin\end{matrix}(m\varphi') I_q(\varphi') d\varphi', \\ I_{cq}^{s} &= \int_0^{2\pi} \begin{matrix}\sin\\ \cos\end{matrix}(m\varphi') I_q(\varphi') d\varphi', \quad e_q = e^{-jh_1 z_q}, \\ J_{n\pm}^{(1a_q)} &= J_n\left(\eta_{\pm}^{(1)}a_q\right), \quad dJ_{n\pm}^{(1a_q)} = \frac{dJ_n\left(\eta_{\pm}^{(1)}r'\right)}{dr'}\Big|_{r'=a_q}, \\ H_{n\pm}^{(1a_q)} &= H_n^{(1)}\left(\eta_{\pm}^{(1)}a_q\right), \quad dH_{n\pm}^{(1a_q)} = \frac{dH_n^{(1)}\left(\eta_{\pm}^{(1)}r'\right)}{dr'}\Big|_{r'=a_q}, \\ c &= \frac{\omega\mu^{(1)}}{4\left(k_{+}^{(1)}+k_{-}^{(1)}\right)}, \quad q = 1, 2. \end{aligned} \tag{13}$$

where $J_n(\cdot)$ and $H_n^{(1)}(\cdot)$ are the n-order cylindrical Bessel and Hankel functions of the first kind, respectively.

In addition, using the constitutive relations (1a, b) or (3b, c) the corresponding magnetic fields can be obtained, i.e.,

$$\overline{H}_{<}^{(1)} = \overline{H}^{(10)} + \overline{H}^{(1s)} \qquad 0 < r < r_1 \tag{14a}$$

and

$$\overline{H}^{(10)} = \frac{jc}{\sqrt{2}\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \delta_{n0}^{(2)}$$

$$\cdot \begin{cases} \left\{ \frac{\tilde{K}_{+}^{(1)}}{Z_{+}^{(1)}} \overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) \left[e_1 P_{H\pm}^{(1+,1)} + e_2 P_{H\pm}^{(1+,2)} \right] \right. \\ \left. - \frac{\tilde{K}_{-}^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) \left[e_1 P_{H\mp}^{(1-,1)} + e_2 P_{H\mp}^{(1-,2)} \right] \right\} \text{ if } 0 < r < a_1 \\ \left\{ \frac{\tilde{K}_{+}^{(1)}}{Z_{+}^{(1)}} \left[\overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) e_1 P_{J\pm}^{(1+,1)} + \overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) e_2 P_{H\pm}^{(1+,2)} \right] \right. \\ \left. - \frac{\tilde{K}_{-}^{(1)}}{Z_{-}^{(1)}} \left[\overline{W}^{(1)}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) e_1 P_{J\mp}^{(1-,1)} + \overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) e_2 P_{H\mp}^{(1-,2)} \right] \right\} \\ \qquad \text{if } a_1 < r < a_2 \\ \left\{ \frac{\tilde{K}_{+}^{(1)}}{Z_{+}^{(1)}} \overline{V}^{(1)}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) \left[e_1 P_{J\pm}^{(1+,1)} + e_2 P_{J\pm}^{(1+,2)} \right] \right. \\ \left. - \frac{\tilde{K}_{-}^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) \left[e_1 P_{J\mp}^{(1-,1)} + e_2 P_{J\mp}^{(1-,2)} \right] \right\} \text{ if } a_2 < r < r_2 \end{cases} \tag{14b}$$

$$\overline{H}^{(1s)} = \frac{jc}{\sqrt{2}\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \delta_{n0}^{(2)}$$

$$\cdot \left\{ \tilde{K}_{+}^{(1)} \left[\frac{A_1^{(1)}}{Z_{+}^{(1)}} \overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) - \frac{A_2^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) + \frac{A_3^{(1)}}{Z_{+}^{(1)}} \overline{V}_{\substack{o\\e}n\eta_{+}^{(1)}}(h) \right. \right.$$

$$\left. - \frac{A_4^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{o\\e}n\eta_{-}^{(1)}}(h) \right] \left[e_1 P_{J\pm}^{(1+,1)} + e_2 P_{J\pm}^{(1+,2)} \right]$$

$$+ \tilde{K}_{-}^{(1)} \left[\frac{B_1^{(1)}}{Z_{+}^{(1)}} \overline{V}_{\substack{e\\o}n\eta_{+}^{(1)}}(h) - \frac{B_2^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{e\\o}n\eta_{-}^{(1)}}(h) + \frac{B_3^{(1)}}{Z_{+}^{(1)}} \overline{V}_{\substack{o\\e}n\eta_{+}^{(1)}}(h) \right.$$

$$\left. \left. - \frac{B_4^{(1)}}{Z_{-}^{(1)}} \overline{W}_{\substack{o\\e}n\eta_{-}^{(1)}}(h) \right] \left[e_1 P_{J\mp}^{(1-,1)} + e_2 P_{J\mp}^{(1-,2)} \right] \right. \quad 0 < r < r_1 \tag{14c}$$

$$
\begin{aligned}
\overline{H}^{(i)} = {} & \frac{jc}{\sqrt{2}\pi}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \delta_{n0}^{(2)} \\
& \cdot \Bigg\{ \tilde{K}_{+}^{(1)} \left[\frac{A_1^{(i)}}{Z_{+}^{(i)}} \overline{V}_{\substack{e\\o} n\eta_{+}^{(i)}}(h) - \frac{A_2^{(i)}}{Z_{-}^{(i)}} \overline{W}_{\substack{e\\o} n\eta_{-}^{(i)}}(h) + \frac{A_3^{(i)}}{Z_{+}^{(i)}} \overline{V}_{\substack{o\\e} n\eta_{+}^{(i)}}(h) \right. \\
& - \frac{A_4^{(i)}}{Z_{-}^{(i)}} \overline{W}_{\substack{o\\e} n\eta_{-}^{(i)}}(h) + \frac{A_5^{(i)}}{Z_{+}^{(i)}} \overline{V}^{(1)}_{\substack{e\\o} n\eta_{+}^{(i)}}(h) - \frac{A_6^{(i)}}{Z_{-}^{(i)}} \overline{W}^{(1)}_{\substack{e\\o} n\eta_{-}^{(i)}}(h) \\
& \left. + \frac{A_7^{(i)}}{Z_{+}^{(i)}} \overline{V}^{(1)}_{\substack{o\\e} n\eta_{+}^{(i)}}(h) - \frac{A_8^{(i)}}{Z_{-}^{(i)}} \overline{W}^{(1)}_{\substack{o\\e} n\eta_{-}^{(i)}}(h) \right] \left[e_1 P_{J\pm}^{(1+,1)} + e_2 P_{J\pm}^{(1+,2)} \right] \\
& + \tilde{K}_{-}^{(1)} \left[\frac{B_1^{(i)}}{Z_{+}^{(i)}} \overline{V}_{\substack{e\\o} n\eta_{+}^{(i)}}(h) - \frac{B_2^{(i)}}{Z_{-}^{(i)}} \overline{W}_{\substack{e\\o} n\eta_{-}^{(i)}}(h) + \frac{B_3^{(i)}}{Z_{+}^{(i)}} \overline{V}_{\substack{o\\e} n\eta_{+}^{(i)}}(h) \right. \\
& - \frac{B_4^{(i)}}{Z_{-}^{(i)}} \overline{W}_{\substack{o\\e} n\eta_{-}^{(i)}}(h) + \frac{B_5^{(i)}}{Z_{+}^{(i)}} \overline{V}^{(1)}_{\substack{e\\o} n\eta_{+}^{(1)}}(h) - \frac{B_6^{(i)}}{Z_{-}^{(i)}} \overline{W}^{(1)}_{\substack{e\\o} n\eta_{-}^{(i)}}(h) \\
& \left. + \frac{B_7^{(i)}}{Z_{+}^{(i)}} \overline{V}^{(1)}_{\substack{o\\e} n\eta_{+}^{(i)}}(h) - \frac{B_8^{(i)}}{Z_{-}^{(i)}} \overline{W}^{(1)}_{\substack{o\\e} n\eta_{-}^{(i)}}(h) \right] \left[e_1 P_{J\mp}^{(1-,1)} + e_2 P_{J\mp}^{(1-,2)} \right] \Bigg\} \\
& \qquad r_{i-1} \le r \le r_i \ (i = 2, \cdots, N)
\end{aligned} \tag{16}
$$

$$
\begin{aligned}
\overline{H}^{(0)} = {} & \frac{jc}{\sqrt{2}\pi Z_0}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \delta_{n0}^{(2)} \\
& \cdot \Big\{ \tilde{K}_{+}^{(1)} \Big[A_1^{(0)} \overline{V}^{(1)}_{\substack{e\\o} n\eta_0}(h) - A_2^{(0)} \overline{W}^{(1)}_{\substack{o\\e} n\eta_0}(h) + A_3^{(0)} \overline{V}^{(1)}_{\substack{o\\e} n\eta_0}(h) \\
& - A_4^{(0)} \overline{W}^{(1)}_{\substack{o\\e} n\eta_0}(h) \Big] \left[e_1 P_{J\pm}^{(1+,1)} + e_2 P_{J\pm}^{(1+,2)} \right] \\
& + \tilde{K}_{-}^{(1)} \Big[B_1^{(0)} \overline{V}^{(1)}_{\substack{e\\o} n\eta_0}(h) - B_2^{(0)} \overline{W}^{(1)}_{\substack{e\\o} n\eta_0}(h) + B_3^{(0)} \overline{V}^{(1)}_{\substack{o\\e} n\eta_0}(h) \\
& - B_4^{(0)} \overline{W}^{(1)}_{\substack{o\\e} n\eta_0}(h) \Big] \left[e_1 P_{J\mp}^{(1-,1)} + e_2 P_{J\mp}^{(1-,2)} \right] \Big\} \quad r \ge r_N
\end{aligned} \tag{17}
$$

and

$$
Z_{\pm}^{(i)} = \frac{1}{\varepsilon^{(i)}} \left[\pm \frac{j\left(\xi^{(i)} + \eta^{(i)}\right)}{2} + \sqrt{\varepsilon^{(i)}\mu^{(i)} - \frac{\left(\xi^{(i)} + \eta^{(i)}\right)^2}{4}} \right] \tag{18}
$$

are the characteristic impedances of RCP (+) and LCP (-) modes in the i th biisotropic layer, respectively, $Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$. It should be noted that the field expressions (9)–(12) and (14)–17) are valid for any current distribution on the thin circular loop antenna and any observation point except that $r = a_q (q = 1, 2)$. From above the scalar forms of all field components with respect to the cylindrical coordinate system (r, φ, z) can also be derived but are suppressed here. Now we pay our attention to the following current distributions:

1. Fourier Series Form [1–3], i.e.,

$$I_q(\varphi') = \sum_{p=1}^{\infty} I_{pq} \cos(p\varphi') \tag{19}$$

Substituting (19) into (13), the radiated electric fields (9)–(12) can be expressed as

$$\begin{aligned}\overline{E}^{(10)}_{r<a_1} = &- \frac{c}{\sqrt{2}} \int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}^{(2)}_{n0} \\ &\cdot \left\{ \tilde{K}^{(1)}_{+} \left\{ \overline{V}_{en\eta^{(1)}_{+}}(h) \left[-\tilde{H}^{(1)}_{+} \right] + \overline{V}_{on\eta^{(1)}_{+}}(h) \left[-\tilde{H}^{(2)}_{+} \right] \right\} \right. \\ &\left. + \tilde{K}^{(1)}_{-} \left\{ \overline{W}_{en\eta^{(1)}_{-}}(h) \left[-\tilde{H}^{(1)}_{-} \right] + \overline{W}_{on\eta^{(1)}_{-}}(h) \left[-\tilde{H}^{(2)}_{-} \right] \right\} \right\}\end{aligned} \tag{20a}$$

where $\tilde{\delta}^{(2)}_{n0} = (2 - \delta_{n0})(1 + \delta_{n0})$, $\tilde{H}^{(1)}_{\pm} = \sum_{q=1}^{2} e_q d H^{(1a_q)}_{n+} I_{nq}$, $\tilde{H}^{(2)}_{\pm} = \sum_{q=1}^{2} e_q \frac{jhn}{k^{(1)}_{+} a_q} H^{(1a_q)}_{n+} I_{nq}$.

$$\begin{aligned}\overline{E}^{(10)}_{a_1<r<a_2} = &-\frac{c}{\sqrt{2}} \int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}^{(2)}_{n0} \\ &\cdot \left\{ \tilde{K}^{(1)}_{+} \left[-\overline{V}^{(1)}_{en\eta^{(1)}_{+}}(h) e_1 d J^{(1a_1)}_{n+} I_{n1} - \overline{V}_{en\eta^{(1)}_{+}}(h) e_2 d H^{(1a_2)}_{n+} I_{n2} \right. \right. \\ &\left. -\overline{V}^{(1)}_{on\eta^{(1)}_{+}}(h) e_1 \frac{jhn}{k^{(1)}_{+} a_1} J^{(1a_1)}_{n+} I_{n1} - \overline{V}_{on\eta^{(1)}_{+}}(h) e_2 \frac{jhn}{k^{(1)}_{+} a_2} H^{(1a_2)}_{n+} I_{n2} \right] \\ &+ \tilde{K}^{(1)}_{-} \left[-\overline{W}^{(1)}_{en\eta^{(1)}_{-}}(h) e_1 d J^{(1a_1)}_{n-} I_{n1} - \overline{W}_{on\eta^{(1)}_{-}}(h) e_2 d H^{(1a_2)}_{n-} I_{n2} \right. \\ &\left. \left. +\overline{W}^{(1)}_{on\eta^{(1)}_{-}}(h) e_1 \frac{jhn}{k^{(1)}_{-} a_1} J^{(1a_1)}_{n-} I_{n1} + \overline{W}_{on\eta^{(1)}_{-}}(h) e_2 \frac{jhn}{k^{(1)}_{-} a_2} H^{(1a_2)}_{n-} I_{n2} \right] \right\}\end{aligned} \tag{20b}$$

$$\overline{E}^{(10)}_{a_2<r<r_2} = -\frac{c}{\sqrt{2}}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}^{(2)}_{n0}$$
$$\cdot \left\{ \tilde{K}^{(1)}_{+} \left\{ \overline{V}^{(1)}_{en\eta^{(1)}_{+}}(h) \left[-\tilde{J}^{(1)}_{+} \right] + \overline{V}^{(1)}_{on\eta^{(1)}_{+}}(h) \left[-\tilde{J}^{(2)}_{+} \right] \right\} \right.$$
$$\left. + \tilde{K}^{(1)}_{-} \left\{ \overline{W}^{(1)}_{en\eta^{(1)}_{-}}(h) \left[-\tilde{J}^{(1)}_{-} \right] + \overline{W}^{(1)}_{on\eta^{(1)}_{-}}(h) \left[-\tilde{J}^{(2)}_{-} \right] \right\} \right\} \tag{20c}$$

$$\overline{E}^{(1s)} = -\frac{c}{\sqrt{2}}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}^{(2)}_{n0}$$
$$\cdot \left\{ \tilde{K}^{(1)}_{+} \left\{ \left[A^{(1)}_1 \overline{V}_{en\eta^{(1)}_{+}}(h) + A^{(1)}_2 \overline{W}_{en\eta^{(1)}_{-}}(h) + A^{(1)}_3 \overline{V}_{on\eta^{(1)}_{+}}(h) \right.\right.\right.$$
$$\left. + A^{(1)}_4 \overline{W}_{on\eta^{(1)}_{-}}(h) \right] \left[-\tilde{J}^{(1)}_{+} \right] + \left[A^{(1)}_1 \overline{V}_{on\eta^{(1)}_{+}}(h) + A^{(1)}_2 \overline{W}_{on\eta^{(1)}_{-}}(h) \right.$$
$$\left.\left. + A^{(1)}_3 \overline{V}_{en\eta^{(1)}_{+}}(h) + A^{(1)}_4 \overline{W}_{en\eta^{(1)}_{-}}(h) \right] \left[-\tilde{J}^{(2)}_{+} \right] \right\} \}$$
$$+ \tilde{K}^{(1)}_{-} \left\{ \left[B^{(1)}_1 \overline{V}_{en\eta^{(1)}_{+}}(h) + B^{(1)}_2 \overline{W}_{en\eta^{(1)}_{-}}(h) + B^{(1)}_3 \overline{V}_{on\eta^{(1)}_{+}}(h) \right.\right.$$
$$\left. + B^{(1)}_4 \overline{W}_{on\eta^{(1)}_{-}}(h) \right] \left[-\tilde{J}^{(1)}_{-} \right] + \left[B^{(1)}_1 \overline{V}_{on\eta^{(1)}_{+}}(h) + B^{(1)}_2 \overline{W}_{on\eta^{(1)}_{-}}(h) \right.$$
$$\left.\left. + B^{(1)}_3 \overline{V}_{en\eta^{(1)}_{+}}(h) + B^{(1)}_4 \overline{W}_{en\eta^{(1)}_{-}}(h) \right] \tilde{J}^{(2)}_{-} \right\} \} \quad 0 < r < r_1 \tag{20d}$$

$$\overline{E}^{(i)} = -\frac{c}{\sqrt{2}}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}^{(2)}_{n0}$$
$$\cdot \left\{ \tilde{K}^{(1)}_{+} \left\{ \left[A^{(i)}_1 \overline{V}_{en\eta^{(i)}_{+}}(h) + A^{(i)}_2 \overline{W}_{en\eta^{(i)}_{-}}(h) + A^{(i)}_3 \overline{V}_{on\eta^{(i)}_{+}}(h) \right.\right.\right.$$
$$+ A^{(i)}_4 \overline{W}_{on\eta^{(i)}_{-}}(h) + A^{(i)}_5 \overline{V}^{(1)}_{en\eta^{(i)}_{+}}(h) + A^{(i)}_6 \overline{W}^{(1)}_{en\eta^{(i)}_{-}}(h)$$
$$\left. + A^{(i)}_7 \overline{V}^{(1)}_{on\eta^{(i)}_{+}}(h) + A^{(i)}_8 \overline{W}^{(1)}_{on\eta^{(i)}_{-}}(h) \right] \left[-\tilde{J}^{(1)}_{+} \right]$$
$$+ \left[A^{(i)}_1 \overline{V}_{on\eta^{(i)}_{+}}(h) + A^{(i)}_2 \overline{W}_{on\eta^{(i)}_{-}}(h) + A^{(i)}_3 \overline{V}_{en\eta^{(i)}_{+}}(h) \right.$$
$$+ A^{(i)}_4 \overline{W}_{en\eta^{(i)}_{-}}(h) + A^{(i)}_5 \overline{V}^{(1)}_{on\eta^{(i)}_{+}}(h) + A^{(i)}_6 \overline{W}^{(1)}_{on\eta^{(i)}_{-}}(h)$$
$$\left. + A^{(i)}_7 \overline{V}^{(1)}_{en\eta^{(i)}_{+}}(h) + A^{(i)}_8 \overline{W}^{(1)}_{en\eta^{(i)}_{-}}(h) \right] \left[-\tilde{J}^{(2)}_{+} \right]$$
$$+ \tilde{K}^{(1)}_{-} \left\{ \left[B^{(i)}_1 \overline{V}_{en\eta^{(i)}_{+}}(h) + B^{(i)}_2 \overline{W}_{en\eta^{(i)}_{-}}(h) + B^{(i)}_3 \overline{V}_{on\eta^{(i)}_{+}}(h) \right.\right.$$

$$+ B_4^{(i)}\overline{W}_{on\eta_-^{(i)}}(h) + B_5^{(i)}\overline{V}^{(1)}_{en\eta_+^{(i)}}(h) + B_6^{(i)}\overline{W}^{(1)}_{en\eta_-^{(i)}}(h)$$
$$+B_7^{(i)}\overline{V}^{(1)}_{on\eta_+^{(i)}}(h) + B_8^{(i)}\overline{W}^{(1)}_{on\eta_-^{(i)}}(h)\Big]\left[-\tilde{J}_-^{(1)}\right]$$
$$+\Big[B_1^{(i)}\overline{V}_{on\eta_+^{(i)}}(h) + B_2^{(i)}\overline{W}_{on\eta_-^{(i)}}(h) + B_3^{(i)}\overline{V}_{en\eta_+^{(i)}}(h)$$
$$+ B_4^{(i)}\overline{W}_{en\eta_-^{(i)}}(h) + B_5^{(i)}\overline{V}^{(1)}_{on\eta_+^{(i)}}(h) + B_6^{(i)}\overline{W}^{(1)}_{on\eta_-^{(i)}}(h)$$
$$+B_7^{(i)}\overline{V}^{(1)}_{en\eta_+^{(i)}}(h) + B_8^{(i)}\overline{W}^{(1)}_{en\eta_-^{(i)}}(h)\Big]\tilde{J}_-^{(2)}\Big\}\Big\}$$
$$i = 2, \cdots, N, \; r_{i-1} \le r \le r_i \tag{21}$$

$$\overline{E}^{(0)} = -\frac{c}{\sqrt{2}}\int_{-\infty}^{+\infty} dh \sum_{n=0}^{\infty} \tilde{\delta}_{n0}^{(2)}$$
$$\cdot\Big\{\tilde{K}_+^{(1)}\Big\{\Big[A_1^{(0)}\overline{V}^{(1)}_{en\eta_0}(h) + A_2^{(0)}\overline{W}^{(1)}_{en\eta_0}(h) + A_3^{(0)}\overline{V}^{(1)}_{on\eta_0}(h)$$
$$+ A_4^{(0)}\overline{W}^{(1)}_{on\eta_0}(h)\left[-\tilde{J}_+^{(1)}\right] + \Big[A_1^{(0)}\overline{V}^{(1)}_{on\eta_0}(h) + A_2^{(0)}\overline{W}^{(1)}_{on\eta_0}(h)$$
$$+A_3^{(0)}\overline{V}^{(1)}_{en\eta_0}(h) + A_4^{(0)}\overline{W}^{(1)}_{en\eta_0}(h)\Big]\left[-\tilde{J}_+^{(2)}\right]\Big\}$$
$$+ \tilde{K}_-^{(1)}\Big\{\Big[B_1^{(0)}\overline{V}^{(1)}_{on\eta_0}(h) + B_2^{(0)}\overline{W}^{(1)}_{on\eta_0}(h) + B_3^{(0)}\overline{V}^{(1)}_{en\eta_0}(h)$$
$$+B_4^{(0)}\overline{W}^{(1)}_{en\eta_0}(h)\Big]\left[-\tilde{J}_-^{(1)}\right] + \Big[B_1^{(0)}\overline{V}^{(1)}_{en\eta_0}(h) + B_2^{(0)}\overline{W}^{(1)}_{en\eta_0}(h)$$
$$+B_3^{(0)}\overline{V}^{(1)}_{on\eta_0}(h) + B_4^{(0}\overline{W}^{(1)}_{on\eta_0}(h)\Big]\tilde{J}_-^{(2)}\Big\}\Big\} \quad r \ge r_N \tag{22}$$

and $\tilde{J}_\pm^{(1)} = \sum_{q=1}^{2} e_q d J_{n\pm}^{(1a_q)} I_{nq}$, $\tilde{J}_\pm^{(2)} = \sum_{q=1}^{2} e_q \frac{jhn}{k_\pm^{(1)} a_q} J_{n\pm}^{(1a_q)} I_{nq}$, while the corresponding magnetic field expressions are understood and suppressed here. As a special case, the electromagnetic fields in both near and far field zones can be obtained when two thin circular-loop antennas are excited by sinusoidal currents, i.e.,

$$I_q(\varphi') = I_{pq}\cos(p\varphi') \tag{23}$$

In this case only the factor I_{nq} above should be replaced by $I_{nq} = I_{pq}\delta_{np}$.

2. Uniform Current Distribution [1–3], i.e.,

$$I_q(\varphi') = I_{0q} \tag{24}$$

For such uniform current excitations, both the electric and the magnetic fields in each layer of the multilayered biisotropic cylinder do not include the odd-mode component because the unknown coefficients $A_3^{(i)}$, $A_4^{(i)}$, $B_3^{(i)}$, $B_4^{(i)}$, $A_7^{(i)}$, $A_8^{(i)}$, $B_7^{(i)}$, $B_8^{(i)}$, $A_3^{(0)}$, $A_4^{(0)}$, $B_3^{(0)}$, $B_4^{(0)}$, are all zero. Therefore, the above field expressions can be greatly simplified, and the electric fields in the scalar form are presented in APPENDIX 1. Obviously, there exist six radiated field components, while for isotropic case only exist three field components and $E_{r,z}$ and H_φ are all zero. So the magnetoelectric cross-coupling parameters cause new components in the radiated fields in the near or far field zone. In addition, it should be pointed out that the above integration with respect to h can be evaluated using the technique of saddle-point integration. Mathematically, the unknown coefficients $A_1^{(1)}$, $A_2^{(1)}$, $B_1^{(1)}$, $B_2^{(1)}$, $A_1^{(0)}$, $A_2^{(0)}$, $B_1^{(0)}$ and $B_2^{(0)}$ above can be determined by solving the following two sets of linear equations:

$$\left[M^{(1,1)}\right]\begin{bmatrix} A_1^{(1)} \\ A_2^{(1)} \end{bmatrix} + \left[M^{(2,0)}\right]\begin{bmatrix} A_1^{(0)} \\ A_2^{(0)} \end{bmatrix} = \begin{bmatrix} \partial H_{0+}^{(1r_1)} \\ \eta_+^{(1)^2} H_{0+}^{(1r_1)}/k_+^{(1)} \\ \partial H_{0+}^{(1r_1)}/Z_+^{(1)} \\ \eta_+^{(1)^2} H_{0+}^{(1r_1)}/\left(k_+^{(1)} Z_+^{(1)}\right) \end{bmatrix} \tag{25a}$$

$$\left[M^{(1,1)}\right]\begin{bmatrix} B_1^{(1)} \\ B_2^{(1)} \end{bmatrix} + \left[M^{(2,0)}\right]\begin{bmatrix} B_1^{(0)} \\ B_2^{(0)} \end{bmatrix} = \begin{bmatrix} \partial H_{0-}^{(1r_1)} \\ -\eta_-^{(1)^2} H_{0-}^{(1r_1)}/k_-^{(1)} \\ -\partial H_{0-}^{(1r_1)}/Z_-^{(1)} \\ \eta_-^{(1)^2} H_{0-}^{(1r_1)}/\left(k_-^{(1)} Z_-^{(1)}\right) \end{bmatrix} \tag{25b}$$

and

$$\left[M^{(2,0)}\right] = \left[M^{(2,1)}\right]\left[M^{(2,2)}\right]^{-1}\left[M^{(3,2)}\right]\cdots \left[M^{(i,i)}\right]^{-1}\left[M^{(i+1,i)}\right]\cdots\left[M^{(N,N)}\right]^{-1}\left[M^{(0,0)}\right] \tag{25c}$$

where the relevant matrices are defined in APPENDIX 2.

For uniform current distribution we further consider two cases:

1° . Electrically Large Circular Loop Antennas $(k_\pm^{(1)}(\eta_\pm^{(1)} a_q) \gg 1)$

Since two thin circular loop antennas are placed in the inner region $(a_q \le r_1)$, the condition $k_\pm^{(1)}(\eta_\pm^{(1)} a_q) \gg 1$ also implies that

$x_\pm^{(i)} = \eta_\pm^{(i)} r_i \gg 1$. Under such circumstances, the cylindrical Bessel and Hankel functions can be stated as follows:

$$J_n\left(x_\pm^{(i)}\right) \approx \sqrt{\frac{2}{\pi x_\pm^{(i)}}} \cos\left[x_\pm^{(i)} - \frac{(2n+1)\pi}{4}\right],$$
$$H_n^{(1)}\left(x_\pm^{(i)}\right) \approx \sqrt{\frac{2}{\pi x_\pm^{(i)}}} e^{j\left[x_\pm^{(i)} - \frac{(2n+1)\pi}{4}\right]} \tag{26a, b}$$

These approximate expressions can be exploited for reducing the coefficient equations greatly for the unknown coefficients in (25). In the far field zone $(\eta_0 r \gg 1)$, we have

$$\overline{V}_{en\eta_0}^{(1)}(h) \approx (-j)^{\frac{1}{2}} \sqrt{\frac{\eta_0}{\pi r}} e^{j(\eta_0 r + hz)} \left(-j\overline{e}_\varphi + \frac{\eta_0 \overline{e}_z - h\overline{e}_r}{k_0}\right) \tag{27}$$
$$\overline{W}_{en\eta_0}^{(1)}(h) \approx (-j)^{\frac{1}{2}} \sqrt{\frac{\eta_0}{\pi r}} e^{j(\eta_0 r + hz)} \left(-j\overline{e}_\varphi + \frac{\eta_0 \overline{e}_z - h\overline{e}_r}{k_0}\right) \tag{28}$$

and by means of the technique of saddle-point integration, i.e.,

$$\int_\Omega g(h) e^{jkf(h)} dh \approx \left[\frac{\pi}{2kf''(h_0)}\right]^{\frac{1}{2}} g(h_0) e^{j\left[kf(h_0) - \frac{\pi}{4}\right]} \tag{29}$$

where Ω stands for the steepest-decent path, it is found that in the far field zone $r \geq r_N$,

$$\overline{E}^{(0)} \approx \frac{\omega\mu^{(1)} k_0 e^{jk_0 R}}{2\sqrt{2\pi}\left(k_+^{(1)} + k_-^{(1)}\right) R} \sin\theta \left\{ \frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}} \left[\frac{e_1}{\sqrt{a_1}} I_{01} \cos X_{+1}^{(1)} \right.\right.$$
$$\left. + \frac{e_2}{\sqrt{a_2}} I_{02} \cos X_{+2}^{(1)}\right] \left[A_1^{(0)} \overline{R}_H - A_2^{(0)} \overline{R}_L\right] + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}} \left[\frac{e_1}{\sqrt{a_1}} I_{01} \cos X_{-1}^{(1)} \right.$$
$$\left.\left. + \frac{e_2}{\sqrt{a_2}} I_{02} \cos X_{-2}^{(1)}\right] \left[B_1^{(0)} \overline{R}_H - B_2^{(0)} \overline{R}_L\right] \right\} \tag{30}$$

$$\overline{H}^{(0)} \approx -j \frac{\omega^2 \mu^{(1)} \varepsilon_0 e^{jk_0 R} \sin\theta}{2\sqrt{2\pi}\left(k_+^{(1)} + k_-^{(1)}\right) R} \left\{ \frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}} \left[\frac{e_1}{\sqrt{a_1}} I_{01} \cos X_{+1}^{(1)} \right.\right.$$

$$+\frac{e_2}{\sqrt{a_2}} I_{02} \cos X_{+2}^{(1)}\Big] \left[A_1^{(0)}\overline{R}_H - A_2^{(0)}\overline{R}_L\right] + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}} \left[\frac{e_1}{\sqrt{a_1}} I_{01} \cos X_{-1}^{(1)}\right.$$

$$\left. + \frac{e_2}{\sqrt{a_2}} I_{02} \cos X_{-2}^{(1)}\right] \left[B_1^{(0)}\overline{R}_H - B_2^{(0)}\overline{R}_L\right] \Bigg\} \tag{31}$$

where $\overline{R}_H = \overline{e}_\theta + j\overline{e}_\varphi$, $\overline{R}_L = \overline{e}_\theta - j\overline{e}_\varphi$, $\eta_\pm^{(1)} = \sqrt{k_\pm^{(1)^2} - k_0^2 \cos^2\theta}$, $h = k_0 \cos\theta$, $X_{\pm 1,2}^{(1)} = \eta_\pm^{(1)} a_{1,2} - \frac{3\pi}{4}$. Obviously, the radiated fields are just the combination of RCP and LCP modes. But, it should be pointed out that the effects of proximity that occurs when the saddle point is near the singularities of the coefficients $A_1^{(0)} - A_2^{(0)}$ and $B_1^{(0)} - B_2^{(0)}$ have not taken into account, and this means that the radiated fields (30) and (31) only contain the contribution of the space wave and are not valid for observation points such that θ is near $0°$ and $180°$.

2°. Electrically Small Circular Loops $(\eta_\pm^{(1)} a_q \ll 1,\ but\ \eta_\pm^{(i)} r_i \gg 1)$

For this extreme case, since $J_1(\eta_\pm^{(1)} a_q) \approx \frac{\eta_\pm^{(1)} a_q}{2}$, and using (27–28), the radiated fields become

$$\overline{E}^{(0)} \approx \frac{\omega\mu^{(1)} k_0 e^{jk_0 R}}{8\left(k_+^{(1)} + k_-^{(1)}\right) R} \left[e_1 a_1 I_{01} + e_2 a_2 I_{02}\right] \sin\theta$$
$$\cdot \left\{k_+^{(1)} \left[A_1^{(0)}\overline{R}_H - A_2^{(0)}\overline{R}_L\right] + k_-^{(1)} \left[B_1^{(0)}\overline{R}_H - B_2^{(0)}\overline{R}_L\right]\right\} \tag{32}$$

$$\overline{H}^{(0)} \approx -j\frac{\omega^2\mu^{(1)} \varepsilon_0 e^{jk_0 R}}{8\left(k_+^{(1)} + k_-^{(1)}\right) R} \left[e_1 a_1 I_{01} + e_2 a_2 I_{02}\right] \sin\theta$$
$$\cdot \left\{k_+^{(1)} \left[A_1^{(0)}\overline{R}_H + A_2^{(0)}\overline{R}_L\right] + k_-^{(1)} \left[B_1^{(0)}\overline{R}_H + B_2^{(0)}\overline{R}_L\right]\right\} \tag{33}$$

Also, the polarized state of the radiated fields can be represented by the point on the Poincare's sphere with latitude 2χ [7], and for the case of electrically large thin circular loops,

$$\sin 2\chi = \frac{\left|\frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}}C_+^{(1)}A_1^{(0)} + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}}C_-^{(1)}B_1^{(0)}\right|^2 - \left|\frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}}C_+^{(1)}A_2^{(0)} + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}}C_-^{(1)}B_2^{(0)}\right|^2}{\left|\frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}}C_+^{(1)}A_1^{(0)} + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}}C_-^{(1)}B_1^{(0)}\right|^2 + \left|\frac{k_+^{(1)}}{\eta_+^{(1)\frac{3}{2}}}C_+^{(1)}A_2^{(0)} + \frac{k_-^{(1)}}{\eta_-^{(1)\frac{3}{2}}}C_-^{(1)}B_2^{(0)}\right|^2} \tag{34}$$

where $C_\pm^{(1)} = \left[\frac{e_1}{\sqrt{a_1}}I_{01}\cos X_{\pm 1}^{(1)} + \frac{e_2}{\sqrt{a_2}}I_{02}\cos X_{\pm 2}^{(1)}\right]$, $\sin 2\chi \in [-1, 1]$ is the ellipticity of the polarization ellipse, and it can indicate the right- and left-hand elliptically, circularly, and linear polarized waves of the far fields, respectively.

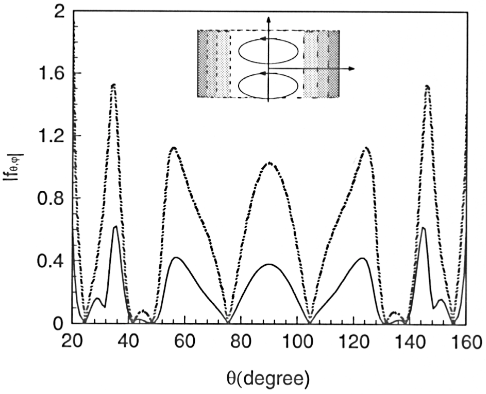
4. NUMERICAL EXAMPLES

Based on the developed mathematical formulation, numerical calculations are carried out to investigate the combined effects of the magnetoelectric cross-coupling parameters on the radiated characteristics of two thin circular loop antennas corresponding to different sizes and positions of two circular loop antennas.

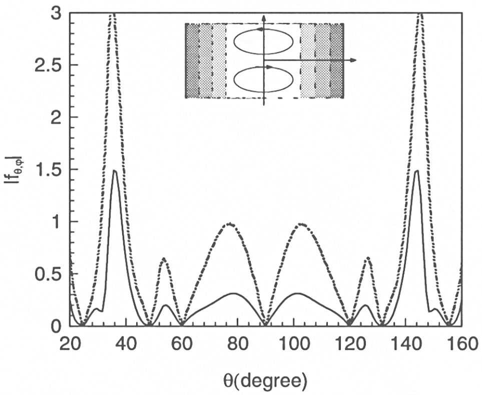
In Figs. 2(a)–(d), the inner region $r \le r_1$ is assumed to be the free space and only the angle range of $\theta \in [20°, 160°]$ is shown. Since the radii of two circular loop antennas are chosen to be: $a_1 = a_2 = 1.5\lambda$ or $a_2 = 1.0\lambda$, they belong to the electrically large case. It is obvious that strong cross-polarized phenomena can be observed for above cases, and usually we have $|E_\theta| < |E_\varphi|$. Changing the geometrical sizes or the positions of two loops, the relative magnitude between the co- and cross-polarized field components can be adjusted effectively.

Fig. 3 depicts the three-dimensional pattern of the ellipticity for two thin circular loop antennas in a three-layer biisotropic shell, and the parameters are assumed to be the same as in Fig. 2(b) with $r_2 = r_1 + 0.25\lambda$, $r_3 = r_1 + 0.5\lambda$, $r_4 = r_1 + 0.75\lambda$.

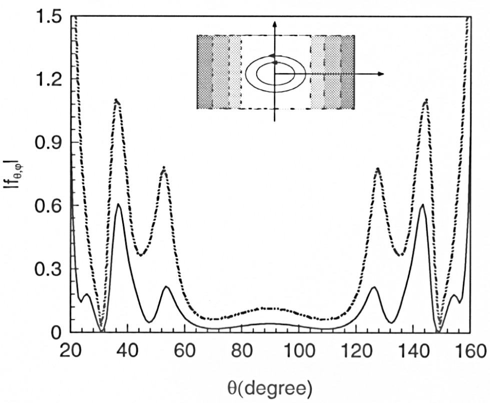
In Fig. 3, only the angle range of $\theta \in [40°, 140°]$ is considered, and $\sin 2\chi$ is symmetrical about the direction of $\theta = 90°$. It is clear that the polarized state of the radiated field for two electrically large circular loop antennas (or electrically small) is in the right- or left-handed elliptically polarized state.



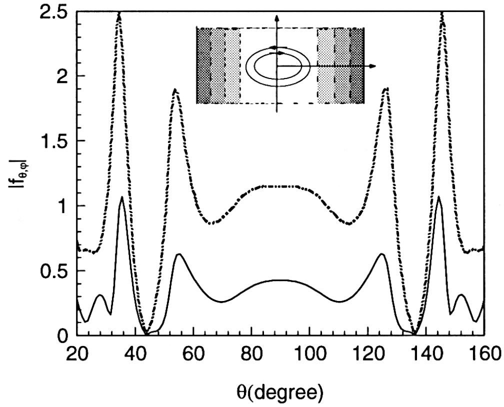
(a)



(b)



(c)



(d)

Figure 2. The radiation patterns of two thin circular loop antennas in a three-layer biisotropic shell $(\overline{E} \sim f_\theta \overline{e}_\theta + f_\varphi \overline{e}_\varphi)$.

$$\begin{aligned}
&\varepsilon^{(1)} = 1.0\varepsilon_0, && \mu^{(1)} = 1.0\mu_0, && \xi^{(2)} = \eta^{(2)} = 0.0, \\
&\varepsilon^{(2)} = 3.5\varepsilon_0, && \mu^{(2)} = 1.5\mu_0, && \xi^{(2)} = \eta^{(2)^*} = j0.8\sqrt{\mu_0\varepsilon_0}, \\
&\varepsilon^{(3)} = 2.5\varepsilon_0, && \mu^{(3)} = 1.2\mu_0, && \xi^{(3)} = \eta^{(3)^*} = j0.6\sqrt{\mu_0\varepsilon_0}, \\
&\varepsilon^{(4)} = 2.0\varepsilon_0, && \mu^{(4)} = 1.0\mu_0, && \xi^{(4)} = \eta^{(4)^*} = j0.4\sqrt{\mu_0\varepsilon_0},
\end{aligned}$$

$r_1/\lambda = 2.0$, $r_2/\lambda = 2.25$, $r_3/\lambda = 2.5$, $r_4/\lambda = 2.75$, λ is the operating wavelength.
(a) $a_1/\lambda = a_2/\lambda = 1.5$, $I_{01} = I_{02} = 1.0A$, $z_1 = -z_2 = 1.0\lambda$.
(b) The parameters are the same as (a), except that $I_{01} = -I_{02} = 1.0A$.
(c) $a_1/\lambda = 1.5$, $a_2/\lambda = 1.0$, $I_{01} = I_{02} = 1.0A$, $z_1 = z_2 = 0.0$.
(d) The parameters are the same as (c), except that $I_{01} = -I_{02} = 1.0A$.
solid line: f_θ (cross-polarized component), dotted line: f_φ (co-polarized component).

5. CONCLUSIONS

The technique of dyadic Greens function in terms of the normalized cylindrical vector wave functions has been employed to examine the electromagnetic field characteristics of two thin circular loop antennas

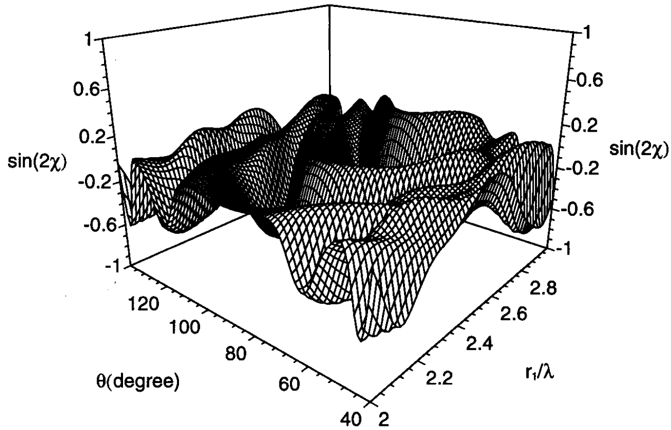


Figure 3. The three-dimensional ellipticity of the polarization ellipse as a function of θ and the radii of three-layer biisotropic shell with the same parameters as in Fig. 2(b).

in a radially multilayered biisotropic cylinder. Corresponding to different current distributions, the radiated fields in both near and far field zones are derived and all expressed in a analytical form. Numerical results have also been presented to demonstrate the cross-polarized effect in the radiated fields resulted from the magneto- electric cross-coupling parameters of biisotropic media.

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APPENDIX 1: In the near field zone $0 < r < a_1$,

$$E^{(10)}_{r,0<r<a_1} = jc\int_{-\infty}^{+\infty} he^{jhz}dh\left[J^{(1r)}_{1+}\tilde{H}_{1+} - J^{(1r)}_{1-}\tilde{H}_{1-}\right]$$

$$E^{(10)}_{\varphi,0<r<a_1} = -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[k^{(1)}_{+}J^{(1r)}_{1+}\tilde{H}_{1+} + k^{(1)}_{-}J^{(1r)}_{1-}\tilde{H}_{1-}\right]$$

$$E_{z,0<r<a_1}^{(10)} = -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[\eta_+^{(1)}J_{0+}^{(1r)}\tilde{H}_{1+} - \eta_-^{(1)}J_{0-}^{(1r)}\tilde{H}_{1-}\right] \quad \text{(A1)}$$

$$\text{and } \tilde{H}_{1\pm} = \sum_{q=1}^{2} e_q H_{1\pm}^{(1a_q)} I_{0q}.$$

In the near field zone $a_1 < r < a_2$,

$$\begin{aligned}
E_{r,a_1<r<a_2}^{(10)} &= jc\int_{-\infty}^{+\infty} he^{jhz}dh\left[\tilde{L}_{1+} - \tilde{L}_{1-}\right] \\
E_{\varphi,a_1<r<a_2}^{(10)} &= -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[k_+^{(1)}\tilde{L}_{1+} + k_-^{(1)}\tilde{L}_{1-}\right] \\
E_{z,a_1<r<a_2}^{(10)} &= -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[\eta_+^{(1)}\tilde{L}_{1+}^{(2)} - \eta_-^{(1)}\tilde{L}_{1-}^{(2)}\right] \qquad \text{(A2)} \\
\text{and } \tilde{L}_{1\pm} &= H_{1\pm}^{(1r)}e_1J_{1\pm}^{(1a_1)}I_{01} + J_{1\pm}^{(1r)}e_2H_{1\pm}^{(1a_1)}I_{02}, \\
\tilde{L}_{1\pm}^{(2)} &= H_{0\pm}^{(1r)}e_1J_{1\pm}^{(1a_1)}I_{01} + J_{0\pm}^{(1r)}e_2H_{1\pm}^{(1a_1)}I_{02}.
\end{aligned}$$

In the near field zone $a_2 < r < r_1$,

$$\begin{aligned}
E_{r,a_1<r<r_2}^{(10)} &= jc\int_{-\infty}^{+\infty} he^{jhz}dh\left[H_{1+}^{(1r)}\tilde{G}_{1+} - H_{1-}^{(1r)}\tilde{G}_{1-}\right] \\
E_{\varphi,a_2<r<r_1}^{(10)} &= -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[k_+^{(1)}H_{1+}^{(1r)}\tilde{G}_{1+} + k_-^{(1)}H_{1-}^{(1r)}\tilde{G}_{1-}\right] \\
E_{z,a_2<r<r_1}^{(10)} &= -c\int_{-\infty}^{+\infty} e^{jhz}dh\left[\eta_+^{(1)}H_{0+}^{(1r)}\tilde{G}_{1+} - \eta_-^{(1)}H_{0-}^{(1r)}\tilde{G}_{1-}\right] \text{(A3)} \\
\text{and } \tilde{G}_{1\pm} &= \sum_{q=1}^{2} e_q J_{1\pm}^{(1a_q)} I_{0q}.
\end{aligned}$$

In the near field zone $0 < r < r_1$,

$$\begin{aligned}
E_{r,0<r<r_1}^{(1s)} =& jc\int_{-\infty}^{+\infty} he^{jhz}dh \\
&\cdot\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\left[A_1^{(1)}\frac{\eta_+^{(1)}}{k_+^{(1)}}J_{1+}^{(1r)} - A_2^{(1)}\frac{\eta_-^{(1)}}{k_-^{(1)}}J_{1-}^{(1r)}\right]\tilde{G}_{1+}\right. \\
&\left. + \frac{k_-^{(1)}}{\eta_-^{(1)}}\left[B_1^{(1)}\frac{\eta_+^{(1)}}{k_+^{(1)}}J_{1+}^{(1r)} - B_2^{(1)}\frac{\eta_-^{(1)}}{k_-^{(1)}}J_{1-}^{(1r)}\right]\tilde{G}_{1-}\right\}
\end{aligned}$$

$$\begin{aligned}
E^{(1s)}_{\varphi,0<r<r_1} =& -c\int_{-\infty}^{+\infty} e^{jhz}dh\\
&\cdot\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\left[A_1^{(1)}\eta_+^{(1)}J_{1+}^{(1r)}+A_2^{(1)}\eta_-^{(1)}J_{1-}^{(1r)}\right]\tilde{G}_{1+}\right.\\
&\left.+\frac{k_-^{(1)}}{\eta_-^{(1)}}\left[B_1^{(1)}\eta_+^{(1)}J_{1+}^{(1r)}+B_2^{(1)}\eta_-^{(1)}J_{1-}^{(1r)}\right]\tilde{G}_{1-}\right\}\\
E^{(1s)}_{z,0<r<r_1} =& -c\int_{-\infty}^{+\infty} e^{jhz}dh\\
&\cdot\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\left[A_1^{(1)}\frac{\eta_+^{(1)^2}}{k_+^{(1)}}J_{0+}^{(1r)}-A_2^{(1)}\frac{\eta_-^{(1)^2}}{k_-^{(1)}}J_{0-}^{(1r)}\right]\tilde{G}_{1+}\right.\\
&\left.+\frac{k_-^{(1)}}{\eta_-^{(1)}}\left[B_1^{(1)}\frac{\eta_+^{(1)^2}}{k_+^{(1)}}J_{0+}^{(1r)}-B_2^{(1)}\frac{\eta_-^{(1)^2}}{k_-^{(1)}}J_{0-}^{(1r)}\right]\tilde{G}_{1-}\right\}
\end{aligned}\tag{A4}$$

In the region $r_i \le r \le r_{i+1}$,

$$\begin{aligned}
E_r^{(i)} =& ic\int_{-\infty}^{+\infty} he^{jhz}dh\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\left[A_1^{(i)}\frac{\eta_+^{(i)}}{k_+^{(i)}}J_{1+}^{(ir)}-A_2^{(i)}\frac{\eta_-^{(i)}}{k_-^{(i)}}J_{1-}^{(ir)}\right.\right.\\
&\left.+A_5^{(i)}\frac{\eta_+^{(i)}}{k_+^{(i)}}H_{1+}^{(ir)}-A_6^{(i)}\frac{\eta_-^{(i)}}{k_-^{(i)}}H_{1-}^{(ir)}\right]\tilde{G}_{1+}\\
&+\frac{k_-^{(1)}}{\eta_-^{(1)}}\left[B_1^{(i)}\frac{\eta_+^{(i)}}{k_+^{(i)}}J_{1+}^{(ir)}-B_2^{(i)}\frac{\eta_-^{(i)}}{k_-^{(i)}}J_{1-}^{(ir)}\right.\\
&\left.\left.+B_5^{(i)}\frac{\eta_+^{(i)}}{k_+^{(i)}}H_{1+}^{(ir)}-B_6^{(i)}\frac{\eta_-^{(i)}}{k_-^{(i)}}H_{1-}^{(ir)}\right]\tilde{G}_{1-}\right\}\\
E_\varphi^{(i)} =& -c\int_{-\infty}^{+\infty} e^{jhz}dh\left\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\left[A_1^{(i)}\eta_+^{(i)}J_{1+}^{(ir)}+A_2^{(i)}\eta_-^{(i)}J_{1-}^{(ir)}\right.\right.\\
&\left.+A_5^{(i)}\eta_+^{(i)}H_{1+}^{(ir)}+A_6^{(i)}\eta_-^{(i)}H_{1-}^{(ir)}\right]\tilde{G}_{1+}\\
&+\frac{k_-^{(1)}}{\eta_-^{(1)}}\left[B_1^{(i)}\eta_+^{(i)}J_{1+}^{(ir)}+B_2^{(i)}\eta_-^{(i)}J_{1-}^{(ir)}+B_5^{(i)}\eta_+^{(i)}H_{1+}^{(ir)}\right.
\end{aligned}$$

$$+ B_6^{(i)}\eta_-^{(i)}H_{1-}^{(ir)}\Big]\tilde{G}_{1-}\Bigg\}$$

$$\begin{aligned}
E_z^{(i)} = &- c\int_{-\infty}^{+\infty} e^{jhz}dh\Bigg\{\frac{k_+^{(1)}}{\eta_+^{(1)}}\Bigg[A_1^{(i)}\frac{{\eta_+^{(i)}}^2}{k_+^{(i)}}J_{0+}^{(ir)} - A_2^{(i)}\frac{{\eta_-^{(i)}}^2}{k_-^{(i)}}J_{0-}^{(ir)} \\
&+ A_5^{(i)}\frac{{\eta_+^{(i)}}^2}{k_+^{(i)}}H_{0+}^{(ir)} - A_6^{(i)}\frac{{\eta_-^{(i)}}^2}{k_-^{(i)}}H_{0-}^{(ir)}\Bigg]\tilde{G}_{1+} \\
&+ \frac{k_-^{(1)}}{\eta_-^{(1)}}\Bigg[B_1^{(i)}\frac{{\eta_+^{(i)}}^2}{k_+^{(i)}}J_{0+}^{(ir)} - B_2^{(i)}\frac{{\eta_-^{(i)}}^2}{k_-^{(i)}}J_{0-}^{(ir)} \\
&+ B_5^{(i)}\frac{{\eta_+^{(i)}}^2}{k_+^{(i)}}H_{0+}^{(ir)} - B_6^{(i)}\frac{{\eta_-^{(i)}}^2}{k_-^{(i)}}H_{0-}^{(ir)}\Bigg]\tilde{G}_{1-}\Bigg\} \qquad \text{(A5)}
\end{aligned}$$

In the region $r \geq r_N$,

$$\begin{aligned}
E_r^{(0)} &= jc\int_{-\infty}^{+\infty} h\eta_0 H_{10}^{(0r)}e^{jhz}dh \\
&\cdot\left[\frac{k_+^{(1)}}{\eta_+^{(1)}}(A_1^{(0)} - A_2^{(0)})\tilde{G}_{1+} + \frac{k_-^{(1)}}{\eta_-^{(1)}}(B_1^{(0)} - B_2^{(0)})\tilde{G}_{1-}\right] \\
E_\varphi^{(0)} &= -c\int_{-\infty}^{+\infty} \eta_0 H_{10}^{(0r)}e^{jhz}dh \\
&\cdot\left[\frac{k_+^{(1)}}{\eta_+^{(1)}}(A_1^{(0)} + A_2^{(0)})\tilde{G}_{1+} + \frac{k_-^{(1)}}{\eta_-^{(1)}}(B_1^{(0)} + B_2^{(0)})\tilde{G}_{1-}\right] \\
E_z^{(0)} &= -c\int_{-\infty}^{+\infty} \eta_0^2 H_{10}^{(0r)}e^{jhz}dh \\
&\cdot\left[\frac{k_+^{(1)}}{\eta_+^{(1)}}(A_1^{(0)} - A_2^{(0)})\tilde{G}_{1+} + \frac{k_-^{(1)}}{\eta_-^{(1)}}(B_1^{(0)} - B_2^{(0)})\tilde{G}_{1-}\right] \qquad \text{(A6)}
\end{aligned}$$

The shorthand notations in (A1–A6) are

$$\begin{aligned}
&J_{1\pm}^{(1r)} = J_1(\eta_\pm^{(1)}r), \qquad J_{0\pm}^{(1r)} = J_0(\eta_\pm^{(1)}r), \\
&H_{1\pm}^{(1r)} = H_1^{(1)}(\eta_\pm^{(1)}r), \qquad J_{1\pm}^{(ir)} = J_1(\eta_\pm^{(i)}r), \\
&H_{1\pm}^{(ir)} = H_1^{(1)}(\eta_\pm^{(i)}r), \qquad H_{00}^{(0r)} = H_0^{(1)}(\eta_0 r), \qquad H_{10}^{(0r)} = H_1^{(1)}(\eta_0 r),
\end{aligned}$$

and the magnetic fields in the scalar form are understood and suppressed.

APPENDIX 2: In (25),

$$\left[M^{(i,i)}\right] = \begin{bmatrix} \dfrac{\partial J_{0+}^{(ir_i)}}{\eta_+^{(i)^2} J_{0+}^{(ir_i)}} & -\dfrac{\partial J_{0-}^{(ir_i)}}{\eta_-^{(i)^2} J_{0-}^{(ir_i)}} & \dfrac{\partial H_{0+}^{(ir_i)}}{\eta_+^{(i)^2} H_{0+}^{(ir_i)}} & -\dfrac{\partial H_{0-}^{(ir_i)}}{\eta_-^{(i)^2} H_{0-}^{(ir_i)}} \\ \overline{k_+^{(i)}} & \overline{k_-^{(i)}} & \overline{k_+^{(i)}} & \overline{k_-^{(i)}} \\ \dfrac{\partial J_{0+}^{(ir_i)}}{Z_+^{(i)}} & -\dfrac{\partial J_{0-}^{(ir_i)}}{Z_-^{(i)}} & \dfrac{\partial H_{0+}^{(ir_i)}}{Z_+^{(i)}} & -\dfrac{\partial H_{0-}^{(ir_i)}}{Z_-^{(i)}} \\ \dfrac{\eta_+^{(i)^2} J_{0+}^{(ir_i)}}{Z_+^{(i)} k_+^{(i)}} & \dfrac{\eta_-^{(i)^2} J_{0-}^{(ir_i)}}{Z_-^{(i)} k_-^{(i)}} & \dfrac{\eta_+^{(i)^2} H_{0+}^{(ir_i)}}{Z_+^{(i)} k_+^{(i)}} & \dfrac{\eta_-^{(i)^2} H_{0-}^{(ir_i)}}{Z_-^{(i)} k_-^{(i)}} \end{bmatrix}$$

$$i = 2, \ldots, N$$

$$\left[M^{(i+1,i)}\right] = \left[\begin{matrix} \dfrac{\partial J_{0+}^{((i+1)r_i)}}{\eta_+^{(i+1)^2} J_{0+}^{((i+1)r_i)}} & -\dfrac{\partial J_{0-}^{((i+1)r_i)}}{\eta_-^{(i+1)^2} J_{0-}^{((i+1)r_i)}} \\ \overline{k_+^{(i+1)}} & \overline{k_-^{(i+1)}} \\ \dfrac{\partial J_{0+}^{((i+1)r_i)}}{Z_+^{(i+1)}} & -\dfrac{\partial J_{0-}^{((i+1)r_i)}}{Z_-^{(i+1)}} \\ \dfrac{\eta_+^{(i+1)^2} J_{0+}^{((i+1)r_i)}}{Z_+^{(i+1)} k_+^{(i+1)}} & \dfrac{\eta_-^{(i+1)^2} J_{0-}^{((i+1)r_i)}}{Z_-^{(i+1)} k_-^{(i+1)}} \end{matrix}\right.$$

$$\left.\begin{matrix} \dfrac{\partial H_{0+}^{((i+1)r_i)}}{\eta_+^{(i+1)^2} H_{0+}^{((i+1)r_i)}} & -\dfrac{\partial H_{0-}^{((i+1)r_i)}}{\eta_-^{(i+1)^2} H_{0-}^{((i+1)r_i)}} \\ \overline{k_+^{(i+1)}} & \overline{k_-^{(i+1)}} \\ \dfrac{\partial H_{0+}^{((i+1)r_i)}}{Z_+^{(i+1)}} & -\dfrac{\partial H_{0-}^{((i+1)r_i)}}{Z_-^{(i+1)}} \\ \dfrac{\eta_+^{(i+1)^2} H_{0+}^{((i+1)r_i)}}{Z_+^{(i+1)} k_+^{(i+1)}} & \dfrac{\eta_-^{(i+1)^2} H_{0-}^{((i+1)r_i)}}{Z_-^{(i+1)} k_-^{(i+1)}} \end{matrix}\right] \quad i = 1, \ldots, N-1$$

where

$$J_{0\pm}^{(ir_i)} = J_0(\eta_\pm^{(i)} r_i), \;\; \partial J_{0\pm}^{(ir_i)} = \left.\frac{\partial J_0(\eta_\pm^{(i)} r)}{\partial r}\right|_{r=r_i}$$

$$H_{0\pm}^{(ir_i)} = H_0^{(1)}(\eta_\pm^{(i)} r_i),\ \partial H_{0\pm}^{(ir_i)} = \left.\frac{\partial H_0^{(1)}(\eta_\pm^{(i)} r)}{\partial r}\right|_{r=r_i}$$

$$J_{0\pm}^{((i+1)r_i)} = J_0(\eta_\pm^{(i+1)} r_i),\ \partial J_{0\pm}^{((i+1)r_i)} = \left.\frac{\partial J_0(\eta_\pm^{(i+1)} r)}{\partial r}\right|_{r=r_i}$$

$$H_{0\pm}^{((i+1)r_i)} = H_0^{(1)}(\eta_\pm^{(i+1)} r_i),\ \partial H_{0\pm}^{((i+1)r_i)} = \left.\frac{\partial H_0^{(1)}(\eta_\pm^{(i+1)} r)}{\partial r}\right|_{r=r_i} \quad \text{(A7)}$$

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