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COUPLED-MODE ANALYSIS OF COUPLED MICROSTRIP TRANSMISSION LINES USING A SINGULAR PERTURBATION TECHNIQUE

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1. INTRODUCTION

Coupled microstrip transmission lines in multilayered dielectric medium are widely used in the design of microwave and millimeter-wave integrated circuits. One of the important subjects on such transmission systems is to evaluate efficiently as well as accurately the high frequency electromagnetic coupling between nearby lines [1–3], which affects seriously the circuit performance in high speed operation. The transmission characteristics of coupled microstrip lines can be rigorously analyzed using various numerical techniques [4]. However those direct solution methods become very involved both analytically and numerically when the number of coupled lines increases.

To avoid such difficulty, several approximate solution methods have been implemented for multilayered and multiple coupled microstrip lines. The full-wave perturbation theory [5] assumes the isolated eigenmodes of different lines to be nearly equal and approximates the coupled electric-field integral equations by the reduced first-order equations, using the Taylor's series expansions around the isolated eigenmode solutions. The conventional coupled-mode theory [6] approximates the fields of coupled lines by a linear combination of the eigenmode fields of isolated lines and deduces the coupled-differential equations for the modal amplitudes by using the reciprocity theorem. Both approaches are very powerful and efficient for the calculation of approximate propagation constants of coupled microstrip lines. However they have a defect in the approximation of the coupled-current distributions on the lines. The current distributions in each line under the coupled situation are assumed to be same as those in the isolated situation.

In this paper, we present a self-consistent coupled-mode theory for coupled microstrip lines which enables us to calculate systematically the coupled-current distributions as well as the propagation constants. The theory is based on a singular perturbation technique in the spectral domain and is an extension of the coupled-mode approach [7] for optical waveguides. The total fields of coupled microstrip lines are decomposed into elementary fields associated with the induced currents on the individual lines. A small parameter that is a measure of interaction between nearby lines is introduced for the perturbation analysis. The elementary fields and the induced currents are expanded using the multiple space-scales and are solved in spectral domain so that the total fields satisfy the boundary conditions on the original coupled system. This analytical procedure leads to the coupled-mode equations which determine the propagation constants of coupled-modes and the coupled-current distributions. The coupling coefficients between adjacent lines are calculated by a simple matrix algebra using the current spectra of each isolated single line. The proposed theory is applied to the analysis of two identical coupled microstrip lines in single plane. It is shown that the dispersion characteristics and the coupled-current distributions of symmetric and asymmetric modes are in good agreement with those of the rigorous Galerkin's moment method solutions.

2. FORMULATION OF THE PROBLEM

To illustrate the formulation process, we consider two coupled microstrip lines as shown in Fig. 1. Two microstrips a and b of infinitesimal thickness are situated with a spacing $2d$ on the substrate-cover interface in a trilayered structure, which consists of a ground plane, a dielectric substrate of thickness h and relative permittivity ε_r, and a cover layer of free space. The widths of two microstrips are $2w_a$ and $2w_b$. The microstrips and ground plane are assumed to be perfect conductors. The geometry is uniform in the z direction. Let $\varepsilon(y)$ be the distribution of relative permittivity of the structure and introduce two potential functions which satisfy the scalar Helmholtz equations

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\varepsilon(y)\right]\phi(x,y,z)=0 \tag{1}$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\varepsilon(y)\right]\psi(x,y,z)=0 \tag{2}$$

where k is the wavenumber in free space, and $\phi(x,y,z)$ and $\psi(x,y,z)$ represent the x components of the electric and magnetic Hertz vectors. Then the electric and magnetic field vectors are described in terms of $\phi(x,y,z)$ and $\psi(x,y,z)$ as follows:

$$\mathbf{E}=\hat{\mathbf{x}}\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\phi+\nabla_t\frac{\partial}{\partial x}\phi+i\omega\mu_0\hat{\mathbf{x}}\times\nabla_t\psi \tag{3}$$

$$\mathbf{H}=\hat{\mathbf{x}}\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\psi+\nabla_t\frac{\partial}{\partial x}\psi-i\omega\varepsilon_0\varepsilon(y)\hat{\mathbf{x}}\times\nabla_t\phi \tag{4}$$

where

$$\nabla_t=\hat{\mathbf{y}}\frac{\partial}{\partial y}+\hat{\mathbf{z}}\frac{\partial}{\partial z}. \tag{5}$$

We decompose the potential functions $\phi(x,y,z)$, $\psi(x,y,z)$, and the field vectors as follows:

$$\phi(x,y,z)=\phi_a(x,y,z)+\phi_b(x,y,z) \tag{6}$$

$$\psi(x,y,z)=\psi_a(x,y,z)+\psi_b(x,y,z) \tag{7}$$

$$\mathbf{E}(x,y,z)=\mathbf{E}_a(x,y,z)+\mathbf{E}_b(x,y,z) \tag{8}$$

$$\mathbf{H}(x,y,z)=\mathbf{H}_a(x,y,z)+\mathbf{H}_b(x,y,z) \tag{9}$$

where $(\phi_a,\psi_a,\mathbf{E}_a,\mathbf{H}_a)$ and $(\phi_b,\psi_b,\mathbf{E}_b,\mathbf{H}_b)$ are the potential functions and elementary fields which satisfy the equations:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}+k^2\varepsilon(y)\right]\phi_\nu(x,y,z)=0 \quad (\nu=a,b) \tag{10}$$

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k^2\varepsilon(y)\right] \psi_\nu(x, y, z) = 0 \tag{11}$$

$$\mathbf{E}_\nu = \hat{\mathbf{x}} \left[\frac{\partial^2}{\partial x^2} + k^2\varepsilon(y)\right] \phi_\nu + \nabla_t \frac{\partial}{\partial x}\psi_\nu + i\omega\mu_0 \hat{\mathbf{x}} \times \nabla_t \psi_\nu \tag{12}$$

$$\mathbf{H}_\nu = \hat{\mathbf{x}} \left[\frac{\partial^2}{\partial x^2} + k^2\varepsilon(y)\right] \psi_\nu + \nabla_t \frac{\partial}{\partial x}\psi_\nu - i\omega\varepsilon_0\varepsilon(y) \hat{\mathbf{x}} \times \nabla_t \phi_\nu \tag{13}$$

Note that $(\phi_\nu, \psi_\nu, \mathbf{E}_\nu, \mathbf{H}_\nu)$ $(\nu = a, b)$ are associated with the induced currents $\mathbf{J}_\nu$ on the line ν under the coupled situation. When the elementary fields $(\mathbf{E}_\nu, \mathbf{H}_\nu)$ are solved to satisfy the required boundary conditions on the coupled system, the total fields $(\mathbf{E}, \mathbf{H})$ also satisfy the same boundary conditions. This approach of analysis is rigorous one for the coupled structure. In what follows, we shall develop a perturbation approach by regarding the interaction between two lines as a small perturbation.

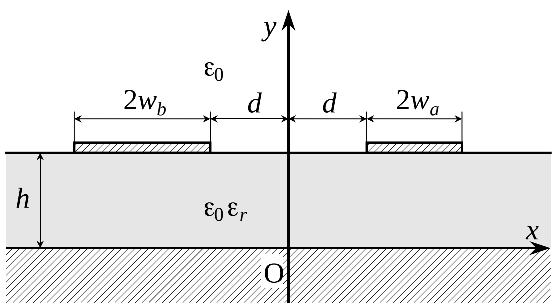


Figure 1. Cross section of two coupled microstrip lines in a single plane.

When the coupling between two lines is moderate, the elementary fields $(\mathbf{E}_\nu, \mathbf{H}_\nu)$ are concentrated near the microstrip ν. Then the effect of the elementary fields on the induced current $\mathbf{J}_\mu$ of the second microstrip μ is assumed to be the order of δ in magnitude. We introduce a multiple space-scales; $z_0 = z$, $z_1 = \delta z$, and expand the potential functions, elementary fields, and induced currents as follows:

$$\phi_\nu(x, y, z) = \phi_\nu^{(0)}(x, y; z_0, z_1) + \delta\phi_\nu^{(1)}(x, y; z_0, z_1) \quad (\nu = a, b) \tag{14}$$

$$\psi_\nu(x, y, z) = \psi_\nu^{(0)}(x, y; z_0, z_1) + \delta\psi_\nu^{(1)}(x, y; z_0, z_1) \tag{15}$$

$$\mathbf{E}_\nu = \mathbf{E}_\nu^{(0)}(x, y; z_0, z_1) + \delta\mathbf{E}_\nu^{(1)}(x, y; z_0, z_1) \tag{16}$$

$$\mathbf{H}_\nu = \mathbf{H}_\nu^{(0)}(x, y; z_0, z_1) + \delta\mathbf{H}_\nu^{(1)}(x, y; z_0, z_1) \tag{17}$$

$$\mathbf{J}_\nu = \mathbf{J}_\nu^{(0)}(x, h; z_0, z_1) + \delta\mathbf{J}_\nu^{(1)}(x, h; z_0, z_1). \tag{18}$$

Substituting Eqs. (14)–(18) into Eqs. (10)–(13) and making use of a relation of the derivative expansion; $\partial/\partial z = \partial/\partial z_0 + \delta\partial/\partial z_1$, we obtain a set of equations to be solved in respective orders of perturbation (see APPENDIX). The sets of equations are solved in the substrate region and in the cover region under the required boundary conditions at $y = 0$ and $y = h$.

3. COUPLED-MODE EQUATIONS

Since the effect of the elementary fields $(\mathbf{E}_\nu, \mathbf{H}_\nu)$ on the induced current $\mathbf{J}_\mu$ of the second microstrip μ is assumed to be the order of δ, the tangential $\mathbf{H}_{\nu,t}^{(0)}$ field is continuous across the microstrip μ. Then the wave equations and boundary conditions for two elementary fields are decoupled. Following the standard procedure in the spectral domain method [4], the zero-order electric fields are given in the Fourier transformed domain as follows:

$$\begin{bmatrix} \widetilde{E}_{\nu,x}^{(0)}(\zeta, h; z_0, z_1) \\ \widetilde{E}_{\nu,z}^{(0)}(\zeta, h; z_0, z_1) \end{bmatrix} = I_\nu^{(0)}(z_1)\exp(-i\beta_\nu z_0) \begin{bmatrix} \widetilde{G}_{xx}(\zeta, \beta_\nu) & \widetilde{G}_{xz}(\zeta, \beta_\nu) \\ \widetilde{G}_{zx}(\zeta, \beta_\nu) & \widetilde{G}_{zz}(\zeta, \beta_\nu) \end{bmatrix} \begin{bmatrix} \widetilde{J}_{\nu,x}^{(0)}(\zeta) \\ \widetilde{J}_{\nu,z}^{(0)}(\zeta) \end{bmatrix} \tag{19}$$

where $I_\nu^{(0)}(z_1)(\nu = a, b)$ denote the slowly varying amplitudes of zero-order induced currents, β_ν is the propagation constant of the isolated single microstrip line, $[\widetilde{\mathbf{G}}(\zeta, \beta_\nu)]$ is the spectral dyadic Green's function [4]. The tangential electric field $\mathbf{E}_{\nu,t}^{(0)}$ in the space domain should vanish on the surface of strip ν at $y = h$. The resulting integral equations for $\mathbf{J}_a^{(0)}(x)$ and $\mathbf{J}_b^{(0)}(x)$ are solved using Galerkin's moment method in the spectral domain. The spectra of transverse and longitudinal current components are expanded as follows:

$$\widetilde{J}_{\nu,x}^{(0)}(\zeta) = \sum_{n=1}^{N} a_{\nu,xn}^{(0)}\tilde{p}_{\nu,n}(\zeta) \quad (\nu = a, b) \tag{20}$$

$$\widetilde{J}_{\nu,z}^{(0)}(\zeta) = \sum_{n=1}^{N} a_{\nu,zn}^{(0)} \tilde{q}_{\nu,n}(\zeta) \tag{21}$$

where $\tilde{p}_{\nu,n}(\zeta)$ and $\tilde{q}_{\nu,n}(\zeta)$ are the Fourier transforms of the corresponding basis functions $p_{\nu,n}(x)$ ad $q_{\nu,n}(x)$ in the space domain. Thus the zero-order problem is finally rendered into the matrix equations

$$\begin{bmatrix} A_{\nu,xx}^{mn}(\beta_\nu) & A_{\nu,xz}^{mn}(\beta_\nu) \\ A_{\nu,zx}^{mn}(\beta_\nu) & A_{\nu,zz}^{mn}(\beta_\nu) \end{bmatrix} \begin{bmatrix} a_{\nu,x1}^{(0)} \\ \cdot \\ \cdot \\ \cdot \\ a_{\nu,xN}^{(0)} \\ a_{\nu,z1}^{(0)} \\ \cdot \\ \cdot \\ \cdot \\ a_{\nu,zN}^{(0)} \end{bmatrix} = \mathbf{0} \tag{22}$$

with

$$A_{\nu,xx}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{p}_{\nu,m}(\zeta) \widetilde{G}_{xx}(\zeta,\beta_\nu) \tilde{p}_{\nu,n}(\zeta) d\zeta \tag{23}$$

$$A_{\nu,xz}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{p}_{\nu,m}(\zeta) \widetilde{G}_{xz}(\zeta,\beta_\nu) \tilde{q}_{\nu,n}(\zeta) d\zeta \tag{24}$$

$$A_{\nu,zx}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{q}_{\nu,m}(\zeta) \widetilde{G}_{zx}(\zeta,\beta_\nu) \tilde{p}_{\nu,n}(\zeta) d\zeta \tag{25}$$

$$A_{\nu,zz}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{q}_{\nu,m}(\zeta) \widetilde{G}_{zz}(\zeta,\beta_\nu) \tilde{q}_{\nu,n}(\zeta) d\zeta \tag{26}$$

where $[\mathbf{A}_\nu(\beta_\nu)]$ is Galerkin's matrix [4] for the isolated single microstrip located at $y = h$. The propagation constant β_ν is obtained as the eigenvalue satisfying

$$\det[\mathbf{A}_\nu(\beta_\nu)] = 0 \quad (\nu = a, b). \tag{27}$$

The associated solutions $\{a_{\nu,xn}^{(0)}\}$ and $\{a_{\nu,zn}^{(0)}\}$ of Eq. (22) determine the expansion coefficients for the eigenmode currents.

For the first-order problem, we assume that $|\beta_a - \beta_b|/\beta_a$ is the order of δ in magnitude. This implies that the two microstrip lines are nearly degenerate when in isolation. Note that the coupling between two microstrips is negligible when the isolated propagation constants are noticeably different from each other. The zero-order solutions are substituted into Eqs. (66) and (67), and the first-order wave equations are solved. The results are used in Eqs. (68)–(71) to derive the first-order fields $\mathbf{E}_\nu^{(1)}$ and $\mathbf{H}_\nu^{(1)}$. Following the similar procedure as in the zero-order problem, after tedious but straightforward manipulations, the first-order electric fields in the Fourier transformed domain are expressed as

$$\begin{aligned}&\begin{bmatrix}\widetilde{E}_{\nu,x}^{(1)}(\zeta,h;z_0,z_1)\\ \widetilde{E}_{\nu,z}^{(1)}(\zeta,h;z_0,z_1)\end{bmatrix}\\ &\quad=\begin{bmatrix}\widetilde{G}_{xx}(\zeta,\beta_\nu) & \widetilde{G}_{xz}(\zeta,\beta_\nu)\\ \widetilde{G}_{zx}(\zeta,\beta_\nu) & \widetilde{G}_{zz}(\zeta,\beta_\nu)\end{bmatrix}\begin{bmatrix}\widetilde{J}_{\nu,x}^{(1)}(\zeta,z_1)\\ \widetilde{J}_{\nu,z}^{(1)}(\zeta,z_1)\end{bmatrix}\exp(-i\beta_\nu z_0)\\ &\qquad+i\frac{\partial}{\partial z_1}I_\nu^{(0)}(z_1)\begin{bmatrix}\frac{\partial}{\partial\beta_\nu}\widetilde{G}_{xx}(\zeta,\beta_\nu) & \frac{\partial}{\partial\beta_\nu}\widetilde{G}_{xz}(\zeta,\beta_\nu)\\ \frac{\partial}{\partial\beta_\nu}\widetilde{G}_{zx}(\zeta,\beta_\nu) & \frac{\partial}{\partial\beta_\nu}\widetilde{G}_{zz}(\zeta,\beta_\nu)\end{bmatrix}\\ &\qquad\begin{bmatrix}\widetilde{J}_{\nu,x}^{(0)}(\zeta)\\ \widetilde{J}_{\nu,z}^{(0)}(\zeta)\end{bmatrix}\exp(-i\beta_\nu z_0). \end{aligned}\tag{28}$$

The first-order boundary conditions in the space domain require that

$$\mathbf{E}_{a,t}^{(1)}(x,h;z_0,z_1)+\mathbf{E}_{b,t}^{(0)}(x,h;z_0,z_1)=0 \text{ (on microstrip } a) \tag{29}$$

$$\mathbf{E}_{b,t}^{(1)}(x,h;z_0,z_1)+\mathbf{E}_{a,t}^{(0)}(x,h;z_0,z_1)=0 \text{ (on microstrip } b). \tag{30}$$

From Eqs. (19) and (28)–(30), we have the integral equations for $\mathbf{J}_a^{(1)}(x)$ and $\mathbf{J}_b^{(1)}(x)$. To solve the integral equations, the spectra of the first-order currents are expanded as follows:

$$\widetilde{J}_{\nu,x}^{(1)}(\zeta,z_1)=\sum_{n=1}^{N}a_{\nu,xn}^{(1)}(z_1)\tilde{p}_{\nu,n}(\zeta)\quad(\nu=a,b) \tag{31}$$

$$\widetilde{J}_{\nu,z}^{(1)}(\zeta,z_1)=\sum_{n=1}^{N}a_{\nu,zn}^{(1)}(z_1)\tilde{q}_{\nu,n}(\zeta) \tag{32}$$

where $\{a_{\nu,xn}^{(1)}\}$ and $\{a_{\nu,zn}^{(1)}\}$ are unknown expansion coefficients. Using Galerkin's moment method again, the first-order problem is reduced to the following matrix equations:

$$[\mathbf{A}_a(\beta_a)] \cdot \mathbf{a}_a^{(1)} = -i\mathbf{U}_a(\beta_a)\frac{\partial}{\partial z_1} I_a^{(0)}(z_1) - \mathbf{V}_b(\beta_b) I_b^{(0)}(z_1)\exp(i\Delta\beta z_1) \tag{33}$$

$$[\mathbf{A}_b(\beta_b)] \cdot \mathbf{a}_b^{(1)} = -i\mathbf{U}_b(\beta_b)\frac{\partial}{\partial z_1} I_b^{(0)}(z_1) - \mathbf{V}_a(\beta_a) I_a^{(0)}(z_1)\exp(-i\Delta\beta z_1) \tag{34}$$

with

$$\mathbf{U}_\nu(\beta_\nu) = \left[\frac{\partial}{\partial \beta_\nu}\mathbf{A}(\beta_\nu)\right] \cdot \mathbf{a}_\nu^{(0)} \tag{35}$$

$$\mathbf{V}_\nu(\beta_\nu) = [\mathbf{M}_{\mu\nu}(\beta_\nu)] \cdot \mathbf{a}_\nu^{(0)} \ (\nu \neq \mu) \tag{36}$$

$$[\mathbf{M}_{\mu\nu}(\beta_\nu)] = \begin{bmatrix} M_{\mu\nu,xx}^{mn}(\beta_\nu) & M_{\mu\nu,xz}^{mn}(\beta_\nu) \\ M_{\mu\nu,zx}^{mn}(\beta_\nu) & M_{\mu\nu,zz}^{mn}(\beta_\nu) \end{bmatrix} \tag{37}$$

$$M_{\mu\nu,xx}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{p}_{\mu,m}(\zeta)\widetilde{G}_{xx}(\zeta,\beta_\nu)\tilde{p}_{\nu,n}(\zeta)d\zeta \tag{38}$$

$$M_{\mu\nu,xz}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{p}_{\mu,m}(\zeta)\widetilde{G}_{xz}(\zeta,\beta_\nu)\tilde{q}_{\nu,n}(\zeta)d\zeta \tag{39}$$

$$M_{\mu\nu,zx}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{q}_{\mu,m}(\zeta)\widetilde{G}_{zx}(\zeta,\beta_\nu)\tilde{p}_{\nu,n}(\zeta)d\zeta \tag{40}$$

$$M_{\mu\nu,zz}^{mn}(\beta_\nu) = \int_{-\infty}^{\infty} \tilde{q}_{\mu,m}(\zeta)\widetilde{G}_{zz}(\zeta,\beta_\nu)\tilde{q}_{\nu,n}(\zeta)d\zeta \tag{41}$$

where $\Delta\beta = \beta_a - \beta_b$, and $\mathbf{a}_\nu^{(0)}$ and $\mathbf{a}_\nu^{(1)}$ denote the column vectors with the elements $(a_{\nu,xn}^{(0)}, b_{\nu,zn}^{(0)})$ and $(a_{\nu,xn}^{(1)}, a_{\nu,zn}^{(1)})$, respectively. The inhomogeneous system of linear equations (33) and (34) are singular, because $\det[\mathbf{A}_\nu(\beta_\nu)] = 0$ as shown in Eq. (27). Then the solutions to the first-order problem are allowed only when a solvability condition is satisfied [7]. After several manipulations, the solvability condition leads to the first-order coupled-mode equations for $I_a^{(0)}$ and $I_b^{(0)}$ as follows:

$$\frac{d}{dz} I_a^{(0)} = -iK_{ab} I_b^{(0)} \exp(i\Delta\beta z) \tag{42}$$

$$\frac{d}{dz} I_b^{(0)} = -iK_{ba} I_a^{(0)} \exp(-i\Delta\beta z) \tag{43}$$

with

$$K_{ab} = -\frac{\mathbf{L}_a \cdot \mathbf{V}_b}{\mathbf{L}_a \cdot \mathbf{U}_a} \tag{44}$$

$$K_{ba} = -\frac{\mathbf{L}_b \cdot \mathbf{V}_a}{\mathbf{L}_b \cdot \mathbf{U}_b} \tag{45}$$

where $\mathbf{L}_a$ and $\mathbf{L}_b$ are the right eigenvectors that satisfy $[\mathbf{A}_a(\beta_a)]^T \cdot \mathbf{L}_a = 0$ and $[\mathbf{A}_b(\beta_b)]^T \cdot \mathbf{L}_b = 0$, respectively, and the slow space-scale z_1 has been transformed back into the original space-scale z by letting $\delta = 1$. The solutions to Eqs. (42) and (43) give the perturbed propagation constants in the presence of adjacent microstrips. In order to determine the unknown expansion coefficients $\{a_{\nu,xn}^{(1)}\}$ and $\{a_{\nu,zn}^{(1)}\}$, the condition $\det[\mathbf{A}_\nu(\beta_\nu)] = 0$ is used and Eqs. (33) and (34) are rearranged as follows:

$$\begin{aligned}[\mathbf{A}''_a(\beta_a)] \cdot \mathbf{a'}_a^{(1)} &= -a_{a,z1}^{(1)} \mathbf{s}_a - i\mathbf{U}'_a(\beta_a)\frac{\partial}{\partial z} I_a^{(0)}(z) \\ &\quad - \mathbf{V}'_b(\beta_b) I_b^{(0)}(z) \exp(i\Delta\beta z)\end{aligned} \tag{46}$$

$$\begin{aligned}[\mathbf{A}''_b(\beta_b)] \cdot \mathbf{a'}_b^{(1)} &= -a_{b,z1}^{(1)} \mathbf{s}_b - i\mathbf{U}'_b(\beta_b)\frac{\partial}{\partial z} I_b^{(0)}(z) \\ &\quad - \mathbf{V}'_a(\beta_a) I_a^{(0)}(z) \exp(-i\Delta\beta z)\end{aligned} \tag{47}$$

with

$$\mathbf{s}_\nu = \left[A_{\nu,xz}^{11} \cdots A_{\nu,xz}^{N1} A_{\nu,zz}^{21} \cdots A_{\nu,zz}^{N1}\right]^T \quad (\nu = a, b) \tag{48}$$

$$\mathbf{a'}_\nu^{(1)} = \left[a_{\nu,x1}^{(1)} \cdots a_{\nu,xN}^{(1)} a_{\nu,z2}^{(1)} \ldots a_{\nu,zN}^{(1)}\right]^T \tag{49}$$

$$\mathbf{U}'_\nu = \left[\frac{\partial}{\partial \beta_\nu} \mathbf{A}'(\beta_\nu)\right] \cdot \mathbf{a}_\nu^{(0)} \tag{50}$$

$$\mathbf{V}'_\nu = \left[\mathbf{M}'_{\mu\nu}(\beta_\nu)\right] \cdot \mathbf{a}_\nu^{(0)} \tag{51}$$

where $[\mathbf{A}'_\nu(\beta_\nu)]$ and $[\mathbf{A}''_\nu(\beta_\nu)]$ denote the matrices deduced by eliminating the $(N+1)$-th row elements from $[\mathbf{A}_\nu(\beta_\nu)]$ and the $(N+1)$-th column elements from $[\mathbf{A}'_\nu(\beta_\nu)]$, respectively, and the slow space-scale z_1 has been transformed back into the original space-scale z by letting $\delta = 1$. Retaining only the terms being independent of the known expansion coefficients $\mathbf{a}_\nu^{(0)}$ for the zero-order current distributions, Eqs. (46) and (47) yield the first-order solutions as follows:

$$\mathbf{a'}_a^{(1)} = -i\left[\mathbf{A}''_a(\beta_a)\right]^{-1}\left(\mathbf{U}'_a + \frac{\mathbf{V}'_b}{K_{ab}}\right)\frac{d}{dz} I_a^{(0)}(z) \tag{52}$$

$$\mathbf{a}'^{(1)}_b = -i\left[\mathbf{A}''_b(\beta_b)\right]^{-1}\left(\mathbf{U}'_b + \frac{\mathbf{V}'_a}{K_{ba}}\right)\frac{d}{dz}I_b^{(0)}(z) \tag{53}$$

where Eqs. (42) and (43) have been used. Thus the unknown expansion coefficients $\mathbf{a}'^{(1)}_\nu = \left[a^{(1)}_{\nu,x1}\cdots a^{(1)}_{\nu,xN}a^{(1)}_{\nu,z2}\cdots a^{(1)}_{\nu,zN}\right]^T$ for the first-order current distribution were obtained. The remaining unknown $a^{(1)}_{\nu,z1}$ is determined from the requirement that $\mathbf{a}^{(1)}_\nu$ should be orthogonal to $\mathbf{a}^{(0)}_\nu$ as follows:

$$a^{(1)}_{\nu,z1} = -\frac{\mathbf{a}'^{(0)}_a\cdot\mathbf{a}'^{(1)}_a}{a^{(1)}_{\nu,z0}} \quad (\nu = a, b). \tag{54}$$

Using the solutions to Eqs. (52)–(54), the current distributions on line ν perturbed by the presence of the adjacent line μ are given as follows:

$$J_{\nu,x}(x) = \sum_{n=1}^{N}\left[a^{(0)}_{\nu,xn} + \frac{a^{(1)}_{\nu,xn}}{I^{(0)}_\nu(z)}\right]p_{\nu,n}(x) \quad (\nu = a, b) \tag{55}$$

$$J_{\nu,z}(x) = \sum_{n=1}^{N}\left[a^{(0)}_{\nu,zn} + \frac{a^{(1)}_{\nu,zn}}{I^{(0)}_\nu(z)}\right]q_{\nu,n}(x) \tag{56}$$

under the solvability conditions (42) and (43). The coupled-current distributions (55) and (56) are calculated for each of two independent coupled-modes obtained from Eqs. (42) and (43). Note that $a^{(1)}_{\nu,xn}/I^{(0)}_\nu(z)$ and $a^{(1)}_{\nu,zn}/I^{(0)}_\nu(z)$ in Eqs. (55) and (56) are in proportion to $i(d/dz)I^{(0)}_\nu(z)/I^{(0)}_\nu(z)$, which is finally replaced by the perturbed propagation constant in each of the coupled-modes.

4. NUMERICAL EXAMPLES

To validate the proposed coupled-mode theory, two identical coupled microstrip lines with $w_a = w_b = w$ in Fig. 1 is chosen as the model for numerical computations. For this symmetric structure, the coupled-mode equations (42) and (43) are reduced to

$$\frac{d}{dz}I^{(0)}_a = -iKI^{(0)}_b \tag{57}$$

$$\frac{d}{dz}I^{(0)}_b = -iKI^{(0)}_a \tag{58}$$

where $\beta_0 = \beta_a = \beta_b$ and $K = K_{ab} = K_{ba}$. Equations (57) and (58) reveal that the two identical coupled microstrip line support two coupled modes, symmetric and asymmetric modes, with the propagation constants $\beta = \beta_0 + K$ and $\beta = \beta_0 - K$ perturbed symmetrically from their isolated limits β_0. The currents in the two microstrips are in the same (opposite) direction for the symmetric (asymmetric) mode. The eigenmode current $\mathbf{J}_\nu^{(0)}(x)$ and propagation constants β_0 for each isolated single microstrip, which are the basis of the present coupled-mode analysis, were calculated by Galerkin's moment method with the Chebyshev polynomial basis functions weighted by appropriate edge factors. For comparison, the same coupled problem was also rigorously solved by using the direct Galerkin's moment method in spectral domain.

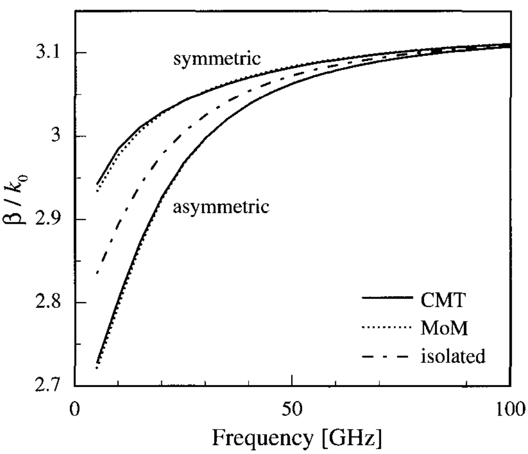


Figure 2. Dispersion curves of the symmetric and asymmetric modes of two identical coupled microstrip lines with the separation $d/w = 0.3$. The values of other parameters are the same as those given in Table 1.

The normalized propagation constants β/k of the symmetric and asymmetric modes are given in Table 1 for $w = 1.5$ mm, $h = 0.635$ mm, $\varepsilon_r = 9.8$, $f = 5 \sim 20$ GHz, and four different separations d/w, and are compared with those of the direct Galerkin's moment method solutions. Figure 2 shows the dispersion curves of the symmetric and asymmetric modes for $d/w = 0.3$ with other parameters same as those given in Table 1. CMT and MoM refer to the present coupled-mode theory and the direct Galerkin's moment method. From the comparison in Table 1 and Fig. 2, we can see that the coupled-mode approximations are in good agreement with the rigorous Galerkin's moment method solutions over a broad range of weak to strong coupling.

(a) $f = 5$ GHz and $\beta_0/k = 2.83466$.

	Symmetric mode				Asymmetric Mode			
d/W	0.1	0.3	0.5	1.0	0.1	0.3	0.5	1.0
CMT	2.97461	2.94187	2.91223	2.87066	2.69471	2.72746	2.75710	2.79866
MoM	2.95881	2.93332	2.90836	2.86994	2.68003	2.72062	2.75351	2.79775

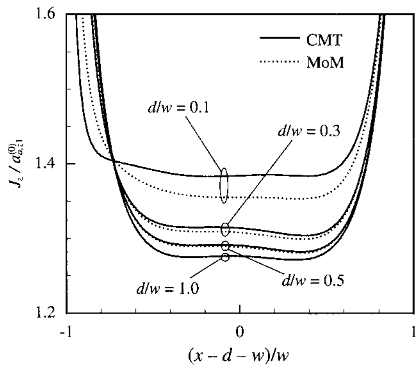
(b) $f = 10$ GHz and $\beta_0/k = 2.89439$.

	Symmetric mode				Asymmetric Mode			
d/W	0.1	0.3	0.5	1.0	0.1	0.3	0.5	1.0
CMT	3.02205	2.98471	2.95335	2.91415	2.76672	2.80407	2.83542	2.87462
MoM	3.00504	2.97713	2.95055	2.91397	2.75194	2.79690	2.83183	2.87393

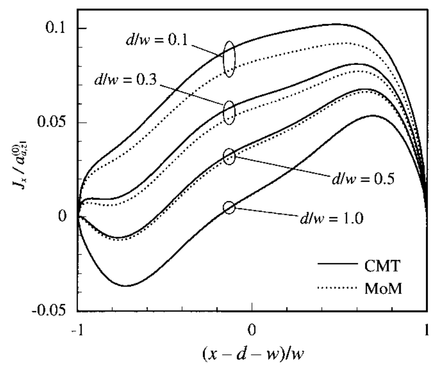
(c) $f = 20$ GHz and $\beta_0/k = 2.97776$.

	Symmetric mode				Asymmetric Mode			
d/W	0.1	0.3	0.5	1.0	0.1	0.3	0.5	1.0
CMT	3.06429	3.02850	3.00403	2.98210	2.89124	2.92702	2.95149	2.97343
MoM	3.05437	3.02685	3.00422	2.98227	2.88632	2.92450	2.95011	2.97320

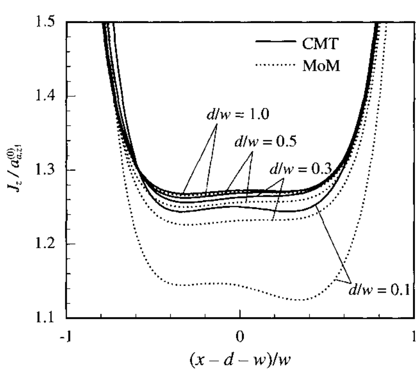
Table 1. Normalized propagation constants β/k of the symmetric and asymmetric modes of two identical coupled microstrip lines for $f = 5 \sim 20$ GHz with $w_a = w_b = w = 1.5$ mm, $h = 0.635$ mm, $\varepsilon_r = 9.8$, and four different separation distances d/w. β_0 is the propagation constant of the isoalted mode. CMT and MoM refer to the coupled-mode theory and the direct Galerkin's moment method.



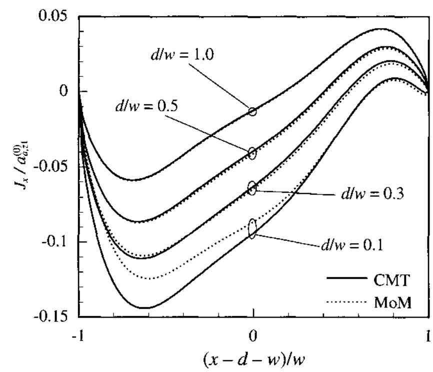
(a) J_z for the symmetric mode.



(b) J_x for the symmetric mode.



(c) J_z for the asymmetric mode.



(d) J_x for the asymmetric mode.

Figure 3. Normalized longitudinal and transverse current distributions on the microstrip of the right hand side for the symmetric and asymmetric modes at $f = 10$ GHz. The values of parameters are the same as those given in Table 1. The solid and dotted lines indicate the results of the present coupled-mode theory and the direct Galerkin's moment method, respectively.

Figure 3 shows the longitudinal and transverse current distributions calculated by Eqs. (55) and (56) for $f = 10$ GHz and four different separations d/w. The distributions are plotted only for the microstrip of the right-hand side by taking into account of the symmetry of structure. To acquire a clear physical picture of the coupled-current, the amplitude is normalized by $a_{a,z1}^{(0)}$ and the vertical scale is magnified. The coupled-current distributions exhibit significant changes as the separation distance decreases. The results of the coupled-mode theory are again in good agreement with the rigorous coupled-current distributions obtained by the direct Galerkin's moment method. When the two microstrips are closely spaced with $d/w = 0.1$, the relative discrepancy of 10% at maximum was observed in the current distributions calculated by the two approaches.

Due to the simpler matrix equation involved, the numerical procedure of the coupled-mode analysis is much more efficient than that of the direct Galerkin's moment method. For the same computation of the propagation constants and current distributions, the coupled-mode analysis requires about 8% of the computer time needed by the direct method.

5. CONCLUDING REMARKS

A coupled-mode theory for coupled microstrip lines has been developed by using the singular perturbation technique in the spectral domain. The theory provides a powerful analytical and numerical technique for approximating the coupling between adjacent microstrip lines with a good physical justification. The numerical procedure is much simpler than the direct numerical solution methods and therefore the computation time is greatly reduced. Not only the propagation constants of the coupled modes but also the coupled-current distributions on the lines is calculated with the same accuracy from a simple matrix algebra using the spectra of currents in each isolated single line. This is a distinct advantage of the present theory compared with the conventional perturbation theory [5] and the coupled-mode theory based on the reciprocity theorem [6], which have some defect in the approximation of the coupled-current distributions. A better approximation of the coupled-current distributions leads to a better approximation of characteristic mode impedances of the coupled lines. To confirm the validity of the proposed theory, two identical coupled microstrip lines were analyzed. The numerical results of the propagation constants and the coupled-current distributions for the symmetric and asymmetric modes are in good agreement with those obtained by the direct Galerkin's moment method over a broad range of weak to strong coupling. The extension of the theory to multilayered and multiple-coupled microstrip lines is straightforward.

APPENDIX

δ^0-Order Equations:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z_0^2}+k^2\varepsilon(y)\right]\phi_\nu^{(0)}(x,y;z_0,z_1)=0 \tag{59}$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z_0^2}+k^2\varepsilon(y)\right]\psi_\nu^{(0)}(x,y;z_0,z_1)=0 \tag{60}$$

$$E_{\nu,x}^{(0)}=\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\phi_\nu^{(0)}(x,y;z_0,z_1) \tag{61}$$

$$E_{\nu,z}^{(0)}=\frac{\partial}{\partial x}\frac{\partial}{\partial z_0}\phi_\nu^{(0)}(x,y;z_0,z_1)+i\omega\mu_0\frac{\partial}{\partial y}\psi_\nu^{(0)}(xy;z_0,z_1) \tag{62}$$

$$H_{\nu,x}^{(0)}=\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\psi_\nu^{(0)}(x,y;z_0,z_1) \tag{63}$$

$$H_{\nu,z}^{(0)}=\frac{\partial}{\partial x}\frac{\partial}{\partial z_0}\psi_\nu^{(0)}(x,y;z_0,z_1)-i\omega\varepsilon_0\varepsilon(y)\frac{\partial}{\partial y}\phi_\nu^{(0)}(x,y;z_0,z_1) \tag{64}$$

δ^1-Order Equations:

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z_0^2}+k^2\varepsilon(y)\right]\phi_\nu^{(1)}(x,y;z_0,z_1) = -2\frac{\partial}{\partial z_0}\frac{\partial}{\partial z_1}\phi_\nu^{(0)}(x,y;z_0,z_1) \tag{65}$$

$$\left[\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z_0^2}+k^2\varepsilon(y)\right]\psi_\nu^{(1)}(x,y;z_0,z_1) = -2\frac{\partial}{\partial z_0}\frac{\partial}{\partial z_1}\psi_\nu^{(0)}(x,y;z_0,z_1) \tag{66}$$

$$E_{\nu,x}^{(1)}=\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\phi_\nu^{(1)}(x,y;z_0,z_1) \tag{67}$$

$$E_{\nu,z}^{(1)}=\frac{\partial}{\partial x}\frac{\partial}{\partial z_0}\phi_\nu^{(1)}(x,y;z_0,z_1)+\frac{\partial}{\partial x}\frac{\partial}{\partial z_1}\phi_\nu^{(0)}(x,y;z_0,z_1) +i\omega\mu_0\frac{\partial}{\partial y}\psi_\nu^{(1)}(x,y;z_0,z_1) \tag{68}$$

$$H_{\nu,x}^{(1)}=\left[\frac{\partial^2}{\partial x^2}+k^2\varepsilon(y)\right]\psi_\nu^{(1)}(x,y;z_0,z_1) \tag{69}$$

$$H_{\nu,z}^{(1)} = \frac{\partial}{\partial x}\frac{\partial}{\partial z_0}\psi_\nu^{(1)}(x,y;z_0,z_1) + \frac{\partial}{\partial x}\frac{\partial}{\partial z_1}\psi_\nu^{(0)}(x,y;z_0,z_1) - i\omega\varepsilon_0\varepsilon(y)\frac{\partial}{\partial y}\phi_\nu^{(1)}(x,y;z_0,z_1) \tag{70}$$

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