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AXIS EXPANSION METHOD FOR NEARLY TWO DIMENSIONAL OBJECTS

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1. INTRODUCTION

We denote by "nearly two dimensional object" a three dimensional (3D) homogeneous scattering object which is close to a two dimensional (2D) one. In other words, this object is almost invariant by translation with respect to a privileged direction. This kind of object is commonplace in Electromagnetics and in Optics. For instance, some gratings can present slight variations of profile along the direction of the grooves. In the same way, 2D photonic crystals made by photolithography [1] can present slight variations of section. Furthermore, airplane wings or helicopter blades (which has a vital importance in the calculations of the Radar Cross Section) and many biologic objects (arm and leg bones for examples) that can be observed by microwave imaging, satisfy this property.

In general, two kinds of approaches are developed for these objects. The first one reduces the problem to a 2D problem by neglecting the section variation of the object along the privileged axis. In contrast, the second one uses a classical 3D theory. Unfortunately, the numerical treatment of rigorous 3D problems requires long computation times and large memory storage. This fact explains why these methods become more and more difficult to implement as the size of the diffracting object increases.

Thus we thought it of interest to develop a new theory which should be able to solve with precision the problem of scattering from this kind of object by using codes devoted to 2D objects, without neglecting the variation of section along the privileged axis.

The first basic idea is to cut the nearly two-dimensional object into different cross section, perpendicular to its privileged axis, then to consider that the local behaviour of the object is the same as that of the 2D object having the same cross-ection. Of course, this method (called section method in the following) reduces the scattering problem to the solution of several 2D problems. However, we thought that this rudimentary approach could be improved. With this aim, we have tried to elaborate a theory which keeps the basic features of a rigorous electromagnetic theory when necessary (i.e. in the directions perpendicular to the privileged axis) and adapts the point of view of the physical optics approximation (or Kirchhoff approximation) along this last axis [2,3]. The main advantage of the classical physical optics approximation lies in its simplicity, which entails a straightforward numerical treatment and a short computation time. On the other hand, it has been shown that this method may fail in resonance conditions, when the wavelength of the incident field has the same order of magnitude as the dimensions of the diffracting object [4]. Due to this reason the variation of the section of the nearly 2D object on two planes orthogonal to the privileged axis and separated by one wavelength must be small with respect to the wavelength.

The starting point of our theory is the rigorous Waterman formalism [5]. Our basic equations are obtained by analysing the asymptotic behaviour of Waterman equations when the nearly 2D object is expanded in the direction of the privileged axis. In addition, this classical theory will provide rigorous numerical data for 3D objects, in order to estimate the precision of our approximate theory.

In the present paper, the Axis Expansion Method (A.E.M.) is developed in the case of perfectly conducting diffraction grating problems. It will be shown that it can provide very precise results with very short computation time, using codes devoted to 1D gratings only (2D problems). It can deal with gratings with small slopes along the privileged axis and arbitrary shapes along the other axis of periodicity.

2. PRESENTATION OF THE PROBLEM

We consider an homogeneous monochromatic plane wave incident on a doubly-periodic perfectly conducting grating (Fig. 1). In accordance with the conventional choice of axes for classical diffraction grating problems, the y axis of the cartesian rectangular coordinate system is orthogonal to the mean plane of the grating surface $\mathcal{P}$, given by $y = F(x,z)$. The two periods of the grating are denoted by $d_x = 2\pi/K_x$, the period along the x axis, and $d_z = 2\pi/K_z$, the period along the z axis, with $d_z >> d_x$, d_x being of the same order as the wavelength λ of the incident light. The region above the profile $\mathcal{P}$ contains air (region R^+) and thus its index will be assumed to be equal to 1. The unit normal to $\mathcal{P}$ oriented towards this region is denoted by $\hat{\mathbf{n}}$.

The direction of the incoming plane wave is specified by two angles θ and ϕ (Fig. 2). The first angle θ is the angle between the incident wave vector $\mathbf{k}$ and the y axis, ϕ is the angle between the projection of $\mathbf{k}$ onto the $x-z$ plane and the x axis. Thus, the wave vector of the incident beam writes

$$\mathbf{k} = \alpha\hat{\mathbf{x}} - \beta\hat{\mathbf{y}} + \gamma\hat{\mathbf{z}}, \tag{1}$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ are the unit vectors of x, y and z axes respectively, with

$$\begin{cases} \alpha = k\sin\theta\cos\phi, \\ \beta = k\cos\theta, \\ \gamma = k\sin\theta\sin\phi, \end{cases} \tag{2}$$

and $k = |\mathbf{k}| = \mathbf{2}\pi/\lambda$ is the wave number of the incident light.

Suppressing the time dependence $\exp(-i\omega t)$, the expression for the electric field of the incident wave is given by

$$\mathbf{E}^i = \mathbf{A}_{00}\exp(i\alpha x - i\beta y + i\gamma z). \tag{3}$$

In order to define the polarization of the incident beam, we introduce a third angle δ, such that when $\delta = 90°$, the vector amplitude of

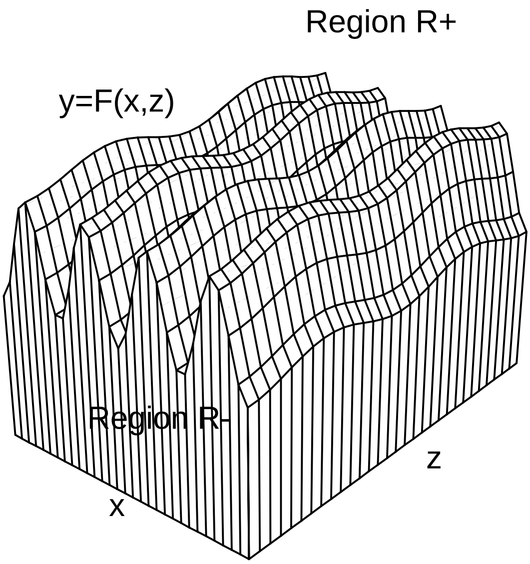

Figure 1. Schematic representation of a nearly 2D diffraction grating.

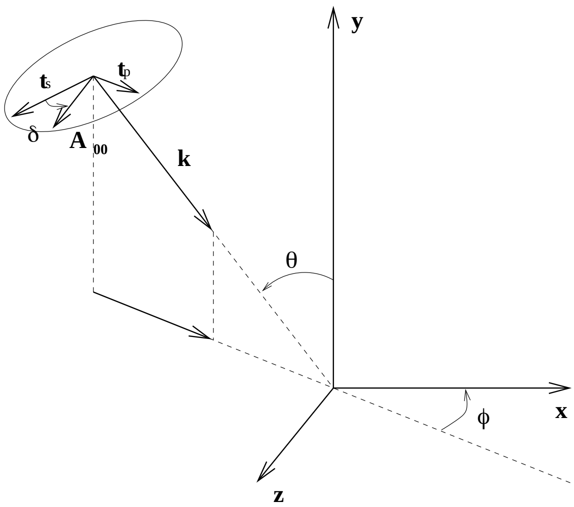

Figure 2. Definition of the incidence angles θ , ϕ and the polarization angle δ . $\mathbf{A}_{00} = \cos\delta \mathbf{t}_s + \sin\delta \mathbf{t}_p$.

the incident electric field $\mathbf{A}_{00} = \mathbf{t}_p$ lies in the plane of incidence (p polarization) and when $\delta = 0°$, $\mathbf{A}_{00} = \mathbf{t}_s$ is orthogonal to the plane of incidence (s polarization). If we normalize the magnitude of $\mathbf{A}_{00}$ to unity, then

$$\begin{cases} A_{x,00} = -\cos\delta\sin\phi + \sin\delta\cos\theta\cos\phi, \\ A_{y,00} = \sin\delta\sin\theta, \\ A_{z,00} = \cos\delta\cos\phi + \sin\delta\cos\theta\sin\phi. \end{cases} \tag{4}$$

The diffracted field is defined above the surface of the grating by $\mathbf{E}^d = \mathbf{E} - \mathbf{E}^i$, where $\mathbf{E}$ represents the total field. Above the top of the grooves, the diffracted field can be represented by the so-called Rayleigh expansion [6–8]:

$$\mathbf{E}^d = \sum_n \sum_m \mathbf{B}_{nm} \exp(i\alpha_n x + i\beta_{nm} y + i\gamma_m z), \ \ if \ \ y > y_M, \tag{5}$$

where y_M is the maximal value of $F(x,z)$ and $\sum_n$ denoting a sum from $-\infty$ to $+\infty$, α_n , β_{nm} and γ_m denoting the propagation constants of the diffracted orders :

$$\alpha_n = \alpha + nK_x, \tag{6}$$

$$\gamma_m = \gamma + mK_z, \tag{7}$$

$$\beta_{nm} = \begin{cases} (k^2 - \alpha_n^2 - \gamma_m^2)^{1/2} & \text{if } (n,m) \in U \\ i(\alpha_n^2 + \gamma_m^2 - k^2)^{1/2} & \text{otherwise,} \end{cases} \tag{8}$$

U is a finite set of integers (n,m) for which $k^2 - \alpha_n^2 - \gamma_m^2$ is positive, the corresponding plane waves being the propagating (non-evanescent) orders of the grating.

We denote efficiency $e_{n,m}$, in the (n,m) order, the diffracted energy to incident energy ratio, deduced from the $\mathbf{B}_{nm}$:

$$if \ \ (n,m) \in U, \quad e_{nm} = (|B_{x,nm}|^2 + |B_{y,nm}|^2 + |B_{z,nm}|^2)\frac{\beta_{nm}}{\beta}. \tag{9}$$

It is well known that the knowledge of the surface current density $\mathbf{j}_{\mathcal{P}}$ flowing the surface of the grating allows one to compute the amplitudes of the diffracted waves in all the propagative or evanescent orders.

Expressions linking the amplitudes $\mathbf{B}_{nm}$ of the diffracted electric wave in the (n, m) order and the surface current density $\mathbf{j}_{\mathcal{P}}$ are :

$$
\begin{aligned}
(\gamma_m B_{x,nm} - \alpha_n B_{z,nm}) = \\
\frac{1}{2id_x d_z\ \beta_{nm}} \int_0^{d_x} \int_0^{d_z} [\gamma_m \phi_x(x,z) - \alpha_n \phi_z(x,z)] \\
\times \exp[-i\alpha_n x - i\beta_{nm} F(x,z) - i\gamma_m z]\ dxdz,
\end{aligned} \tag{10}
$$

and

$$
\begin{aligned}
(\alpha_n B_{x,nm} + \gamma_m B_{z,nm}) = \\
-\frac{1}{2id_x d_z\ k^2} \int_0^{d_x} \int_0^{d_z} \left\{ \left[\alpha_n \beta_{nm} - (\alpha_n^2 + \gamma_m^2) \frac{\partial F(x,z)}{\partial x} \right] \phi_x(x,z) \right. \\
\left. + \left[\gamma_m \beta_{nm} - (\alpha_n^2 + \gamma_m^2) \frac{\partial F(x,z)}{\partial z} \right] \phi_z(x,z) \right\} \\
\times \exp[-i\alpha_n x - i\beta_{nm} F(x,z) - i\gamma_m z]\ dxdz.
\end{aligned} \tag{11}
$$

where the function $\mathbf{\Phi} = \phi_x \hat{x} + \phi_y \hat{y} + \phi_z \hat{z}$ is closely linked to the surface current density $\mathbf{j}_{\mathcal{P}}$ on the grating surface by :

$$
\mathbf{\Phi}(x,z) = -i\omega\mu_0 \sqrt{1 + \left(\frac{\partial F(x,z)}{\partial x} \right)^2 + \left(\frac{\partial F(x,z)}{\partial z} \right)^2}\ \mathbf{j}_{\mathcal{P}}(x,z). \tag{12}
$$

So, the problem lies in the calculation of the function $\mathbf{\Phi}$.

3. THEORY

Obviously, the mathematical handling of our basic idea, i.e. the synthesis of a rigorous treatment in two directions of space with a physical optics approximation in the third direction is not trivial. In order to understand how it can be made, it is necessary to give to the classical physical optics approximation a new mathematical interpretation. To this end, we have chosen the most simple case: the classical grating.

3.1 Case of 1D Gratings in s-polarization

Let us consider a perfectly conducting grating having a large period with respect to the incident wavelength and a small slope at any point of the profile $y = f(x)$. This grating is illuminated with s_polarized light, the incident wave-vector lying in the $x - y$ plane (Fig. 3). In that case, the total field $\mathbf{E} = E_z \hat{\mathbf{z}}$ and the function $\mathbf{\Phi}(x, z) = \phi_z(x)\hat{\mathbf{z}}$ are parallel to the z axis, with

$$\phi_z(x) = \sqrt{1 + f'(x)^2}\, dE_z/dn. \tag{13}$$

The diffracted electric field $\mathbf{E}^d = E_z^d \hat{\mathbf{z}}$ above the grooves is given by the following expansion:

$$E_z^d = \sum_n B_{z,n} \exp(i\alpha_n x + i\beta_n y) \tag{14}$$

with

$$\alpha_n = \alpha + n\lambda/d, \tag{15}$$

$$\beta_n = \begin{cases} (k^2 - \alpha_n^2)^{1/2} & \text{if } n \in U \\ i(\alpha_n^2 - k^2)^{1/2} & \text{else} \end{cases} \tag{16}$$

U is a finite set of integers n, for which $k^2 - \alpha_n^2$ is positive, the corresponding plane waves being the propagating (non-evanescent) orders of the grating.

So, equation (10) enables one to express $B_{z,n}$ and the efficiencies e_n in the form:

$$B_{z,n} = \frac{1}{2id_x\beta_n} \int_0^{d_x} \exp[-i\alpha_n x - i\beta_n f(x)]\phi_z(x)dx, \tag{17}$$

$$e_n = |\beta_{z,n}|^2\, \frac{\beta_n}{\beta}. \tag{18}$$

The approximate value ϕ_z^K of ϕ_z is obtained from the Kirchhoff approximation in the following way: we shall assume that the normal derivative of the field at a certain point M of $\mathcal{P}$ is the same as the normal derivative of the field which would be obtained by replacing $\mathcal{P}$ by a plane mirror Σ_2 tangent to $\mathcal{P}$ at that same point (Fig. 3). So the "local" value of E_z at the vicinity of $\mathcal{P}$ will be the sum of the incident wave and the wave reflected by the fictitious mirror. An effortless calculation shows that:

$$\phi_z^K(x) = -2i(\beta + \alpha f'(x)) \exp(i\alpha x - i\beta f(x)). \tag{19}$$

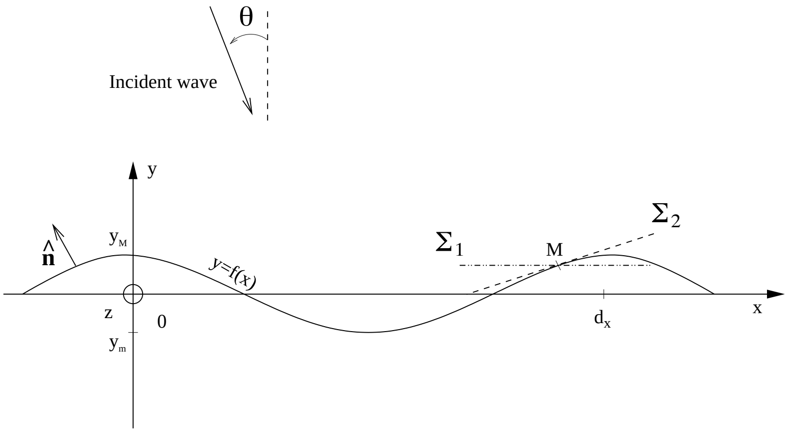

Figure 3. Symbols and notations for a nearly 1D problem.

Let us notice that the section method, in this simple case, consists in replacing the grating surface by the mirror Σ_1 , parallel to the x axis, which shows obviously the superiority of the physical optics approximation.

The aim of this subsection is to show that equation (19) can be retrieved from a mathematical approach based on the Waterman method [5]. We start from the Waterman formula

$$\forall n, \quad \frac{1}{d_x}\int_0^{d_x} \tilde{\phi}_z(x)\exp[-inK_x x + i\beta_n f(x)]dx = -2i\beta_n\delta_{n,0}, \qquad (20)$$

where $\tilde{\phi}_z(x)$ is the periodized function $\phi_z(x)$, i.e. the function $\phi_z(x)$ multiplied by $\exp(-i\alpha x)$.

Let us recall that the Waterman theory uses the infinite set of equations given by (20) in order to determine $\tilde{\phi}_z$. Indeed, it can be proved that the functions $v_n(x) = \exp[-inK_x x + i\beta_n f(x)]$ form a topological basis [9].

Now, let us introduce the following change of variable:

$$u = K_x x, \qquad (21)$$

We define $p(u) = f(u/K_x)$, a periodic function of period 2π , and $\psi(u) = \tilde{\phi}_z(u/K_x)$. Let us assume that $K_x = 2\pi/d_x$ tends to 0 . Consequently, $\psi(u)$ and β_n can be developed in entire series of K_x :

$$\psi(u) = \psi_0(u) + K_x\psi_1(u) + {K_x}^2\psi_2(u) + \dots \qquad (22)$$

and

$$\beta_n = \beta - \frac{\alpha}{\beta} n K_x + \dots. \tag{23}$$

So, when K_x tends to 0, (20) can be written, at the zeroth order in K_x :

$$\frac{1}{2\pi} \int_0^{2\pi} \psi_0(u) \exp(-inu + i\beta p(u)) du = -2i\beta \delta_{n,0}, \tag{24}$$

and at the first order:

$$\frac{1}{2\pi} \int_0^{2\pi} [\psi_0(u) + K_x \psi_1(u)][1 - i\frac{\alpha}{\beta} n K_x p(u)] \exp(-inu + i\beta p(u)) du$$
$$= -2i\beta \delta_{n,0}. \tag{25}$$

Now, we can see that the left-hand member of equation (24) is nothing but the Fourier coefficient in the nth order of the function:

$$\eta(u) = \psi_0(u) \exp(i\beta p(u)). \tag{26}$$

So, the summation from $n = -\infty$ to $n = +\infty$ of the Fourier series whose coefficients are given by equation (24) enables one to write:

$$\psi_0(u) = -2i\beta \exp(-i\beta p(u)). \tag{27}$$

Then, the first order term in K_x of (25) is :

$$\frac{1}{2\pi} \int_0^{2\pi} [-\frac{i\alpha}{\beta} p(u) n \psi_0(u) + \psi_1(u)] \exp(-inu + i\beta p(u)) du = 0. \tag{28}$$

Here, the left-hand member of this equation represents the Fourier coefficient at the nth order of the function :

$$\zeta(u) = -\frac{\alpha}{\beta} \frac{d}{du} [p(u) \psi_0(u) \exp(i\beta p(u))] + \psi_1(u) \exp(i\beta p(u)). \tag{29}$$

Using (27) in order to replace $\psi_0(u) \exp(i\beta p(u))$ by $-2i\beta$ in (29) and summing the Fourier series from $n = -\infty$ to $n = +\infty$, yield

$$\psi_1(u) = -2i\alpha p'(u) \exp(-i\beta p(u)). \tag{30}$$

so that at the first order,

$$\psi(u) = -2i(\beta + \alpha K_x p'(u)) \exp(-i\beta p(u)), \tag{31}$$

or

$$\phi_z(x) = -2i(\beta + \alpha f'(x)) \exp(i\alpha x - i\beta f(x)) = \phi_z^K(x). \tag{32}$$

It can be concluded that the mathematical expression of the Kirchhoff approximation is the expansion at the first order in K_x of the Waterman formula. This can explain why the Kirchhoff approximation is valid only when d_x is large with respect to the wavelength [4]. The same result can be obtained for 1D gratings in the p polarization case. In other words, for 1D gratings, the A.E.M. reduces to the classical Kirchhoff approximation.

In the next subsection our new formalism will be obtained by considering that K_z tends to 0 in the Waterman equations for 2D gratings and by developing at the first order in K_z .

3.2 Case of 2D Gratings

As for 1D gratings, the starting point of our new method is the Waterman formalism [10]:

$$\forall(n,m),$$

$$\frac{1}{d_x d_z}\int_0^{d_x}\int_0^{d_z}\left\{\left[\beta_{nm} + \alpha_n \frac{\partial F(x,z)}{\partial x}\right]\tilde{\phi}_x(x,z) + \alpha_n \frac{\partial F(x,z)}{\partial z}\tilde{\phi}_z(x,z)\right\}$$
$$\exp[-inK_x x + i\beta_{nm}F(x,z) - imK_z z]dxdz$$
$$= -2i[(k^2-\gamma^2)A_{x,00} + \alpha\gamma A_{z,00}]\,\delta_{n,0}\,\delta_{m,0}, \tag{33}$$

and

$$\frac{1}{d_x d_z}\int_0^{d_x}\int_0^{d_z}\left\{-\alpha\gamma_m\,\tilde{\phi}_x(x,z) + i\gamma_m \frac{\partial\tilde{\phi}_x(x,z)}{\partial x}\right.$$
$$\left. + \left[k^2 - \gamma_m^2 + \gamma_m\beta_{nm}\frac{\partial F(x,z)}{\partial z}\right]\tilde{\phi}_z(x,z)\right\} \tag{34}$$
$$\exp[-inK_x x + i\beta_{nm}F(x,z) - imK_z z]\,dxdz$$
$$= -2ik^2\beta A_{z,00}\,\delta_{n,0}\,\delta_{m,0},$$

where $\tilde{\mathbf{\Phi}}(x,z)$ is a doubly_periodic function :

$$\tilde{\mathbf{\Phi}}(x,z) = \mathbf{\Phi}(x,z)\exp(-i\alpha x - i\gamma z). \tag{35}$$

We proceed like in subsection 3.1, by introducing the variable:

$$v = K_z z, \tag{36}$$

so that, $p(x,v) = F(x, v/K_z)$ is a doubly_periodic function of periods d_x along x and 2π along v, and $\mathbf{\Psi}(x,v) = \tilde{\mathbf{\Phi}}(x, v/K_z)$.

According to the previous subsection, when $d_z >> d_x$, the approximate value of $\mathbf{\Psi}(x,v)$ will be obtained by using an asymptotic process. We shall assume that K_z (and only K_z) tends to 0, and develop $\mathbf{\Psi}(x,v)$ and β_{nm} in a series of K_z in (33) and (34). So we write

$$\psi_x(x,v) = \psi_{x0} + K_z\psi_{x1} + \dots, \tag{37}$$

$$\psi_z(x,v) = \psi_{z0} + K_z\psi_{z1} + \dots, \tag{38}$$

and

$$\beta_{nm} = \beta_{n0} - \frac{\gamma}{\beta_{n0}} m K_z + \dots, \tag{39}$$

in such a way that equation (33) yields:

$$\frac{1}{2\pi d_x}\int_0^{d_x}\int_0^{2\pi}[\eta(x,v) + K_z\zeta(x,v) + imK_z\vartheta(x,v)]\exp(-imv)\ dxdv$$
$$= -2i[(k^2-\gamma^2)A_{x,00} + \alpha\gamma A_{z,00}]\ \delta_{n,0}\ \delta_{m,0}, \tag{40}$$

where

$$\eta(x,v) = \left[\beta_{n0} + \alpha_n\frac{\partial p(x,v)}{\partial x}\right]\psi_{x0}(x,v)\exp(-inK_x x + i\beta_{n0}p(x,v)), \tag{41}$$

$$\zeta(x,v) = \left\{\left[\beta_{n0} + \alpha_n\frac{\partial p(x,v)}{\partial x}\right]\psi_{x1}(x,v) + \alpha_n\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v)\right\}\exp(-inK_x x + i\beta_{n0}p(x,v)), \tag{42}$$

and

$$\vartheta(x,v) = \frac{\gamma}{\beta_{n0}}\left[i - (\beta_{n0} + \alpha_n\frac{\partial p(x,v)}{\partial x})p(x,v)\right] \times \psi_{x0}(x,v)\exp(-inK_x x + i\beta_{n0}p(x,v)), \tag{43}$$

and (34) becomes:

$$\frac{1}{2\pi d_x}\int_0^{d_x}\int_0^{2\pi}[\chi(x,v) + K_z\xi(x,v) + imK_z\varphi(x,v)]\exp(-imv)\ dxdv$$
$$= -2ik^2\beta A_{z,00}\ \delta_{n,0}\ \delta_{m,0}, \tag{44}$$

where

$$\chi(x,v) = \left[-\alpha\gamma\psi_{x0}(x,v) + i\gamma\frac{\partial\psi_{x0}(x,v)}{\partial x} \right.$$
$$\left. + (k^2-\gamma^2)\psi_{z0}(x,v) \right] \exp(-inK_x x + i\beta_{n0}p(x,v)), \tag{45}$$

$$\xi(x,v) = \left[-\alpha\gamma\psi_{x1}(x,v) + i\gamma\frac{\partial\psi_{x1}(x,v)}{\partial x} + (k^2-\gamma^2)\psi_{z1}(x,v) \right.$$
$$\left. + \gamma\beta_{n0}\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v) \right] \exp(-inK_x x + i\beta_{n0}p(x,v)), \tag{46}$$

and

$$\varphi(x,v) = \left\{ \left[1 - i\frac{\gamma^2}{\beta_{n,0}}p(x,v) \right] \left[i\alpha\psi_{x0}(x,v) + \frac{\partial\psi_{x0}(x,v)}{\partial x} \right] \right.$$
$$\left. + \gamma\left[2i - \frac{(k^2-\gamma^2)}{\beta_{n0}}p(x,v) \right]\psi_{z0}(x,v) \right\} \times \exp(-inK_x x + i\beta_{n0}p(x,v)). \tag{47}$$

Bearing in mind that $\frac{1}{2\pi}\int_0^{2\pi}\mathcal{F}(x,v)\exp(-imv)dv$ represents the Fourier coefficient in the mth order of $\mathcal{F}(x,v)$, the summation from $m=-\infty$ to $m=+\infty$ of the Fourier series whose coefficients are given by the left_hand member of equations (40) and (44) enables one to write:

$$\frac{1}{d_x}\int_0^{d_x}\left[\eta(x,v) + K_z\zeta(x,v) + K_z\frac{\partial\vartheta(x,v)}{\partial v} \right] dx$$
$$= -2i[(k^2-\gamma^2)A_{x,00} + \alpha\gamma A_{z,00}]\ \delta_{n,0}, \tag{48}$$

and

$$\frac{1}{d_x}\int_0^{d_x}\left[\chi(x,v) + K_z\xi(x,v) + K_z\frac{\partial\varphi(x,v)}{\partial v} \right] dx = -2ik^2\beta A_{z,00}\ \delta_{n,0}. \tag{49}$$

Then replacing $\eta(x,v)$, $\zeta(x,v)$, $\vartheta(x,v)$, $\chi(x,v)$, $\xi(x,v)$ and $\varphi(x,v)$ by their expressions in (48) and (49), the $0th$ order in K_z yields :

$$\frac{1}{d_x}\int_0^{d_x}\left[\beta_{n0} + \alpha_n\frac{\partial p(x,v)}{\partial x} \right]\ \psi_{x0}(x,v)\exp(-inK_x x + i\beta_{n0}p(x,v))\ dx$$
$$= -2i[(k^2-\gamma^2)A_{x,00} + \alpha\gamma A_{z,00}]\ \delta_{n,0}, \tag{50}$$

and

$$\frac{1}{d_x}\int_0^{d_x}\left[-\alpha\gamma\psi_{x0}(x,v)+i\gamma\frac{\partial\psi_{x0}(x,v)}{\partial x}+(k^2-\gamma^2)\psi_{z0}(x,v)\right] \\ \times\exp(-inK_x x+i\beta_{n0}p(x,v))dx=-2ik^2\beta A_{z,00}\ \delta_{n,0}. \tag{51}$$

It is worth noting that for fixed v, these equations are nothing else than equations verified, in the Waterman theory, by a 1D diffraction grating whose profile is described by $y=p(x,v)$, illuminated by the incident field $\mathbf{E}^i$ given by (3), as it can be easily verified by comparing (50) and (51) to (33) and (34). Here, v is considered as a simple parameter.

The $1th$ order in K_z yields :

$$\frac{1}{d_x}\int_0^{d_x}\left[\beta_{n0}+\alpha_n\frac{\partial p(x,v)}{\partial x}\right]\psi_{x1}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))\ dx= \\ -\frac{d}{dv}\left\{\frac{1}{d_x}\int_0^{d_x}\frac{\gamma}{\beta_{n0}}\left[i-(\beta_{n0}+\alpha_n\frac{\partial p(x,v)}{\partial x})p(x,v)\right]\right. \\ \left.\times\,\psi_{x0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right\} \\ -\frac{1}{d_x}\int_0^{d_x}\alpha_n\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx, \tag{52}$$

$$\frac{1}{d_x}\int_0^{d_x}\left[-\alpha\gamma\psi_{x1}(x,v)+i\gamma\frac{\partial\psi_{x1}(x,v)}{\partial x}\right. \\ \left.+(k^2-\gamma^2)\psi_{z1}(x,v)\right]\exp(-inK_x x+i\beta_{n0}p(x,v))dx= \\ -\frac{d}{dv}\left\{\frac{1}{d_x}\int_0^{d_x}\left[\left(1-i\frac{\gamma^2}{\beta_{n0}}p(x,v)\right)\left(i\alpha\psi_{x0}(x,v)+\frac{\partial\psi_{x0}(x,v)}{\partial x}\right)\right]\right. \\ \left.\times\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right\} \\ -\frac{d}{dv}\left\{\frac{1}{d_x}\int_0^{d_x}\gamma\left[2i-\frac{(k^2-\gamma^2)}{\beta_{n0}}p(x,v)\right]\psi_{z0}(x,v)\right. \\ \left.\times\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right\}$$

$$-\frac{1}{d_x}\int_0^{d_x}\gamma\beta_{n0}\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx. \tag{53}$$

It is very important to notice that equations (52) and (53) are, in the Waterman theory, the equations verified, for fixed v, by a 1D diffraction grating $(y = p(x, v))$, with incident field $\hat{\mathbf{E}}^i(v)$ defined by a sum of plane waves :

$$\hat{\mathbf{E}}^i(v) = \sum_n \hat{\mathbf{A}}_{n0}(v)\exp(i\alpha_n x - i\beta_{n0}y - i\gamma v/K_z), \tag{54}$$

where

$$\hat{A}_{x,n0}(v) = \frac{1}{2i(k^2-\gamma^2)}\left\{\frac{d}{dv}\left[\frac{1}{d_x}\int_0^{d_x}\frac{\gamma}{\beta_{n0}}\left[i-(\beta_{n0}+\alpha_n\frac{\partial p(x,v)}{\partial x})p(x,v)\right]\right.\right.$$
$$\left.\times\,\psi_{x0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right]$$
$$\left.+\frac{1}{d_x}\int_0^{d_x}\alpha_n\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right\}, \tag{55}$$

and

$$\hat{A}_{z,n0}(v) = \frac{1}{2ik^2\beta_{n0}}\left\{\frac{d}{dv}\left[\frac{1}{d_x}\int_0^{d_x}\left(1-i\frac{\gamma^2}{\beta_{n0}}p(x,v)\right)\right.\right.$$
$$\left.\times\left(i\alpha\psi_{x0}(x,v)+\frac{\partial\psi_{x0}(x,v)}{\partial x}\right)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right]$$
$$+\frac{d}{dv}\left[\frac{1}{d_x}\int_0^{d_x}\gamma\left(2i-\frac{(k^2-\gamma^2)}{\beta_{n0}}p(x,v)\right)\right.$$
$$\left.\times\,\psi_{z0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right]$$
$$\left.+\frac{1}{d_x}\int_0^{d_x}\gamma\beta_{n0}\frac{\partial p(x,v)}{\partial v}\psi_{z0}(x,v)\exp(-inK_x x+i\beta_{n0}p(x,v))dx\right\}. \tag{56}$$

In conclusion, the A.E.M. reduces a problem of scattering by a nearly 2D object to the solution of several 2D problems (depending on the parameter v, thus on the value of z) which can be solved by rigorous methods for 1D gratings. It is fundamental to notice that these problems can be solved by using arbitrary rigorous methods for 1D gratings, like the integral method [11] and not necessary by using the Waterman method.

In other words, the Waterman method has been used in order to show that, at the first order in K_z, the problem of scattering by a nearly 2D grating reduces to the solution of several 2D problems with an incident wave that depends on z. Now, the Waterman method is no more necessary for the solution of these 2D problems.

4. NUMERICAL RESULTS

In this section, the profile of the 2D grating is given by :

$$y = \frac{H_x}{2} \cos(K_x x) + \frac{H_z}{2} \cos(K_z z) \tag{57}$$

with $K_z >> K_x$ (see Fig. 1).

In order to check the validity of the A.E.M., we have compared the values of $\mathbf{\Phi}(x, z)$, obtained from this method, to those given by the regularized Waterman method for 2D gratings [10,12]. The relative error on the surface current density is defined by the ratio of the norms in L^2 of the functions $\delta\mathbf{\Phi}(x, z) = \tilde{\mathbf{\Phi}}(x, z) - \mathbf{\Phi}(x, z)$ and $\mathbf{\Phi}(x, z)$:

$$\rho = \frac{\|\tilde{\mathbf{\Phi}} - \mathbf{\Phi}\|}{\|\mathbf{\Phi}\|}. \tag{58}$$

$\tilde{\mathbf{\Phi}}$ and $\mathbf{\Phi}$ are the surface current densities given by the A.E.M and the regularized Waterman method, respectively.

Figure 4(a) shows the error on the surface current density given by the A.E.M. for several values of H_x/d_x and H_z/d_z. For small values of H_x/d_x ($H_x/d_x < 0.06$), if a precision of the order of 1% is required, the method can deal with gratings with $H_z/d_z > H_x/d_x$. Then, for greater values of H_x/d_x, the error on the surface current density increases. For $H_x/d_x > 0.25$, the error on the surface current density remains lower than 1% for $H_z/d_z < 0.025$, but rapidly increases if H_z/d_z is greater. For comparison, Figure 4(b) shows the error on the surface current density given by the section method. Without any doubt, the A.E.M. is much better than the section method, whatever H_x/d_x may be. If a precision of 1% is required, the section method cannot deal with gratings such that $H_z/d_z > 0.011$, for small values of H_x/d_x (instead of $H_z/d_z > 0.12$ for the A.E.M.). With the same precision, the section method cannot deal with gratings with $H_z/d_z > 0.005$ for large values of H_x/d_x (instead of $H_z/d_z > 0.025$ for the new method). It is important to notice that the error in the section method increases linearly with H_z/d_z whilst it explodes above a given value in the A.E.M., as the convergence of an entire series.

In order to study the influence of the period d_z on the relative error ρ on the surface current density given by the A.E.M., we have shown, in Figure 5, this relative error ρ for different values of the period d_z, with a given value of the ratio H_x/d_x.

By comparing Figures 6(a) and 6(b) we can observe the remarkable similarity between the two figures for $H_x/d_x < 0.15$. This fact shows that the A.E.M. behaves like the classical Kirchhoff method for small values of H_x/d_x. On the other hand, the A.E.M. can deal with large ratios H_x/d_x while the classical Kirchhoff method cannot be used as soon as $H_x/d_x > 0.15$. On these figures, we have drawn the approximate limit of the single scattering (white curves). This limit between single scattering and multi scattering is obtained by considering the geometrical reflexion of incident rays on the grating surface, the region of single scattering covering the domain where the reflected rays do not intersect the grating surface.

Figure 6(b) shows that the multi-scattering phenomenon is the reason for the decrease of the precision given by the Kirchhoff method when the ratios H_z/d_z and H_x/d_x increase, which is not the case for the A.E.M.

In conclusion, we have shown that the A.E.M. combines the robustness of a rigorous method (for modulation along the x axis) with simplicity of the Kirchhoff approximation (for modulation along the z axis).

Figure 5(a) represents the case $H_x = 0$, when the A.E.M. reduces to the classical Kirchhoff approximation for a 1D grating periodic along the z axis. We can see the rapid decrease of the error as d_z is increased. Figures 5(b) and 5(c) show that the same behaviour of the error is observed for 2D gratings, although the increase of the value of H_x/d_x damages the precision of the results. These results are not surprising since it is well known that the Kirchhoff approximation may fail in the resonance domain. Moreover, they show that the behaviour of the A.E.M. is similar to that of the classical Kirchhoff approximation for 1D gratings.

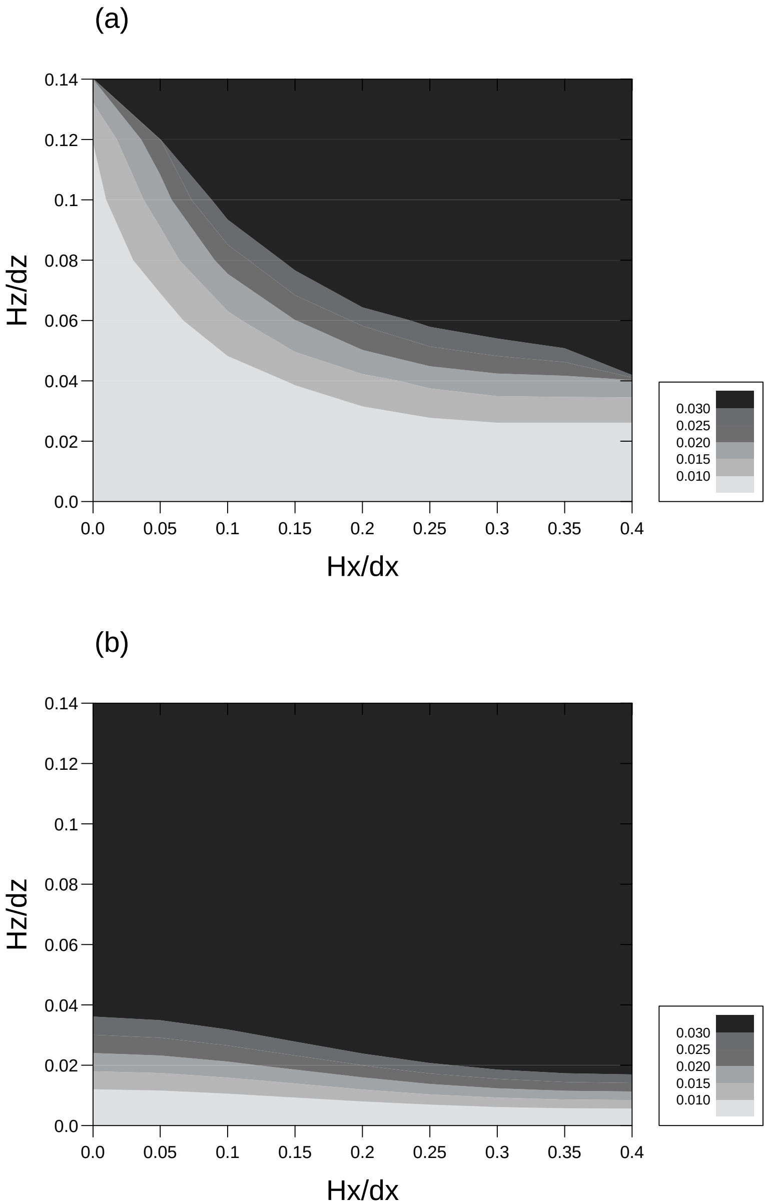

Figure 4. Relative error on the surface current density : (a) with the A.E.M., (b) with the section method. The grating parameters are $d_x = 1\mu m$ and $d_z = 10\mu m$, the wavelength $\lambda = 0.6328\mu m$, the angles of incidence are $\theta = 30^0$ and $\phi = 0^0$, and the angle of polarization is $\delta = 45^0$.

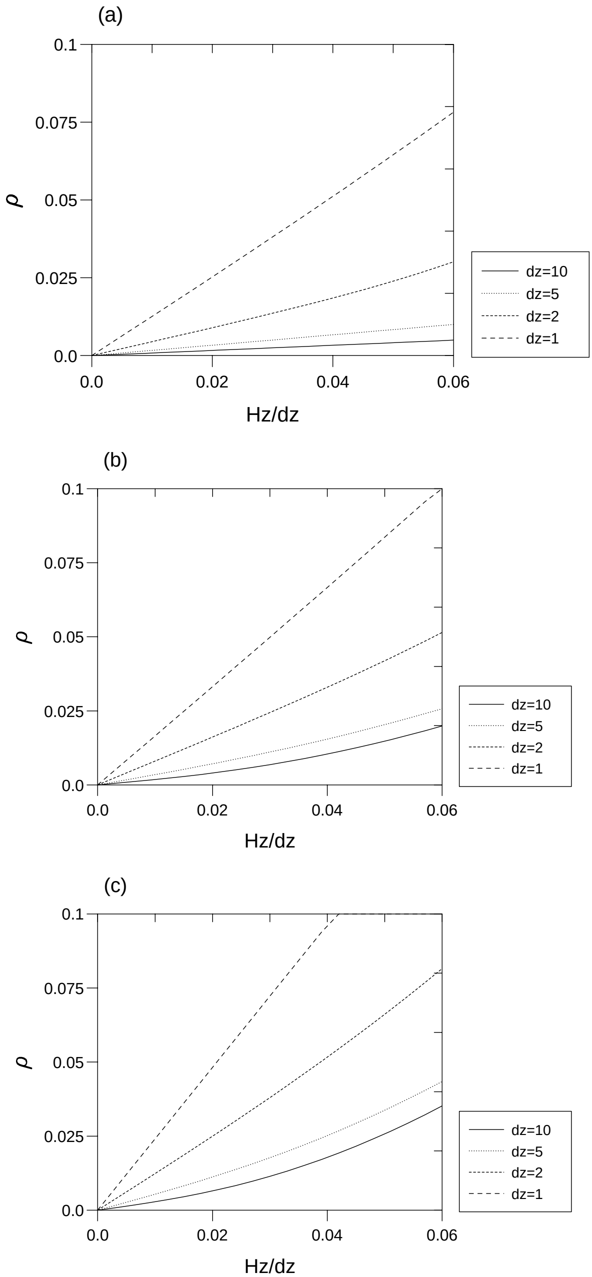

Figure 5. Relative error on the surface current density. The grating parameters are: $d_x = 1\mu m$, $\lambda = 0.6328\mu m$, $\theta = 30^0$, $\phi = 0^0$, $\delta = 45^0$, and (a) $H_x = 0$, (b) $H_x = 0.15\mu m$, (c) $H_x = 0.3\mu m$.

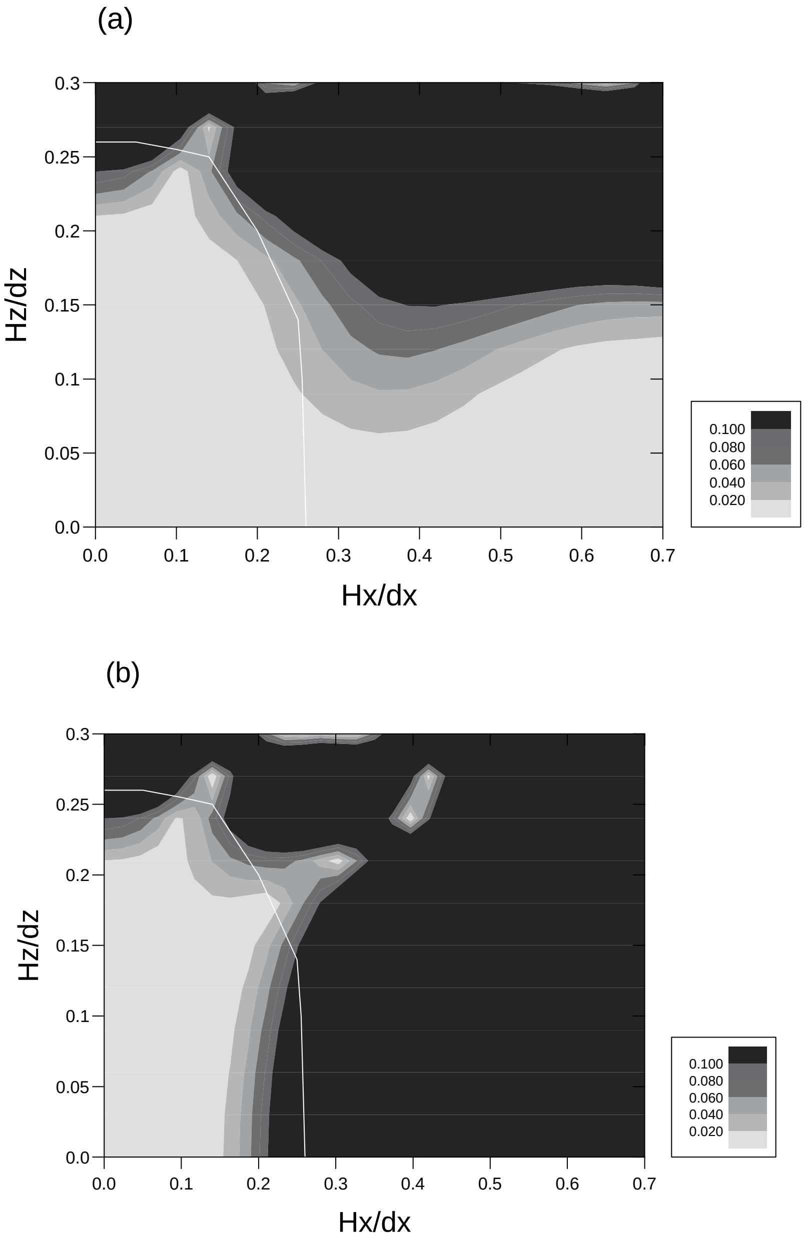

Figure 6. Relative error on the energy balance criterion : (a) with the A.E.M., (b) with the classical Kirchhoff method. The parameters are $d_x = 1\mu m$, $d_z = 5\mu m$, $\lambda = 0.6328\mu m$, $\theta = 0^0$ $\phi = 90^0$, and $\delta = 45^0$. The white curve represents the limit of multi scattering in the conditions of incidence.

Furthermore, we have compared the A.E.M. with the classical Kirchhoff method, for 2D gratings, by implementing the energy balance criterion. We have been led to use this criterion since the regularized Waterman method is unable to treat some of the gratings considered in this study. Figure 6 shows the error on the energy balance criterion for these two methods given by:

$$\rho' = \left| 1 - \sum_{n\in U} \sum_{m\in U} e_{nm} \right| . \tag{58}$$

5. CONCLUSION

In order to study the diffraction by nearly 2D objects, we have developed, for diffraction gratings, a new approximate theory of electromagnetic scattering that is able to solve with precision the problem of scattering from this kind of object, and to give accurate results without using 3D rigorous theories. The basic idea of the A.E.M. is to take into account the slight variation of section of the diffracting object along a privileged axis and to generalize the Kirchhoff approximation to nearly 2D objects. One of the main advantages of this theory lies in its simplicity. It only needs codes devoted to 2D objects.

We have shown that the A.E.M. can provide precise results in a region in which the classical Kirchhoff approximation for 2D surfaces cannot be used. It could be an interesting tool for the theoretical and numerical study of scattering from gratings with slight variations of profile in the direction of the grooves.

Finally, it can be conjectured that the A.E.M. should be used for other kinds of diffracting objects, in Electromagnetics and in Optics, for instance for non-periodic surfaces or bounded objects such as plane wings or helicopter blades. We are presently working in these directions.

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