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THE NEAR- AND FAR-ZONE FIELDS OF PERIODIC SPHERICAL ARRAYS OF DIPOLE ANTENNAS ON SPHERICAL CHIRAL SUBSTRATES

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1. INTRODUCTION

It is known that chiral materials are characterized by three independent constitutive parameters and they exhibit intrinsic handedness in their interaction with electromagnetic waves. Till now significant research progress has been achieved on various electromagnetic characteristics of artificially composite chiral materials, and much theoretical effort has been spent on the dyadic Green's function theory in chiral media [1–14]. Furthermore, the radiation characteristics of canonical sources in various chiral regions have also been investigated, such as (1) dipole antenna radiation in the presence of a homogeneous or a radially inhomogeneous chiral sphere, (2) the radiation and scattering from a thin wire antenna in a unbounded chiral medium, and (3) electromagnetic fields excited by a circular loop antenna in a unbounded chiral medium and above a chiral half space [15–27]. Also, chirolens antennas and chiral slab polarization transformers for aperture antennas have been proposed by some researchers recently [28, 29]. In the previous studies, the dyadic Green's function (DGF) in chiral media has often been employed, and therefore, the formulations of DGF in single layer, two-layer, or radially arbitrary multi-layered cylindrical and spherical chiral regions have been successfully achieved using the eigenfunction expansion method combined with the scattering superposition principle [10–13, 22, 23].

In contrast to the previous work, this contribution aims at solving the problem of electromagnetic radiation from spherical arrays of dipole antennas on spherical chiral substrates. To the authors' best knowledge, the near- and far-field analysis of a spherical array of point dipoles has just been considered recently [30, 31], and the spherical antenna arrays have many practical applications in satellite communication [32, 33]. On the other hand, the interaction of electromagnetic waves with multi-layered spherical structures also has particular applications in the fields of bioelectromagnetics, for instance, the human head can be approximately treated as an inhomogeneous multi-layered sphere [33–36]. In the following sections, the mathematical treatment is based on the DGF in the form of an eigenfunction expansion in terms of the normalized spherical vector wave function. This procedure is very general, compact, and straightforward. Corresponding to different orientations of dipole antenna arrays, the generalized, but analytical form of all field components are derived at first in both far and near zones. Secondly, numerical examples are presented to show the variation characteristics of the far-zone field for some array patterns of dipole antennas and the chiral effect is taken into account, and naturally, this study is also applicable for the isotropic spherical substrate case.

2. GEOMETRIES OF THE PROBLEM

Figs. 1(a) and (b) show two geometries of a periodic dipole antenna array on a chiral sphere and a single layer chiral spherical substrate, respectively.

The constitutive characteristics of the chiral media in the frequency domain with a time dependence $e^{-j\omega t}$ are described by

$$\overline{D} = \varepsilon \overline{E} + j\xi_c \overline{B} \ , \tag{1a}$$

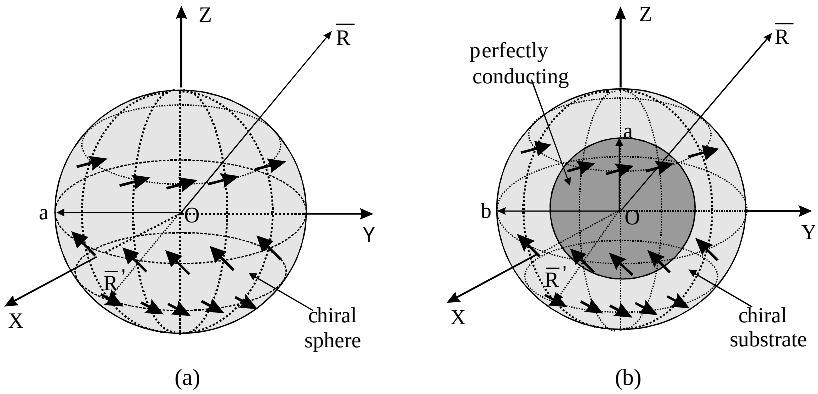


Figure 1. Geometries of a dipole antenna array on a chiral sphere and a single layer spherical chiral substrate.

$$\overline{H} = j\xi_c \overline{E} + \frac{\overline{B}}{\mu}\ , \tag{1b}$$

where ε, μ and ξ_c are the permittivity, permeability and chiral admittance, respectively. The volumetric electric current density of the periodic dipole antenna array on the surface $R = a$ in Fig. 1 (a) is described by

$$\overline{J}(\overline{R}') = \sum_{p=1}^{P} \sum_{q=1}^{Q} \frac{\delta(R' - a)\delta\left(\theta' - \frac{p\pi}{P}\right)\delta\left(\varphi' - \frac{2q\pi}{Q}\right)}{a^2 \sin\theta'} (C_{pq}^{(1)} \overline{e}_\varphi + C_{pq}^{(2)} \overline{e}_\theta) \tag{2}$$

where $p = 1,\ 2, \ldots,\ P$; $q = 1,\ 2,\ \ldots,\ Q$; and $C_{pq}^{(1)}$ and $C_{pq}^{(2)}$ are two direction factors, respectively. For instance, when $C_{pq}^{(2)} = 0$, all the dipole antennas are just in the $\overline{e}_\varphi$-direction. In Fig. 1(b), the region in $0 \le R \le a$ is a perfectly conducting sphere and the one in $a \le R \le b$ is chiral. The volumetric electric current density is supposed to be similar to that in (2), *i.e.*,

$$\overline{J}(\overline{R}') = \sum_{p=1}^{P} \sum_{q=1}^{Q} \frac{\delta(R' - b)\delta\left(\theta' - \frac{p\pi}{P}\right)\delta\left(\varphi' - \frac{2q\pi}{Q}\right)}{b^2 \sin\theta'} (C_{pq}^{(1)} \overline{e}_\varphi + C_{pq}^{(2)} \overline{e}_\theta) \tag{3}$$

3. FIELD DISTRIBUTIONS

3.1 Case 1

In the first case, the electromagnetic fields excited by the periodic dipole antenna array in the inner and outer regions (where $R \leq a$ and $R > a$) are determined as follows:

$$\overline{E}^{(i)} = j\omega\mu \iiint_{\nu} \bar{\bar{G}}_{e}^{(ia)}(\overline{R}|\overline{R}') \cdot \overline{J}(\overline{R}')\, dV' , \quad i = 1, 2 \tag{4a}$$

$$\overline{H}^{(i)} = \iiint_{\nu} \bar{\bar{G}}_{em}^{(ia)}(\overline{R}|\overline{R}') \cdot \overline{J}(\overline{R}')\, dV' \tag{4b}$$

where

$$\bar{\bar{G}}_{em}^{(ia)}(\overline{R}|\overline{R}') = \nabla \times \bar{\bar{G}}_{e}^{(ia)}(\overline{R}|\overline{R}') - \omega\mu\xi_c(1 - \delta_{i1})\bar{\bar{G}}_{e}^{(ia)}(\overline{R}|\overline{R}') \tag{4c}$$

with the subscript ν denoting the volume occupied by the dipole array, and the Kronecker delta defined as $\delta_{i1} = \begin{cases} 1, & i = 1 \\ 0, & i = 0 \end{cases}$. According to the principle of scattering superposition, the electric DGF $\bar{\bar{G}}_{e}^{(1a)}(\overline{R}|\overline{R}')$ is de-composed as:

$$\bar{\bar{G}}_{e}^{(1a)}(\overline{R}|\overline{R}') = \bar{\bar{G}}_{e0}^{(\leq,\geq)}(\overline{R}|\overline{R}') + \bar{\bar{G}}_{es}^{(1a)}(\overline{R}|\overline{R}') \tag{5a}$$

where $\bar{\bar{G}}_{e0}^{(\leq,\geq)}(\overline{R}|\overline{R}')$ is the DGF in the unbounded chiral region and it takes the same form as in [19] except that the irrotational region term there has been omitted. It is further stated as:

$$\bar{\bar{G}}_{e0}^{(\leq,\geq)}(\overline{R}|\overline{R}')$$

$$= -\frac{\overline{e}_R\overline{e}_R\delta(\overline{R} - \overline{R}')}{k^2} + \frac{j}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$

$$\cdot \begin{cases} k_+^2 \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+)\overline{V}'_{\substack{e\\o}mn}(k_+) + k_-^2 \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-)\overline{W}'_{\substack{e\\o}mn}(k_-)\ , & \overline{R} \geq \overline{R}'; \\ k_+^2 \overline{V}_{\substack{e\\o}mn}(k_+)\overline{V}'^{(1)}_{\substack{e\\o}mn}(k_+) + k_-^2 \overline{W}_{\substack{e\\o}mn}(k_-)\overline{W}'^{(1)}_{\substack{e\\o}mn}(k_-)\ , & \overline{R} \leq \overline{R}'; \end{cases} \tag{5b}$$

where

$$D_{mn} = (2 - \delta_{m0}\frac{(2n+1)(n-m)!}{n(n+1)(n+m)!}\ , \tag{5c}$$

$$k_{\pm} = \pm\omega\mu\xi_c + \sqrt{\omega^2\varepsilon\mu + \omega^2\mu^2\xi_c^2}\ ; \tag{5d}$$

while $\overline{e}_R$ is the radial unit vector of the spherical coordinate system (R, θ, φ), and $\delta(\overline{R} - \overline{R}')$ is the Dirac delta function. $k_{\pm}$ are the wavenumbers corresponding to two circularly polarized modes supported in chiral medium, *i.e.*, the right- and left-handed circularly polarized waves (RCP: +; LCP: -), and $k^2 = \omega^2\mu\varepsilon$, $\delta_{m0} = \begin{cases} 1, & m = 0 \\ 0, & m \neq 0 \end{cases}$. The definitions and orthogonal properties of the normalized spherical vector wave functions $\overline{V}^{(1)}_{\substack{e\\o}mn}(k_+)$, $\overline{V}_{\substack{e\\o}mn}(k_+)$ $\overline{W}^{(1)}_{\substack{e\\o}mn}(k_-)$ and $\overline{W}_{\substack{e\\o}mn}(k_-)$ can be found in [19], and the prime in (5) indicates the coordinates (R', θ', φ') of the source. The scattering DGF in different regions in Fig. 1(a) can be constructed according to the multiple transmissions and reflections as follows:

$$\begin{aligned}\bar{\bar{G}}^{(1a)}_{es}(\overline{R}|\overline{R}') = \frac{j}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn} \Big[& A^{(1a)}\overline{V}_{\substack{e\\o}mn}(k_+) \\ & + A^{(2a)}\overline{W}_{\substack{e\\o}mn}(k_-)\Big]\overline{V}'_{\substack{e\\o}mn}(k_+) + \Big[A^{(3a)}\overline{V}_{\substack{e\\o}mn}(k_+) \\ & + A^{(4a)}\overline{W}_{\substack{e\\o}mn}(k_-)\Big]\overline{W}'_{\substack{e\\o}mn}(k_-)\ , \quad R \leq a\ ;\end{aligned} \tag{6a}$$

$$\begin{aligned}\bar{\bar{G}}^{(2a)}_{e}(\overline{R}|\overline{R}') = \frac{j}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn} \Big[& B^{(1a)}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) \\ & + B^{(2a)}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_0)\Big]\overline{V}'_{\substack{e\\o}mn}(k_+) + \Big[B^{(3a)}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) \\ & + B^{(4a)}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_0)\Big]\overline{W}'_{\substack{e\\o}mn}(k_-)\ , \quad 0 \quad R \geq a\ ;\end{aligned} \tag{6b}$$

where $A^{(sa)}$ and $B^{(sa)}(s = 1, \cdots, 4)$ are unknown coefficients to be determined by the boundary conditions at $R = a$, which is shown in Appendix 1. So, by substituting (2), (5) and (6) into (4), the electromagnetic fields excited by the periodic dipole antenna array can be obtained. Here both near and far fields are taken into account, *i.e.*,

$$\overline{E}^{(1a)}_{R,\theta,\varphi}(\overline{R}) = -\frac{\omega\mu}{2\sqrt{2}\,\pi(k_+ + k_-)}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$
$$\cdot\Big\{k_+^2\overline{V}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+)C^{(1+)}_{pq} + k_-^2\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)C^{(1-)}_{pq}$$
$$+\Big[A^{(1a)}\overline{V}^{R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+) + A^{(2a)}\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)\Big]C^{(+)}_{pq}$$
$$+\Big[A^{(3a)}\overline{V}^{R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+) + A^{(4a)}\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)\Big]C^{(-)}_{pq}\Big\}$$
$$R \le a\ ; \qquad (7)$$

$$\overline{E}^{(2a)}_{R,\theta,\varphi}(\overline{R}) = -\frac{\omega\mu}{2\sqrt{2}\,\pi(k_+ + k_-)}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$
$$\cdot\Big\{\Big[B^{(1a)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0) + B^{(2a)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\Big]C^{(+)}_{pq}$$
$$+\Big[B^{(3a)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0) + B^{(4a)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\Big]C^{(-)}_{pq}\Big\}$$
$$R \ge a\ ; \qquad (8)$$

and

$$\overline{H}^{(1a)}_{R,\theta,\varphi}(\overline{R}) = \frac{j\omega\mu}{2\sqrt{2}\,\pi(k_+ + k_-)\eta_c}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$
$$\cdot\Big\{k_+^2\overline{V}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+)C^{(1+)}_{pq} - k_-^2\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)C^{(1-)}_{pq}$$
$$+\Big[A^{(1a)}\overline{V}^{R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+) - A^{(2a)}\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)\Big]C^{(+)}_{pq}$$
$$+\Big[A^{(3a)}\overline{V}^{R,\theta,\varphi)}_{\substack{e\\o}mn}(k_+) - A^{(4a)}\overline{W}^{(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_-)\Big]C^{(-)}_{pq}\Big\}$$
$$R \le a\ ; \qquad (9)$$

$$\overline{H}^{(2a)}_{R,\theta,\varphi}(\overline{R}) = \frac{j\omega\mu}{2\sqrt{2}\,\pi(k_+ + k_-)}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n} D_{mn}$$
$$\cdot\Big\{\Big[B^{(1a)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0) - B^{(2a)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\Big]C^{(+)}_{pq}$$
$$+\Big[B^{(3a)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0) - B^{(4a)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\Big]C^{(-)}_{pq}\Big\}$$
$$R > a\ ; \qquad (10)$$

To simplify the representations, the following shorthand notations are defined:

$$X^{(a1+)(a1-)}_{\substack{e\\o}\varphi} = -\widetilde{P}'_1 C'_s h^a_{+(-)} \mp (\pm)\widetilde{P}'_2 S'_c \partial h^a_{+(-)}\Big|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \quad (11)$$

$$X^{(a1+)(a1-)}_{\substack{e\\o}\theta} = +(-)\widetilde{P}'_1 C'_s \partial h^a_{+(-)} \mp \widetilde{P}'_2 S'_c h^a_{+(-)}\Big|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \quad (12)$$

$$X^{(a1+)(a1-)}_{\substack{e\\o}\varphi} = -\widetilde{P}'_1 C'_s j^a_{+(-)} \mp (\pm)\widetilde{P}'_2 S'_c \partial j^a_{+(-)}\Big|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \quad (13)$$

$$X^{(a1+)(a1-)}_{\substack{e\\o}\theta} = +(-)\widetilde{P}'_1 C'_s \partial j^a_{+(-)} \mp \widetilde{P}'_2 S'_c j^a_{+(-)}\Big|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \quad (14)$$

and

$$\widetilde{P}'_1 = \frac{dP_n^m(\cos\theta')}{d\theta'}, \qquad \widetilde{P}'_2 = \frac{m}{\sin\theta'} P_n^m(\cos\theta'),$$
$$S'_c = \frac{\sin}{\cos}(m\varphi'), \qquad C'_s = \frac{\cos}{\sin}(m\varphi'),$$
$$j^a_\pm = j_n(k_\pm a), \qquad h^a_\pm = h_n^{(1)}(k_\pm a),$$
$$\partial j^a_\pm = \frac{1}{k_\pm a}\frac{\partial}{\partial R'}[R' j_n(k_\pm R')]_{R'=a},$$
$$\partial h^a_\pm = \frac{1}{k_\pm a}\frac{\partial}{\partial R'}[R' h_n^{(1)}(k_\pm R')]_{R'=a},$$
$$C^{(1\pm)}_{pq} = C^{(1)}_{pq} X^{(a1\pm)}_{\substack{e\\o}\varphi} + C^{(2)}_{pq} X^{(a1\pm)}_{\substack{e\\o}\theta},$$
$$C^{(\pm)}_{pq} = C^{(1)}_{pq} X^{(a\pm)}_{\substack{e\\o}\varphi} + C^{(2)}_{pq} X^{(a\pm)}_{\substack{e\\o}\theta},$$
$$\eta_c = 1/\sqrt{\frac{\varepsilon}{\mu} + \xi_c^2},$$

where $j_n(\cdot)$ and $h_n^{(1)}(\cdot)$ are the nth-order spherical Bessel functions of the first and third kinds, respectively; η_c the characteristic impedances of RCP $(+)$ and LCP $(-)$ modes; $P_n^m(\cdot)$, the associated Legendre function, and

$$\widetilde{P}'_1 = \frac{1}{2}\Big[(n-m+1)(n+1)P_n^{m-1}(\cos\theta') - P_n^{m+1}(\cos\theta')\Big], \quad (15)$$

$$\widetilde{P}'_2 = \frac{1}{2\cos\theta'}\Big[(n-m+1)(n+1)P_n^{m-1}(\cos\theta') + P_n^{m+1}(\cos\theta')\Big], \quad (16)$$

are very useful for further reduction. Especially [37],

$$\widetilde{P}_1'|_{\theta'=90^\circ} = \frac{2^{m+1}\sin\left[\frac{1}{2}(n+m)\pi\right]\Gamma\left(\dfrac{n+m}{2}+1\right)}{\sqrt{\pi}\Gamma\left(\dfrac{n-m+1}{2}\right)}, \tag{17}$$

$$P_n^m(0) = \frac{2^{m}\cos\left[\frac{1}{2}(n+m)\pi\right]\Gamma\left(\dfrac{n+m+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\dfrac{n-m}{2}+1\right)}. \tag{18}$$

Obviously, the field expression (7)–(10) is valid for the fields in near- and far-zones, and their scalar forms can be easily obtained from above. As a special case, when the chirality admittance is zero, *i.e.*, $k_+ = k_- = k$, (7)–(10) are reduced to the isotropic case. Now two typical cases are to be discussed:

1. Electrically Large Chiral Sphere $(k_\pm a >> 1)$

Under such circumstances, the following expressions can be exploited for the spherical Hankel function and its derivation, *i.e.*,

$$\begin{aligned} j_\pm^a &\approx (-j)^{n+1}\frac{\cos(k_\pm a)}{k_\pm a}, \quad & \partial j_\pm^a &\approx -(-j)^{n+1}\frac{\sin(k_\pm a)}{k_\pm a}, \\ h_\pm^a &\approx (-j)^{n+1}\frac{e^{jk_\pm a}}{k_\pm a}, \quad & \partial h_\pm^a &\approx (-j)^{n}\frac{e^{jk_\pm a}}{k_\pm a}; \end{aligned} \tag{19}$$

so that the unknown coefficients $A^{(s)}$ and $B^{(s)}$ $(s = 1, \cdots, 4)$ can be simplified from Appendix 1, while the radiation factors are simplified as:

$$X_{\substack{e\\o}\varphi}^{(a1+)(a1-)} \approx (-j)^n\frac{e^{jk_{+(-)}a}}{k_{+(-)}a}\left[j\widetilde{P}_1'C_s' \mp (\pm)\widetilde{P}_2'S_c'\right]\Bigg|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \tag{20}$$

$$X_{\substack{e\\o}\theta}^{(a1+)(a1-)} \approx (-j)^n\frac{e^{jk_{+(-)}a}}{k_{+(-)}a}\left[+(-)\widetilde{P}_1'C_s' \pm j\widetilde{P}_2'S_c'\right]\Bigg|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \tag{21}$$

$$\begin{aligned} X_{\substack{e\\o}\varphi}^{(a+)(a-)} = \frac{(-j)^{n+1}}{k_{+(-)}a}\Big[&-\widetilde{P}_1'C_s'\cos(k_{+(-)}a) \\ &\pm(\mp)\widetilde{P}_2'S_2'\sin(k_{+(-)}a)\Big]\Bigg|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \end{aligned} \tag{22}$$

$$X_{\substack{e\\o}\theta}^{(a+)(a-)} = \frac{(-j)^{n+1}}{k_{+(-)}a}\Big[-(+)\widetilde{P}_1' C_s' \sin(k_{+(-)}a)$$
$$\mp \widetilde{P}_2' S_2' \cos(k_{+(-)}a)\Big]\bigg|_{\theta'=\frac{p\pi}{P},\ \varphi'=\frac{2q\pi}{Q}}, \tag{23}$$

2. In the Far-Field Zone $(k_0 R >> 1)$
Since $k_o R >> 1$, we approximately have

$$\overline{V}_{\substack{e\\o}mn}^{(1)(\theta,\varphi}(k_o) \approx -\frac{e^{jk_0R}}{\sqrt{2}k_0R}(-j)^n(\mp j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s)(\overline{e}_\theta + j\overline{e}_\varphi)\ , \tag{24a}$$
$$\overline{W}_{\substack{e\\o}mn}^{(1)(\theta,\varphi}(k_o) \approx -\frac{e^{jk_0R}}{\sqrt{2}k_0R}(-j)^n(\mp j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s)(\overline{e}_\theta - j\overline{e}_\varphi)\ , \tag{24b}$$

where the definitions of $\widetilde{P}_1,\ \widetilde{P}_2,\ S_c$ and C_s are similar to those given. Furthermore, the radiated fields become

$$\overline{E}_{\theta,\varphi}^{(2a)}(\overline{R}) = \frac{\omega\mu e^{jk_0R}}{4\pi k_o R(k_+ + k_-)}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn}$$
$$\cdot\Big\{(\pm j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s)[B^{(1a)}C_{pq}^{(+)} + B^{(3a)}C_{pq}^{(-)}](\overline{e}_\theta + j\overline{e}_\varphi)$$
$$+ (\pm j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s)[B^{(2a)}C_{pq}^{(+)} + B^{(4a)}C_{pq}^{(-)}](\overline{e}_\theta - j\overline{e}_\varphi)\Big\}\ , \tag{25}$$

$$\overline{H}_{\theta,\varphi}^{(2a)}(\overline{R}) = -\frac{j\omega\mu e^{jk_0R}}{4\pi k_o R(k_+ + k_-)\eta_0}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn}$$
$$\cdot\Big\{(\pm j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s)[B^{(1a)}C_{pq}^{(+)} + B^{(3a)}C_{pq}^{(-)}](\overline{e}_\theta + j\overline{e}_\varphi)$$
$$- (\pm j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s)[B^{(2a)}C_{pq}^{(+)} + B^{(4a)}C_{pq}^{(-)}](\overline{e}_\theta - j\overline{e}_\varphi)\Big\}\ , \tag{26}$$

Evidently, (25) and (26) indicate that the far fields radiated by the periodic dipole antenna array are just the sum of RCP- and LCP-mode contributions, which are reducible to that of the special case of a single dipole antenna radiation shown in [19].

3.2 Case 2

By following the similar procedure used above, the DGF in the chiral layer and the outer region can be constructed as

$$\bar{\bar{G}}_e^{(1b\le,\ge)}(\overline{R}|\overline{R}') = \bar{\bar{G}}_{e0}^{(\le,\ge)}(\overline{R}|\overline{R}') + \bar{\bar{G}}_{es}^{(1b)}(\overline{R}|\overline{R}') , \qquad a \le R \le b\, ; \quad (27a)$$

where

$$\begin{aligned}
&\bar{\bar{G}}_{es}^{(1b)}(\overline{R}|\overline{R}') \\
&\quad = \frac{j}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\
&\qquad \cdot \Big\{ \Big[A^{(1b)} \overline{V}_{\substack{e\\o}mn}(k_+) + A^{(2b)} \overline{W}_{\substack{e\\o}mn}(k_-) \\
&\qquad + A^{(3b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+) + A^{(4b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-) \Big] \overline{V}'_{\substack{e\\o}mn}(k_+) \\
&\qquad + \Big[A^{(5b)} \overline{V}_{\substack{e\\o}mn}(k_+) + A^{(6b)} \overline{W}_{\substack{e\\o}mn}(k_-) \\
&\qquad + A^{(7b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+) + A^{(8b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-) \Big] \overline{W}'_{\substack{e\\o}mn}(k_-) \\
&\qquad + \Big[A^{(9b)} \overline{V}_{\substack{e\\o}mn}(k_+) + A^{(10b)} \overline{W}_{\substack{e\\o}mn}(k_-) \\
&\qquad + A^{(11b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+) + A^{(12b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-) \Big] \overline{V}^{(1)\,\prime}_{\substack{e\\o}mn}(k_-) \\
&\qquad + \Big[A^{(13b)} \overline{V}_{\substack{e\\o}mn}(k_+) + A^{(14b)} \overline{W}_{\substack{e\\o}mn}(k_-) \\
&\qquad + A^{(15b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_+) + A^{(16b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_-) \Big] \overline{W}^{(1)\,\prime}_{\substack{e\\o}mn}(k_-) \Big\} ; \qquad (27b)
\end{aligned}$$

and

$$\begin{aligned}
\bar{\bar{G}}_e^{(2b)}(\overline{R}|\overline{R}') &= \frac{j}{2\pi(k_+ + k_-)} \sum_{n=1}^{\infty} \sum_{m=0}^{n} D_{mn} \\
&\quad \cdot \Big\{ \Big[B^{(1b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) + B^{(2b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_0) \Big] \overline{V}'_{\substack{e\\o}mn}(k_+) \\
&\quad + \Big[B^{(3b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) + B^{(4b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_0) \Big] \overline{W}'_{\substack{e\\o}mn}(k_-) \\
&\quad + \Big[B^{(5b)} \overline{V}^{(1)}_{\substack{e\\o}mn}(k_0) + B^{(6b)} \overline{W}^{(1)}_{\substack{e\\o}mn}(k_0) \Big] \overline{V}^{(1)\,\prime}_{\substack{e\\o}mn}(k_+)
\end{aligned}$$

$$+\left[B^{(7b)}\overline{V}^{(1)}_{\substack{e\\o}mn}(k_0)+B^{(8b)}\overline{W}^{(1)}_{\substack{e\\o}mn}(k_0)\right]\overline{W}^{(1)\,\prime}_{\substack{e\\o}mn}(k_-)\Bigg\}, \qquad R\geq b; \tag{28}$$

where the unknown coefficients $A^{(sb)}$ and $B^{(lb)}$ (with $s=1,\cdots,16$; $l=1,\ \cdots,8$) are also determined by the boundary conditions at $R=a$ and b, and their expressions are shown in Appendix 2. So, inserting (28), (29) and (3) into (4) leads to the electromagnetic fields excited by the periodic dipole antenna array given by:

$$\begin{aligned}\overline{E}^{(2b)}_{R,\theta,\varphi}(\overline{R}) = &-\frac{\omega\mu}{2\sqrt{2}\pi(k_++k_-)}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}\\
&\cdot\Bigg\{\left[B^{(1b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)+B^{(2b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(+)}_{pq}\\
&+\left[B^{(3b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)+B^{(4b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(-)}_{pq}\\
&+\left[B^{(5b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)+B^{(6b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(+)}_{pq}\\
&+\left[B^{(7b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)+B^{(8b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(-)}_{pq}\Bigg\},\\
&R\geq b;\end{aligned} \tag{29}$$

and

$$\begin{aligned}\overline{K}^{(2b)}_{R,\theta,\varphi}(\overline{R}) = &\frac{j\omega\mu}{2\sqrt{2}\pi(k_++k_-)\eta_0}\sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}D_{mn}\\
&\cdot\Bigg\{\left[B^{(1b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)-B^{(2b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(+)}_{pq}\\
&+\left[B^{(3b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)-B^{(4b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(-)}_{pq}\\
&+\left[B^{(5b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)-B^{(6b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(+)}_{pq}\\
&+\left[B^{(7b)}\overline{V}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)-B^{(8b)}\overline{W}^{(1)(R,\theta,\varphi)}_{\substack{e\\o}mn}(k_0)\right]\widetilde{C}^{(-)}_{pq}\Bigg\},\\
&R\geq b;\end{aligned} \tag{30}$$

where

$$\widetilde{C}_{pq}^{(1\pm)} = C_{pq}^{(1)} X_{\substack{e\\o}\varphi}^{(b1\pm)} + C_{pq}^{(2)} X_{\substack{e\\o}\theta}^{(b1\pm)}, \quad \widetilde{C}_{pq}^{(\pm)} = C_{pq}^{(1)} X_{\substack{e\\o}\varphi}^{(b\pm)} + C_{pq}^{(2)} X_{\substack{e\\o}\theta}^{(b\pm)},$$
$$X_{\substack{e\\o}\varphi,\theta}^{(b1\pm)} = X_{\substack{e\\o}\varphi,\theta}^{(a1\pm)}\Big|_{a\to b}, \qquad X_{\substack{e\\o}\varphi,\theta}^{(b\pm)} = X_{\substack{e\\o}\varphi,\theta}^{(a\pm)}\Big|_{a\to b}. \tag{31}$$

In the far-field zone $(k_0 R >> 1)$, the radiated fields become

$$\begin{aligned}\overline{E}_{\theta,\varphi}^{(2b)}(\overline{R}) = \frac{\omega\mu e^{jk_oR}}{4\pi k_0 R(k_+ + k_-)} \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn} \\ \cdot\Bigg\{(\pm j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s)\Big[B^{(1b)}\widetilde{C}_{pq}^{(+)} + B^{(3b)}\widetilde{C}_{pq}^{(-)} \\ + B^{(5b)}\widetilde{C}_{pq}^{(1+)} + B^{(7b)}\widetilde{C}_{pq}^{(1-)}\Big](\overline{e}_\theta + j\overline{e}_\varphi) \\ + (\pm j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s)\Big[B^{(2b)}\widetilde{C}_{pq}^{(+)} + B^{(4b)}\widetilde{C}_{pq}^{(-)} \\ + B^{(6b)}\widetilde{C}_{pq}^{(1+)} + B^{(8b)}\widetilde{C}_{pq}^{(1-)}\Big](\overline{e}_\theta - j\overline{e}_\varphi)\Bigg\},\end{aligned} \tag{32}$$

$$\begin{aligned}\overline{H}_{\theta,\varphi}^{(2b)}(\overline{R}) = -\frac{j\omega\mu e^{jk_oR}}{4\pi k_0 R(k_+ + k_-)\eta_0} \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn} \\ \cdot\Bigg\{(\pm j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s)\Big[B^{(1b)}\widetilde{C}_{pq}^{(+)} + B^{(3b)}\widetilde{C}_{pq}^{(-)} \\ + B^{(5b)}\widetilde{C}_{pq}^{(1+)} + B^{(7b)}\widetilde{C}_{pq}^{(1-)}\Big](\overline{e}_\theta + j\overline{e}_\varphi) \\ - (\pm j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s)\Big[B^{(2b)}\widetilde{C}_{pq}^{(+)} + B^{(4b)}\widetilde{C}_{pq}^{(-)} \\ + B^{(6b)}\widetilde{C}_{pq}^{(1+)} + B^{(8b)}\widetilde{C}_{pq}^{(1-)}\Big](\overline{e}_\theta - j\overline{e}_\varphi)\Bigg\},\end{aligned} \tag{33}$$

Again, (32) and (33) indicate that the far fields due to the periodic dipole antenna array are just the sum of RCP and LCP modes. In addition, if $k_\pm b >> 1$, all the unknown coefficients $\{A^{(sb)}, B^{(lb)}\}$ and the quantities $\{X_{\substack{e\\o}\varphi,\theta}^{(b1+)(b1-)}, X_{\substack{e\\o}\varphi,\theta}^{(b+)(b-)}\}$ are simplified greatly, *i.e.*,

$$X_{\substack{e\\o}\varphi}^{(b1+)(b1-)} \approx (-j)^n \frac{e^{jk_{+(-)}b}}{k_{+(-)}b}\Big[j\widetilde{P}_1' C_s' \mp (\pm)\widetilde{P}_2' S_c'\Big]\Bigg|_{\theta'=\frac{p\pi}{P},\varphi'=\frac{2q\pi}{Q}}, \tag{34}$$

$$X^{(b1+)(b1-)}_{\substack{e\\o}\theta} \approx (-j)^n \frac{e^{jk_{+(-)}b}}{k_{+(-)}b}\left[+(-)\widetilde{P}'_1 C'_s \pm j\widetilde{P}'_2 S'_c\right]\Bigg|_{\theta'=\frac{p\pi}{P},\varphi'=\frac{2q\pi}{Q}}, \quad (35)$$

$$X^{(b+)(b-)}_{\substack{e\\o}\varphi} = \frac{(-j)^{n+1}}{k_{+(-)}b}\left[-\widetilde{P}'_1 C'_s \cos(k_{+(-)}b)\right.$$
$$\left.\pm(\mp)\widetilde{P}'_2 S'_c \sin(k_{+(-)}b)\right]\Bigg|_{\theta'=\frac{p\pi}{P},\varphi'=\frac{2q\pi}{Q}}, \quad (36)$$

$$X^{(b+)(b-)}_{\substack{e\\o}\theta} = \frac{(-j)^{n+1}}{k_{+(-)}b}\left[-(+)\widetilde{P}'_1 C'_s \sin(k_{+(-)}b)\right.$$
$$\left.\mp \widetilde{P}'_2 S'_c \cos(k_{+(-)}b)\right]\Bigg|_{\theta'=\frac{p\pi}{P},\varphi'=\frac{2q\pi}{Q}}, \quad (37)$$

Correspondingly, the polarization state of the radiated fields can be represented by the point on the Poincare's sphere with a latitude χ of

$$\sin\chi = \frac{\left|L^{(b)}_-\right|^2 - \left|L^{(b)}_+\right|^2}{\left|L^{(b)}_-\right|^2 + \left|L^{(b)}_+\right|^2}; \quad (38a)$$

and

$$L^{(b)}_+ = \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn}\cdot\left\{\left(\mp j\widetilde{P}_1 S_c - \widetilde{P}_2 C_s\right)\left[B^{(1b)}\widetilde{C}^{(+)}_{pq}\right.\right.$$
$$\left.\left.+B^{(3b)}\widetilde{C}^{(-)}_{pq} + B^{(5b)}\widetilde{C}^{(1+)}_{pq} + B^{(7b)}\widetilde{C}^{(1-)}_{pq}\right]\right\}, \quad (38b)$$

$$L^{(b)}_- = \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{n=1}^{\infty}\sum_{m=0}^{n}(-j)^n D_{mn}\cdot\left\{\left(\mp j\widetilde{P}_1 S_c + \widetilde{P}_2 C_s\right)\left[B^{(2b)}\widetilde{C}^{(+)}_{pq}\right.\right.$$
$$\left.\left.+B^{(4b)}\widetilde{C}^{(-)}_{pq} + B^{(6b)}\widetilde{C}^{(1+)}_{pq} + B^{(8b)}\widetilde{C}^{(1-)}_{pq}\right]\right\}; \quad (38c)$$

where $\sin\chi \in [-1,\ 1]$ is the ellipticity of the polarization ellipse and it indicates the right- and left-handed elliptically, circularly, and linear polarized waves, respectively.

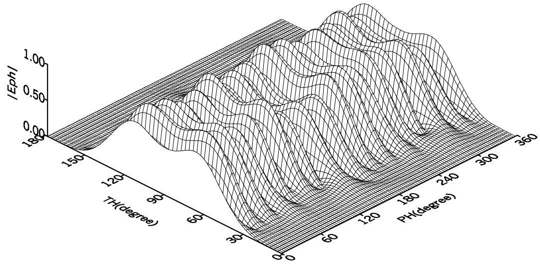
4. NUMERICAL RESULTS

Based on the above derived mathematical formulas of the field distributions resulted in from the above model, the normalized three-dimensional field patterns $E_{\varphi}^{(2b)}$ in the far zone radiated by eight dipole antennas are demonstrated in Fig. 2 at first. Certainly, convergence of the analysis is checked in detail, and forty terms are needed for the summation with respect to the index n of the spherical Bessel and Hankel functions. For mth-order harmonic current excitation, the summation index n should begin from $n = m$, since we must have $n \geq m$ in the associated Legendre function $P_n^m(\cdot)$.

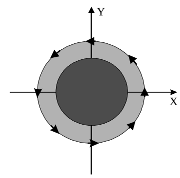
In Figs. 2(a)–(c), the parameters chosen for the calculation are $\varepsilon = 3.5\varepsilon_0$, $\mu = 1.5\mu_o$, $a/\lambda = 1.0$, $b/\lambda = 1.5$, $C_{pq}^{(1)} = 1.0$, and $C_{pq}^{(2)} = 0.0$. The locations of eight dipole antennas are determined by $\theta' = 90°$, $Q = 8$, and $q = 0, 1, 2, \ldots, 7$. Here the lossy effect of the coating is not taken into account, in order that the attenuation by the medium can not mask the chiral effect in the far-zone fields. Actually, the constitutive parameters of chiral materials are the function of the operating frequency, respectively, but the above parameters assumed are all in a physically realizable range. Since $a(b)/\lambda \sim 1.0$, such a single layered (non)-chiral sphere does not have too large electrical size. The field expression in (32) and Appendix 2 are employed to calculate the far-zone field component $E_{\varphi}^{(2b)}$. For case (a), since we let $\xi_c = 0.0$, it is just the example of the ordinary isotropic coating, and the radiated field is nearly confined within the range of $30° \leq \theta \leq 150°$. However, for cases (b) and (c), due to the effects of chirality admittance, the radiated field pattern is spread in all directions and more resonance peaks are observed.

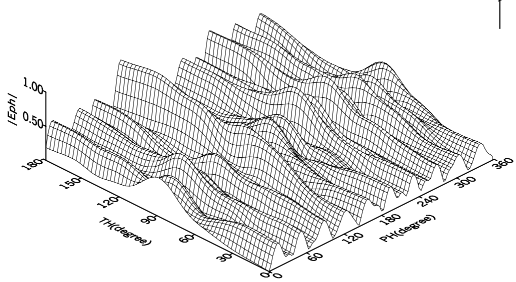
As an another illustrative example, Fig. 3 depicts the normalized radiation pattern $|E_{\varphi}^{(2a)}|$ of four dipole antennas on a chiral sphere, and the parameters chosen for the calculation are $a/\lambda = 1.0$, $\varepsilon = 3.5\varepsilon_0$, $\mu = 1.5\mu_0$, and $\xi_c = 0.003S$. Also, the lossy effect of the chiral sphere is not taken into account here due to the same reason.

In Fig. 3, the locations of four dipole antennas are determined by $\theta' = 30°$, $Q = 4$, $q = 0, 1, 2,$ and 3; and $C_{pq}^{(1)} = 1$ and $C_{pq}^{(2)} = 0$. It is clear that in the forward direction $(\theta = 180°)$, $|E_{\varphi}^{(2a)}|$ often reaches the maximum value for some special angles φ, and similar conclusion can be drawn for the far-zone field component $|E_{\theta}^{(2a)}|$. On the other

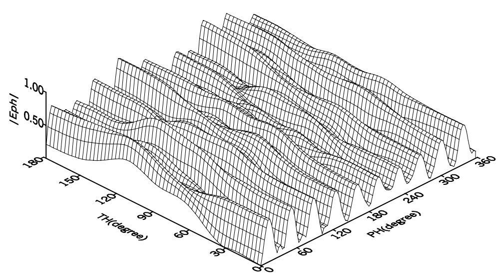


(a) $\xi_c = 0.0$.





(b) $\xi_c = 0.001S$.



(c) $\xi_c = 0.003S$.

Figure 2. Normalized three-dimensional far-zone field patterns $|E_\varphi^{(2b)}|$ for eight dipole antennas on a single layer spherical substrate.

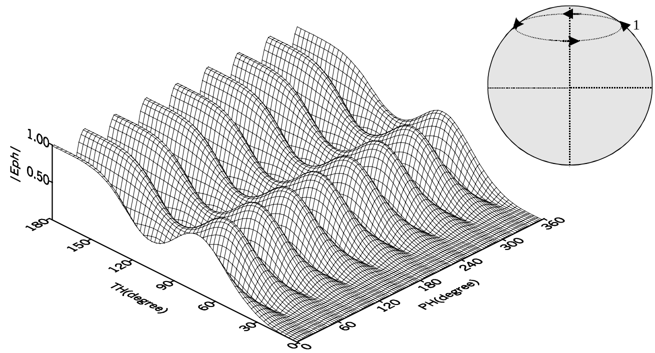


Figure 3. Normalized three-dimensional far-zone field patterns $|E_\varphi^{(2a)}|$ for four dipole antennas on the surface of a chiral sphere.

hand, both $|E_\varphi^{(2a)}|$ and $|E_\theta^{(2a)}|$ can be enhanced with the increasing of the chiral admittance. But practically, the magnitude of ξ_c is also governed by the other parameters of the chiral medium.

Finally, Fig. 4 depicts the ellipticity of the far-zone field in the forward direction of $\theta = 180°$ for eight dipole antennas on a single layer spherical substrate. The parameters are chosen to be $\varepsilon = 3.5\varepsilon_0$, $\mu = 1.5\mu_0$, $a/\lambda = 1.0$, $b/\lambda = 1.5$, $C_{pq}^{(1)} = 1$, and $C_{pq}^{(2)} = 0$.

In Fig. 4, the circular dot line stands for $\xi_c = 0.001S$, while the square dot line corresponds to $\xi_c = 0.003S$. The locations of eight dipole antennas are assumed to be $\theta' = 30°$, $Q = 8$, and $q = 0, 1, 2, \ldots, 7$. Clearly, the far-zone field in the forward direction is usually in the right- or left-handed elliptically polarized state, but for some special azimuth angles, nearly right- or left-handed circularly polarized wave could be achieved $(\sin\chi \approx \pm 1.0)$. Also, for the given chiral admittance $(\xi_c = 0.003S)$, the polarization state of the scattered field can be changed rapidly with the φ.

5. CONCLUSIONS

The canonical problems of radiation by a dipole antenna array in the presence of a chiral sphere and a perfectly conducting sphere coated with a single chiral medium have been formulated and examined, respectively. The technique of dyadic Green's function expressed in terms of the normalized spherical vector wave functions has been imposed in the mathematical analysis. This method is very general as well as

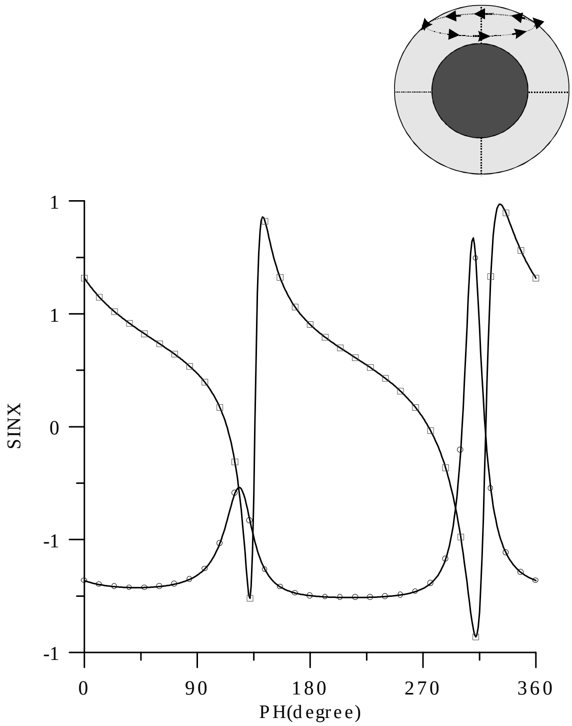


Figure 4. The ellipticity of the polarization ellipse as a function of the azimuth angle φ.

straightforward to apply. Here the orientation of the dipole antenna array can be arbitrary, but the coupling and the phase difference between the adjacent dipole antennas are not taken into account. The formulation performed for calculating the radiated fields in both the near and far zones is also reducible to or valid for that of the simple isotropic case.

APPENDIX 1

Substituting (5) and (6) into the following boundary equations,

$$\overline{e}_R \times \bar{\bar{G}}_e^{(1a)}(\overline{R}|\overline{R}') = \overline{e}_R \times \bar{\bar{G}}_e^{(2a)}(\overline{R}|\overline{R}') \tag{A1}$$

$$\overline{e}_R \times \frac{1}{\mu}\nabla \times \bar{\bar{G}}_e^{(1a)}(\overline{R}|\overline{R}') - \omega\xi_c \bar{\bar{G}}_e^{(1a)}(\overline{R}|\overline{R}') = \overline{e}_R \times \frac{1}{\mu_0}\nabla \times \bar{\bar{G}}_e^{(2a)}(\overline{R}|\overline{R}') \tag{A2}$$

leads to, after tedious mathematical manipulations, the following four sets of equations:

$$[D]\begin{bmatrix} A^{(1a)} \\ A^{(2a)} \\ B^{(1a)} \\ B^{(2a)} \end{bmatrix} = -k_+^2 \begin{bmatrix} h_+^a \\ \partial h_+^a \\ h_+^a \\ \partial h_+^a \end{bmatrix}, \tag{A3}$$

$$[D]\begin{bmatrix} A^{(3a)} \\ A^{(4a)} \\ B^{(3a)} \\ B^{(4a)} \end{bmatrix} = -k_+^2 \begin{bmatrix} -h_-^a \\ \partial h_-^a \\ h_-^a \\ -\partial h_-^a- \end{bmatrix}, \tag{A4}$$

$$[D] = \begin{bmatrix} j_+^a & j_-^a & -h_0^a & -h_0^a \\ \partial j_+^a & -\partial j_-^a & -\partial h_0^a & \partial h_0^a \\ j_+^a & -j_-^a & -l_{10} h_0^a & l_{10} h_0^a \\ \partial j_+^a & \partial j_-^a & -l_{10}\partial h_0^a & -l_{10}\partial h_0^a \end{bmatrix}, \tag{A5}$$

and

$$\begin{aligned} j_\pm^a &= j_n(k_\pm a), \quad h_\pm^a = h_n^{(1)}(k_\pm a), \quad h_0^a = h_n^{(1)}(k_0 a), \\ \partial j_\pm^a &= \frac{1}{k_\pm a}\frac{\partial}{\partial R'}\Big[R' j_n(k_\pm R')\Big]_{R'=a}, \\ \partial h_\pm^a &= \frac{1}{k_\pm a}\frac{\partial}{\partial R'}\Big[R' h_n^{(1)}(k_\pm R')\Big]_{R'=a}, \\ \partial h_0^a &= \frac{1}{k_0 a}\frac{\partial}{\partial R'}\Big[R' h_n^{(1)}(k_0 R')\Big]_{R'=a}, \\ l_{10} &= \sqrt{\frac{\varepsilon_0}{\mu_0}} \Big/ \sqrt{\frac{\varepsilon}{\mu} + \xi_c^2}. \end{aligned}$$

APPENDIX 2

Substituting (30) and (31) into the following boundary equations,

$$\overline{e}_R \times \overline{\overline{G}}_e^{(1b\le)}(\overline{R}|\overline{R}') = 0\,, \quad R = a; \tag{A6}$$

$$\overline{e}_R \times \overline{\overline{G}}_e^{(1b\ge)}(\overline{R}|\overline{R}') = \overline{e}_R \times \overline{\overline{G}}_e^{(2b)}(\overline{R}|\overline{R}')\,, \tag{A7a}$$

$$\begin{aligned} &\overline{e}_R \times \frac{1}{\mu}\nabla \times \overline{\overline{G}}_e^{(1b\ge)}(\overline{R}|\overline{R}') - \omega\xi_c \overline{\overline{G}}_e^{(1b\ge)}(\overline{R}|\overline{R}') \\ &\qquad = \overline{e}_R \times \frac{1}{\mu_0}\nabla \times \overline{\overline{G}}_e^{(2a)}(\overline{R}|\overline{R}')\,; \end{aligned} \tag{A7b}$$

we obtain

$$[D]\begin{bmatrix} A^{(1b)} \\ A^{(2b)} \\ A^{(3b)} \\ A^{(4b)} \\ B^{(1b)} \\ B^{(2b)} \end{bmatrix} = -k_+^2 \begin{bmatrix} 0 \\ 0 \\ h_+^b \\ \partial h_+^b \\ h_+^b \\ \partial h_+^b \end{bmatrix}, \tag{A8}$$

$$[D]\begin{bmatrix} A^{(5b)} \\ A^{(6b)} \\ A^{(7b)} \\ A^{(8b)} \\ B^{(3b)} \\ B^{(4b)} \end{bmatrix} = k_-^2 \begin{bmatrix} 0 \\ 0 \\ -h_-^b \\ \partial h_-^b \\ h_-^b \\ -\partial h_-^b \end{bmatrix}, \tag{A9}$$

$$[D]\begin{bmatrix} A^{(9b)} \\ A^{(10b)} \\ A^{(11b)} \\ A^{(12b)} \\ B^{(5b)} \\ B^{(6b)} \end{bmatrix} = -k_+^2 \begin{bmatrix} j_+^a \\ \partial j_+^a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{A10}$$

$$[D]\begin{bmatrix} A^{(13b)} \\ A^{(14b)} \\ A^{(15b)} \\ A^{(16b)} \\ B^{(7b)} \\ B^{(8b)} \end{bmatrix} = k_-^2 \begin{bmatrix} -j_-^a \\ \partial j_-^a \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \tag{A11}$$

and

$$[D] = \begin{bmatrix} j_+^a & j_-^a & h_+^a & h_-^a & 0 & 0 \\ \partial j_+^a & -\partial j_-^a & \partial h_+^a & -\partial h_-^a & 0 & 0 \\ j_+^b & j_-^b & h_+^b & h_-^b & -h_0^b & -h_0^b \\ \partial j_+^b & -\partial j_-^b & \partial h_+^b & -\partial h_-^b & -\partial h_0^b & \partial h_0^b \\ j_+^b & -j_-^b & h_+^b & -h_-^b & -l_{10} h_0^b & l_{10} h_0^b \\ \partial j_+^b & \partial j_-^b & \partial h_+^b & \partial h_-^b & -l_{10}\partial h_0^b & -l_{10}\partial h_0^b \end{bmatrix}. \tag{A12}$$

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