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A NEW TIME DOMAIN APPROACH FOR ANALYSIS OF VERTICAL MAGNETIC DIPOLE RADIATION IN FRONT OF LOSSY HALF-SPACE

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Abstract–In this paper, the problem of scattering and radiation in the presence of a material half-space is solved using the Transient spectral domain method (TSDM). The TSDM is a general theoretical approach for exact solution to the time-dependent sources radiation in the presence of stratified media. A source used is an electrically short magnetic dipole with impulse current distribution in time domain. It is located in air region in the vicinity of lossy half-space. The method requires the proper spatial Fourier transform for theoretical formulation of the problem. The new closed-form expressions is achieved in terms of spectral domain variable and time. Also a number of special cases are presented for verification of this procedure.

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1. INTRODUCTION

In this paper the classical time domain problem formed by vertically oriented magnetic dipole with impulsive current density distribution in front of lossy half-space is studied theoretically (Sommerfeld's half-space problem).

Time domain solution of Sommerfeld's problem can be used in different areas of electromagnetics, such as evaluation of remote-sensing systems for detection of shallow buried objects [1], and probing of fields in geological media [2]. Also some practical applications can be found for transient solution of this problem in spectral domain [3].

The first exact solution of such a problem was obtained by Van der Pol [4] for vertical dipole source radiation near two dielectric half spaces. For the same geometry, Nikoskinen et al. derived an analytical solution by using Exact Image Theory [5, 6]. Wait [7–9], Haddad et al. [10] and Kooij [11] studied the transient response of a point source with different current distributions over a finitely conducting earth. Also Hsueh-Yuan Pao et al. [12, 13] analyzed plane waves obliquely incident on a conductive half-space. Besides Ezzeddine et al. [14, 15], Mahmoud et al. [16] and Abo Seliem [17] considered the same source radiating near two layered ground and finally Dehoop [18] and Kuester [19] studied the problem of line sources in half space. All of these methods used frequency domain formulation and then performed inverse Laplace or Fourier transforms with proper integration paths for transformation from time harmonic solutions to a transient one, almost in all cases authors used some approximations to give closed form solutions.

Historically, transient solution of this problem is found by applying a Fourier or Laplace transform on the time variable. The boundary conditions are satisfied by taking an expansion of a spherical wave in the terms of cylindrical waves using Fourier-bessel transform [20]. In the present paper a new theoretical approach called TSDM (Transient Spectral Domain Method) is used to solve the problem. In this approach, spatial Fourier transform is applied directly in the time domain, so, a transient formulation of the fields is achieved in terms of spectral functions. This approach has many advantages such as applicability to multi layered media with different current sources and the availability of the simplified closed-form solution for lossless case. The accuracy of this method is verified in two special cases: first, when the interface is removed that is, when the point source is radiating in an unbounded homogenous medium, second, when the lossy half space is supposed to be a prefect magnetic conductor.

2. FORMULATION OF THE PROBLEM USING TSDM

The geometry of the problem is shown in Fig. 1. A plane interface separates two regions (I and II). Region I is considered as free space $(\varepsilon_0,\ \mu_0)$ and region II represents a lossy half-space $(\varepsilon_1,\ \mu_1,\ \sigma)$. The source is located in region I. Assuming homogenous, isotropic and linear medium in each region, we may write the Maxwell's curls equations as [21]:

$$\nabla \times \mathbf{e} = -\mu \frac{\partial \mathbf{h}}{\partial t} - \mathbf{M} \tag{1a}$$

$$\nabla \times \mathbf{h} = \varepsilon \frac{\partial \mathbf{e}}{\partial t} + \sigma \mathbf{e} \tag{1b}$$

Where $\mathbf{M}$ is intrinsic magnetic current density in the medium. The solution of this problem is facilitated by introducing the electric vector potential $\mathbf{F}$, as below:

$$\mathbf{e} = -\frac{1}{\varepsilon} \nabla \times \mathbf{F} \tag{2}$$

Where $\mathbf{F}$ satisfies the inhomogeneous vector Helmholtz equations.

$$\nabla^2 \mathbf{F} - \mu\varepsilon \frac{\partial^2 \mathbf{F}}{\partial t^2} = -\varepsilon \mathbf{M} \qquad z < 0 \tag{3}$$

$$\nabla^2 \mathbf{F} - \mu\varepsilon \frac{\partial^2 \mathbf{F}}{\partial t^2} - \mu\sigma \frac{\partial \mathbf{F}}{\partial t} = 0 \qquad z > 0 \tag{4}$$

The magnetic current density $(\mathbf{M})$ is assumed to have only a z-component with impulse distribution in time and be located in lossless half-space (region I) as shown in Fig. 1. By assuming a z-component for the electric vector potential, i.e., $\mathbf{F} = F_z \mathbf{z}$ we can write the above equation as:

$$\nabla^2 F_z - \mu_0\varepsilon_0 \frac{\partial^2 F_z}{\partial t^2} = -\delta(x)\delta(y)\delta(z - z_0)\delta(t - t_0) \quad z > 0 \tag{5}$$

$$\nabla^2 F_z - \mu_1\varepsilon_1 \frac{\partial^2 F_z}{\partial t^2} + \mu_1\sigma \frac{\partial F_z}{\partial t} = 0 \qquad z < 0 \tag{6}$$

It is readily verified by substitution that the Maxwell's curl equations (1) and the definition of electric vector potential (2) admits the form

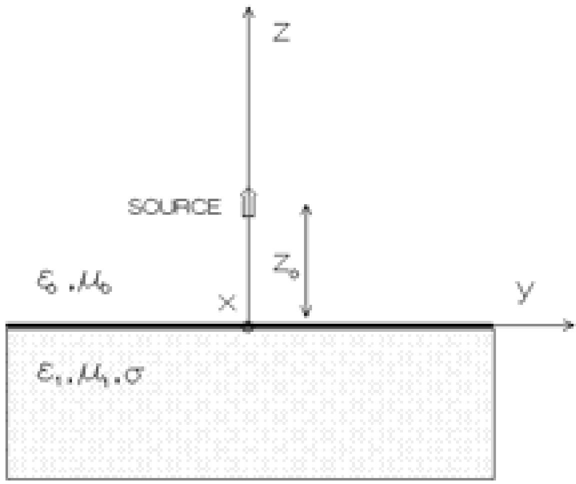

Figure 1. Geometry of the problem.

of electromagnetic fields in terms of $\mathbf{F} = F_z\mathbf{z}$ (Appendix 1):

$$\begin{aligned} e_x &= \frac{1}{\varepsilon}\left(-\frac{\partial F_z}{\partial y}\right) \\ e_y &= \frac{1}{\varepsilon}\left(\frac{\partial F_z}{\partial x}\right) \\ e_z &= 0 \end{aligned} \tag{7}$$

for electric field, and

$$\begin{aligned} h_x &= \frac{1}{\mu\varepsilon}\int_{t_0}^{t} \frac{\partial}{\partial z}\left(\frac{\partial F_z}{\partial x}\right) dt - \frac{1}{\varepsilon}\int_{t_0}^{t} M_x dt \\ h_y &= \frac{1}{\mu\varepsilon}\int_{t_0}^{t} \frac{\partial}{\partial z}\left(\frac{\partial F_z}{\partial y}\right) dt - \frac{1}{\varepsilon}\int_{t_0}^{t} M_y dt \\ h_z &= \frac{1}{\mu\varepsilon}\int_{t_0}^{t} \left(-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2}\right) F_z dt - \frac{1}{\varepsilon}\int_{t_0}^{t} M_z dt \end{aligned} \tag{8}$$

for magnetic field.

By the two dimensional Fourier transformation with respect to x and y, the solution of (5) and (6) can be expressed in terms of cylindrical waves that have the same radial wave number in air and within the lossy half-space [20].

$$\hat{F}_z(k_x, k_y, z, t) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F_z(x, y, z, t)\exp\{jk_x x + jk_y y\}\,dxdy, \quad (9)$$

$$\left\{-(k_x^2 + k_y^2) + \frac{\partial^2}{\partial z^2} - \mu_0\varepsilon_0\frac{\partial^2}{\partial t^2}\right\}\hat{F}_z = -\delta(t - t_0)\delta(z - z_0) \quad z < 0 \quad (10)$$

$$\left\{-(k_x^2 + k_y^2) + \frac{\partial^2}{\partial z^2} - \mu_1\varepsilon_1\frac{\partial^2}{\partial t^2} - \mu_1\sigma\frac{\partial}{\partial t}\right\}\hat{F}_z = 0 \quad z > 0 \quad (11)$$

The exact solution of (9) and (10) is available [22],

$$\begin{aligned}&\hat{F}_z = \frac{1}{2}J_0\left(\sqrt{k_\rho^2[c^2(t - t_0)^2 - (z - z_0)^2]}\right)\\ &k_\rho^2 = k_x^2 + k_y^2, \qquad c = 1/\sqrt{\mu_0\varepsilon_0},\end{aligned} \quad (12)$$

$$\begin{aligned}&\hat{F}_z = \frac{1}{2}\exp\left\{-\frac{\sigma}{2\varepsilon_1}(t - t_0)\right\}J_0\left(\sqrt{k_{\rho\sigma}^2[c^2(t - t_0)^2 - (z - z_0)^2]}\right)\\ &k_{\rho\sigma}^2 = k_\rho^2 - \frac{\mu_1\sigma^2}{4\varepsilon_1}, \qquad c_1 = 1/\sqrt{\mu_1\varepsilon_1},\end{aligned} \quad (13)$$

(12) and (13) are obtained for homogenous medium with $(\varepsilon_0,\ \mu_0)$, and $(\varepsilon_1,\ \mu_1,\ \sigma)$ respectively.

Region of validity for these solutions are:

$$\begin{aligned}c(t - t_0) &> (z - z_0)\\ c(t - t_0) &> -(z - z_0).\end{aligned} \quad (14)$$

In the above equations k_ρ is the spectral domain variable, on the other hand it is defined as cylindrical wave propagation constant.

We use the concept of primary and scattered waves in the spectral domain [20]. That is for $z > 0$:

$$\hat{F}_z^I = \hat{F}_{z^i}^I + \hat{F}_{z^s}^I \quad (15)$$

$$\hat{F}_{z^i}^I = \frac{1}{2}J_0\left(\sqrt{k_\rho^2[c^2(t - t_0)^2 - (z_0 - z)^2]}\right) \quad (16)$$

$$\hat{F}_{z^s}^I = \frac{1}{2}J_0\left(\sqrt{k_\rho^2[c^2(t - t_0)^2 - (z_0 + z)^2]}\right)\cdot R(t_\rho,\ t,\ z_0) \quad (17)$$

Where $\hat{F}_{z^i}^I$ denotes the incident field as the primary wave in the absence of the ground and $\hat{F}_{z^s}^I$ represents the scattered wave due to the presence of the ground in spectral domain. In the ground region $(z < 0)$ we have:

$$\hat{F}_z^{II} = \hat{F}_{z^s}^{II} \tag{18}$$

$$\hat{F}_{z^s}^{II} = T(k_\rho, t, z_0)\frac{1}{2}\exp\left\{-\frac{\sigma}{2\varepsilon_1}(t-t_0)\right\} \cdot J_0\left(\sqrt{k_{\rho\sigma}^2\left[c^2(t-t_0)^2-(z-z_0)^2\right]}\right) \tag{19}$$

Where (16), (17), and (18) are expressed in terms of same radial wave number (k_ρ), which is proper for boundary condition satisfaction.

The continuity of tangential electric and magnetic fields at air-ground interface can be used to find the unknown coefficients R and T (Appendix 2),

$$\frac{1}{\varepsilon_0}\left\{\hat{F}_{z^i}^I + \hat{F}_{z^s}^I\right\}\Big|_{z=0} = \frac{1}{\varepsilon_1}\hat{F}_z^{II}\Big|_{z=0} \tag{20}$$

$$\frac{1}{\varepsilon_0\mu_0}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}(\hat{F}_{z^i}^I + \hat{F}_{z^s}^I)\right\}dt\Bigg|_{z=0} = \frac{1}{\varepsilon_1\mu_1}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}\hat{F}_z^{II}\right\}dt\Bigg|_{z=0} \tag{21}$$

Substituting (16), (17), and (19) in the above equations, we will have:

$$1+R = \frac{\varepsilon_0}{\varepsilon_1}\frac{J_0(u_1)}{J_0(u)}\mathbf{T}, \quad \mathbf{T} = T\exp\left\{-\frac{\sigma}{2\varepsilon_1}(t-t_0)\right\} \tag{22}$$

$$1-R = \frac{\varepsilon_0\mu_0}{\varepsilon_1\mu_1}\frac{J_1(u_1)}{J_1(u)}\frac{u}{u_1}\frac{k_{\rho\sigma}^2}{k_\rho^2}\mathbf{T} \tag{23}$$

where,

$$u = \sqrt{k_\rho^2[c^2(t-t_0)^2 - z_0^2]}, \quad u_1 = \sqrt{k_{\rho\sigma}^2[c_1^2(t-t_0)^2 - z_0^2]} \tag{24}$$

Then, R and T can be easily found as:

$$\mathbf{T} = \frac{2}{\dfrac{\varepsilon_0}{\varepsilon_1}\dfrac{J_0(u_1)}{J_0(u)} + \dfrac{\varepsilon_0\mu_0}{\varepsilon_1\mu_1}\dfrac{J_1(u_1)}{J_1(u)}\dfrac{u}{u_1}\dfrac{k_{\rho\sigma}^2}{k_\sigma^2}} \tag{25}$$

$$R = \frac{\dfrac{\varepsilon_0}{\varepsilon_1}\dfrac{J_0(u_1)}{J_0(u)} - \dfrac{\varepsilon_0\mu_0}{\varepsilon_1\mu_1}\dfrac{J_1(u_1)}{J_1(u)}\dfrac{u}{u_1}\dfrac{k_{\rho\sigma}^2}{k_\sigma^2}}{\dfrac{\varepsilon_0}{\varepsilon_1}\dfrac{J_0(u_1)}{J_0(u)} + \dfrac{\varepsilon_0\mu_0}{\varepsilon_1\mu_1}\dfrac{J_1(u_1)}{J_1(u)}\dfrac{u}{u_1}\dfrac{k_{\rho\sigma}^2}{k_\sigma^2}} \tag{26}$$

if ground region is assumed lossless ($\sigma = 0$), (22), (23), and (24) are changed to:

$$1 + R = \frac{\varepsilon_0}{\varepsilon_1} \frac{J_0(u_1)}{J_0(u)} T \tag{27}$$

$$1 - R = \frac{\varepsilon_0 \mu_0}{\varepsilon_1 \mu_1} \frac{J_1(u_1)}{J_1(u)} \frac{u}{u_1} T \tag{28}$$

$$u = \sqrt{k_\rho^2 [c^2 (t - t_0)^2 - z_0^2]}, \quad u_1 = \sqrt{k_\rho^2 [c_1^2 (t - t_0)^2 - z_0^2]} \tag{29}$$

In lossless case, we can find simple closed-forms for R and T,

$$T = \frac{2}{\dfrac{\varepsilon_0}{\varepsilon_1} \dfrac{J_0(u_1)}{J_0(u)} + \dfrac{\varepsilon_0 \mu_0}{\varepsilon_1 \mu_1} \dfrac{J_1(u_1)}{J_1(u)} \dfrac{u}{u_1}} \tag{30}$$

$$R = \frac{\dfrac{\varepsilon_0}{\varepsilon_1} \dfrac{J_0(u_1)}{J_0(u)} - \dfrac{\varepsilon_0 \mu_0}{\varepsilon_1 \mu_1} \dfrac{J_1(u_1)}{J_1(u)} \dfrac{u}{u_1}}{\dfrac{\varepsilon_0}{\varepsilon_1} \dfrac{J_0(u_1)}{J_0(u)} + \dfrac{\varepsilon_0 \mu_0}{\varepsilon_1 \mu_1} \dfrac{J_1(u_1)}{J_1(u)} \dfrac{u}{u_1}} \tag{31}$$

The scattered waves in spectral domain can be found by substituting (25) and (26) for lossy ground, and (30) and (31) for lossless case to (17) and (19), respectively. The space domain scattered waves can be calculated numerically by the inverse Fourier transformation.

$$F_z(x, y, z, t) = \frac{1}{4\pi^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \hat{F}_z(k_x, k_y, z, t) \cdot \exp\{-jk_x x - jk_y y\}\, dk_x dk_y \tag{32}$$

Efficient numerical evaluation of the inverse Fourier integrals (32) plays an important role in arriving at the final solution of the transient problem in space domain. Such an efficient approach will be presented in a separate paper.

3. VERIFYING SPECIAL CASES

In this section we consider a number of special cases for verifying the presented solution. As the first case let ε_1 and μ_1 approach ε_0 and μ_0 that is, the whole space is filled by a homogenous material $(\varepsilon_0,\ \mu_0)$.

It is seen from (30), (31) that,

$$\lim_{(\varepsilon_1,\mu_1)\to(\varepsilon_0,\mu_0)} T = 1, \qquad \lim_{(\varepsilon_1,\mu_1)\to(\varepsilon_0,\mu_0)} R = 0 \tag{33}$$

Thus, the reflected field vanishes as expected and we have radiation of original source in homogenous unbounded medium. As the second case, let us assume a situation where μ_1 approaches ∞, which corresponds to prefect magnetic conducting half space. It is again observed from (30, 31) that,

$$\lim_{\mu_1\to\infty} T\frac{\varepsilon_0}{\varepsilon_1}\frac{J_0(u_1)}{J_0(u)} = 2, \qquad \lim_{\mu_1\to\infty} R = 1 \tag{34}$$

In this case,

$$\begin{aligned}\hat{F}^I_{z^i} &= \frac{1}{2}J_0\left(\sqrt{k_\rho^2[c^2(t-t_0)^2-(z_0-z)^2]}\right)\\ \hat{F}^I_{z^s} &= \frac{1}{2}J_0\left(\sqrt{k_\rho^2[c^2(t-t_0)^2-(z_0+z)^2]}\right)\\ \hat{F}^{II}_{z^s} &= 0.\end{aligned} \tag{35}$$

which is exactly what we would expect from the classical image method solution of the problem.

4. CONCLUSION

The transient spectral domain method (TSDM) is presented as a versatile procedure for transient analysis of point source radiation near two layered media in spectral domain. In this paper, TSDM is used to analyze electromagnetic radiation from an impulsive vertical magnetic dipole above conducting half space. This procedure leads to a new transient representation of electric vector potential in air and ground regions in spectral domain, also it is simplified to closed-form expressions when the ground medium is assumed to be lossless. A number of special cases are presented for verification.

APPENDIX 1. ELECTRIC FIELD REPRESENTATION IN TERMS OF ELECTRIC VECTOR POTENTIAL

We may write $\mathbf{e}$ in terms of $\mathbf{F}$, as electric vector potential,

$$\mathbf{e} = -\frac{1}{\varepsilon}\nabla\times\mathbf{F} \tag{A1}$$

by combining (1a) and (A1), we have:

$$\nabla\times\nabla\times\mathbf{F}-\varepsilon\mathbf{M}=\varepsilon\mu\frac{\partial\mathbf{h}}{\partial t} \tag{A2}$$

we can express (A2) as:

$$\begin{aligned}
h_x=&\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\left\{\frac{\partial}{\partial y}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial F_z}{\partial x}-\frac{\partial F_x}{\partial z}\right)\right\}dt\\
&-\frac{1}{\mu}\int_{t_0}^{t}M_x dt\\
h_y=&\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\left\{-\frac{\partial}{\partial x}\left(\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)\right\}dt\\
&-\frac{1}{\mu}\int_{t_0}^{t}M_y dt\\
h_z=&\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\left\{-\frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial x}-\frac{\partial F_x}{\partial z}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial y}-\frac{\partial F_y}{\partial z}\right)\right\}dt\\
&-\frac{1}{\mu}\int_{t_0}^{t}M_z dt
\end{aligned} \tag{A3}$$

in Cartesian coordinates.

When $\mathbf{F}$ have only a z-component, we can simplify (A1) and (A3) as:

$$\begin{aligned}
e_x&=-\frac{1}{\varepsilon}\left\{\frac{\partial F_z}{\partial y}\right\}\\
e_y&=\frac{1}{\varepsilon}\left\{\frac{\partial F_z}{\partial x}\right\}\\
e_z&=0
\end{aligned} \tag{A4}$$

$$\begin{aligned}
h_x&=\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\quad\frac{\partial}{\partial z}\left\{\frac{\partial F_z}{\partial x}\right\}dt\\
h_y&=\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\quad\left\{\frac{\partial}{\partial z}\frac{\partial F_z}{\partial y}\right\}dt\\
h_z&=\frac{1}{\varepsilon\mu}\int_{t_0}^{t}\left\{-\frac{\partial}{\partial x}\left(\frac{\partial F_z}{\partial x}\right)-\frac{\partial}{\partial y}\left(\frac{\partial F_z}{\partial y}\right)\right\}dt-\frac{1}{\mu}\int_{t_0}^{t}M_z dt
\end{aligned} \tag{A5}$$

APPENDIX 2. REPRESENTATION OF BOUNDARY CONDITIONS IN TERMS OF F_z

The continuity of tangential electric and magnetic fields at the interface can be represented as:

$$\begin{aligned} e_x^I\big|_{z=0} &= e_x^{II}\big|_{z=0} \\ h_y^I\big|_{z=0} &= h_y^{II}\big|_{z=0} \end{aligned} \tag{A6}$$

Substituting (15) and (18) in (A4) and (A5), and applying the spatial Fourier transform, the electromagnetic fields in terms of $\hat{F}_z$ are represented, as:

$$\begin{aligned} \hat{e}_x^I\big|_{z=0} &= \frac{jk_y}{\varepsilon_0}\left(\hat{F}_{z^i}^I + \hat{F}_{z^s}^I\right)\Big|_{z=0} \\ \hat{e}_x^{II}\big|_{z=0} &= \frac{jk_y}{\varepsilon_0}\hat{F}_{z^s}^{II}\Big|_{z=0} \end{aligned} \tag{A7}$$

$$\begin{aligned} \hat{h}_y^I\Big|_{z=0} &= \frac{jk_y}{\varepsilon_0\mu_0}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}\left(\hat{F}_{z^i}^I + \hat{F}_{z^s}^I\right)\right\}dt\Big|_{z=0} \\ \hat{h}_y^{II}\Big|_{z=0} &= \frac{jk_y}{\varepsilon_1\mu_1}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}\hat{F}_z^{II}\right\}dt\Big|_{z=0} \end{aligned} \tag{A8}$$

Combining (A9) and (A10) with (A8), we have,

$$\frac{1}{\varepsilon_0}\left\{\hat{F}_{z^i}^I + \hat{F}_{z^s}^I\right\}\Big|_{z=0} = \frac{1}{\varepsilon_1}\,\hat{F}_z^{II}\Big|_{z=0} \tag{A9}$$

for electric field and,

$$\frac{1}{\varepsilon_0\mu_0}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}\left(\hat{F}_{z^i}^I + \hat{F}_{z^s}^I\right)\right\}dt\Big|_{z=0} = \frac{1}{\varepsilon_1\mu_1}\int_{t_0}^{t}\left\{\frac{\partial}{\partial z}\hat{F}_z^{II}\right\}dt\Big|_{z=0} \tag{A10}$$

for magnetic field.

REFERENCES

1. Simons, N. R. S., A. Sebak, and G. E. Bridges, "Application of the TLM method to half-space and remote-sensing problems," *IEEE Trans. Geosci. Remote Sensing,* Vol. 33, 759–767, May 1995.
2. Wait, J. R., *Electromagnetic Probing in Geophysics,* The Golem Press, Boulder, Colo., 1971.
3. Park, I., R. Mittra, and M. I. Akusn, "Numerically analysis of planar microstrip configurations using closed form Green's functions," *IEEE Trans. Microwave Theory Tech.,* Vol. 43, 394–400, 1995.
4. Van der Pol, B., "On discontinuous electromagnetic waves and the occurrence of a surface wave," *IRE Trans. Antennas Propagat.,* AP-4, 288–293, 1956.
5. Nikoskinen, K. I., and I. V. Lindell, "Time-domain analysis of Sommerfeld VMD problem with exact image theory," *IEEE Trans. Antennas Propagat.,* Vol. 38, 241–250, Feb. 1990.
6. Nikoskinen, K. I. "Time-domain study of half-space transmission problem with vertical and horizontal dipoles," *IEEE Trans. Antennas Propagat.,* Vol. 41, 1399–1407, Oct. 1993.
7. Wait, J. R., "On the electromagnetic pulse response of a dipole over a plane surface," *Can. J. Phys.,* Vol. 52, 193–196, 1974.
8. Wait, J. R., "On the theory of transient electromagnetic sounding over a stratified earth," *Can. J. Phys.,* Vol. 50, 1055–1061, 1972.
9. Wait, J. R., and K. P. Spies, "Transient fields for an electric dipole in a dissipative media," *Can. J. Phys.,* Vol. 48, 1858–1862, 1970.
10. Haddad, H., and D. C. Cheng, "Transient electromagnetic field generated by a vertical electrical dipole on the surface of a dissipative earth," *Radio Science,* Vol. 16, No. 2, 169–177, 1981.
11. Kooij, B. J., "Transient electromagnetic field of a vertical magnetic dipole above a plane conducting earth," *Radio Science,* Vol. 25, No. 4, 349–356, 1990.
12. Pao, H., S. L. Dvorak, and D. G. Dudley, "An accurate and efficient analysis for transient plane waves obliquely incident on the conductive half space (TE case)," *IEEE Trans. Antennas Propagat.,* Vol. AP-44, 918–924, July 1996.
13. Pao, H., S. L. Dvorak, and D. G. Dudley, "An accurate and efficient analysis for transient plane waves obliquely incident on the conductive half space (TM case)," *IEEE Trans. Antennas Propagat.,* Vol. AP-44, 918–924, July 1996.
14. Ezzeddine, A., J. A. Kong, and L. Tsang, "Transient fields of a vertical electric dipole over a two-layer nondispersive dielectric," *J. Appl. Phys.,* Vol. 52, 1202–1208, March 1981.

15. Ezzeddine, A., and J. A. Kong, "Time response of a vertical dipole over a two layer medium by the double deformation technique," *J. Appl. Phys.,* Vol. 53, 813–822, Feb. 1982.
16. Mahmoud, S. F., A. Z. Botros, and J. R. Wait, "Transient electromagnetic fields of a vertical magnetic dipole on a two-layer earth," *Proc. IEEE,* Vol. 67, 1022–1029, July 1979.
17 Abo Seliem, A. A. S., "Transient electromagnetic field of a vertical electric dipole above an atmospheric surface duct," *Radio Science,* Vol. 31, 833–840, July–August 1996.
18. De Hoop, A. T., "Pulsed electromagnetic radiation from a line source in a two-media configuration," *Radio Science,* Vol. 14, 253–268, March–April 1979.
19. Kuester, E. F., "The transient electromagnetic field of a pulsed line source located above a dispersively reflecting surface," *IEEE Trans. Antennas Propagat.,* Vol. AP-32, 1154–1162, Nov. 1984.
20. Banos, A., *Dipole Radiation in the Presence of a Conducting Half-space,* Pergamon Press, London, 1966.
21. Stratton, J. A., *Electromagnetic Theory,* McGraw-Hill Co., London, 1941.
22. Kevorkian, J., *Partial Differential Equations,* Brooks/cole Co., California, 1990.