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Design of Multi-Beam Rhombus Fractal Array Antenna Using New Geometric Design Methodology

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Abstract—Fractal array antenna design methodology is an artistic type of design methodology. Hence fractal array antennas are also called as artistic array antennas. This article proposes a concentric elliptical ring sub array generator geometric design methodology for a methodical expansion of multi-beam fractal array antennas. Using this new geometric design methodology, any polygon shape can be constructed. This geometric design methodology provides a systematic approach for multiple beams of fractal array antennas, with unit amplitude constriction, using multi-beam sub arrays and without any increase in hardware complication. In this paper, a four-element rhombus fractal array antenna is examined using a proposed design methodology up to four concurrent iterations and for different eccentric values. Due to the recursive nature of the proposed methodology, the rhombus fractal array antenna shows multi-beam performance with abatement of beam width and better side-lobe levels. In the third and fourth iterations of the rhombus fractal array with expansion factor two, beamwidth reached single digit values of 7.2°, 3.6° with side-lobe level angles of 15.5° and 8.1°, respectively. The behavior of the proposed array shows better performance than four-element fractal array antenna generated by concentric circular subarray generator. The proposed fractal array antennas are analyzed and simulated by MATLAB programming.

1. INTRODUCTION

Antenna arrays play an essential role in military applications due to their fast beam scanning, high directive and resolution behaviors [1–4]. Fractal array antennas can find applications in both military and civilian fields due to their wide band, ultrawide band behavior, multi-band characteristics and abated mutual coupling losses [5–7]. Geometric design methodologies play a vital role in the designing of both deterministic and random fractal array antennas. Wide band and multiband performance of fractal array antennas depends on fractal geometric shapes and how they are generated. That is why these arrays are geometry based not material based. But only a few design methodologies are available for the generation and design of linear, planar and conformal fractal array antennas [8, 9]. Concentric circular ring subarray generator is one of the initiate design methodologies for the generation of linear and planar fractal array antennas. Various types of fractal array antennas such as triadic cantor linear and planar fractal arrays, Sierpinski triangular, square fractal arrays, pentagonal, hexagonal, heptagonal and octagonal fractal array antennas with and without central elements for different expansion factors (S) have implemented using this design methodology [10, 11]. Conformal fractal arrays such as a Menger sponge and 3-D Sierpinski gasket arrays are generated by concentric sphere 3-D array generator, and these arrays can be useful for military aircraft, ships, land vehicles and missiles because of their conformal fractal nature [12].

Beyond these methodologies, another type of fractal array antennas is also available. They are nature inspired fractal random arrays [13], and cantor fractal linear array for even number of antenna

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elements [14]. To meet the requirements of satellite communications such as the high end of coverage directivity, less side-lobe level hybrid-fractal direct radiating antenna arrays were presented in [15]. Dragon, flap, twig fractal array antennas are proposed to fulfill the requirements such as abated side-lobe levels and wide variation between the main beam and side lobes for celestial communication surveillance and wireless communication systems. These arrays show better performance in all array factor properties than conventional square and Sierpinski triangular array antennas [16]. The cantor ring array designed by a polyadic design methodology for lower side-lobe levels is also one of the famous fractal arrays, and it is a good example for hybrid array antennas [17]. To avoid overlaps and gaps between the antenna elements as in conventional fractal planar arrays, a novel type of fractal arrays having fractal boundaries named fractile arrays is proposed in [18]. A combination of Sierpinski carpet fractal array with circular ring aperture is proposed in [19] to fulfill the requirements of advanced satellite systems such as multi-beams with direct radiating behavior and good Career to interference ratio. To overcome the basic drawbacks in Sierpinski fractal array such as high side-lobe levels and number of antenna elements, Haferman carpet fractal array antenna is proposed, and this array achieves nearly 40–50% of thinning with lower peak side-lobe levels [20]. Due to their ease of generation and various applications, fractal arrays become more popular, and large amount of research is done on fractal antennas, but articles related to design methodologies are relatively few. Concentric elliptical subarray design methodology is not present in the literature. Some articles investigated and synthesized different types of elliptical array antennas [21].

This paper mainly concentrates on the design of multi-beam rhombus fractal array antennas using concentric elliptical ring subarray generator design methodology. The paper is organized as follows. Section 2 introduces the concentric elliptical ring subarray geometric design methodology. Section 3 explains the expansion of a rhombus fractal array antenna using the proposed design methodology. Section 4 shows results of the proposed fractal array antenna. Finally, conclusion is drawn in Section 5 of this paper.

2. CONCENTRIC ELLIPTICAL RING SUB ARRAY GENERATOR DESIGN METHODOLOGY

New design methodologies may be helpful for the improvement of fractal array antenna performance. On this manner, this paper proposes a concentric elliptical ring subarray generator for the expansion of different types of fractal arrays. This methodology replicates the concentric circular ring subarray generator, but in this case, circular generator is replaced with elliptical generator as shown in Figure 1. Radius is an important parameter in concentric circular subarray generator, but eccentricity is an important parameter in this methodology. Eccentricity can be varied using major axis (a) and minor axis (b). Here only two stages of growth are shown in Figure 1, but stages can be generated up to infinite iterations (P). The basic subarray generator can be repeated again and again according to number of iterations. The number of subarray antenna elements should depend on the expansion factor (S).

The resultant generating array factor for concentric elliptical ring array can be expressed as,

$$GA(\theta,\phi) = \sum_{m=1}^{M}\sum_{n=1}^{N} I_{mn} e^{jk\Psi_{mn}} \tag{1}$$

$$A\cdot F_P(\theta,\phi) = \prod_{P=1}^{P}\left[\sum_{m=1}^{M}\sum_{n=1}^{N} I_{mn} e^{jkS^{P-1}\Psi_{mn}}\right] \tag{2}$$

$$\Psi_{mn} = \sin\theta(a\cos\phi_n\cos\phi + b\sin\phi_n\sin\phi) - \sin\theta_0(a\cos\phi_n\cos\phi_0 + b\sin\phi_n\sin\phi_0) \tag{3}$$

where I_{mn} is the current amplitudes, k the wave number, S the expansion factor, P the iteration number, and θ_0 and ϕ_0 are the beam steering angles, $\phi_{xn} = (n-1)(2\pi/4)$. a is the major axis (λ), b the minor axis (0.7λ) since no grating lobes observed in these range, M the concentric rings, and N the number of elements. This design methodology allows choice in broadening the shape of a beam or for designing multiple beams for any fractal array antenna without involving any amplitude control and without loss in power. This subarray fractal concept can be helpful for ease of practical implementation of the array because only single-feed network is enough for each subarray. Another advantage of this

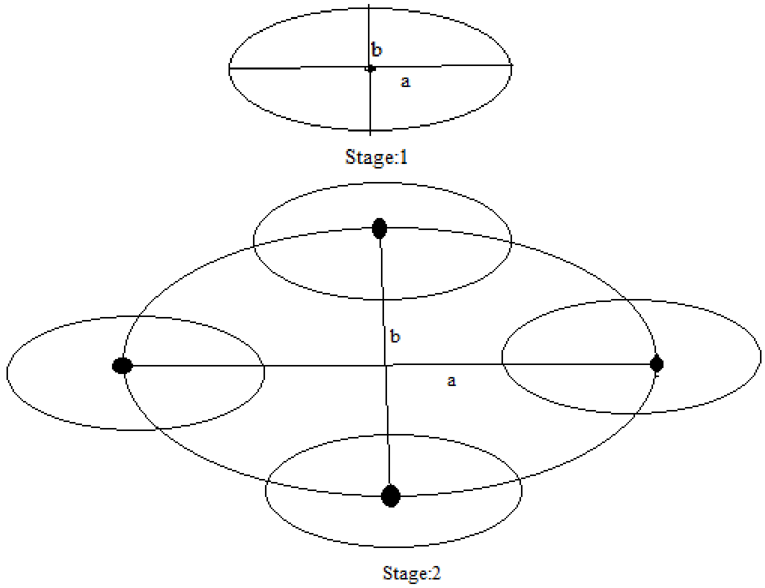

Figure 1. Concentric elliptical ring four-element sub array generator for stage: 1 and stage: 2.

methodology is the generation of both broad and narrow beams, and such broad beams are useful for broadcast or control and narrow beams for data. Actually multiple beams can also be generated with conventional arrays, but they need beam-forming networks. If the number of elements is high, array complexity increases with beam-forming networks and other feeding systems.

3. DESIGN AND ANALYSIS RHOMBUS FRACTAL ARRAY ANTENNA

Basic generator of this array antenna starts with four elements, and it can be generated up to infinite iterations. This paper analyzes up to four iterations having the same distance between the antenna elements maintained for expansion factors (S) of one and two. Nearly one third of the antenna elements can be thinned in each iteration due to the recursive nature of proposed array, and this can lead to multi-beam behavior. Hence this array can also be known as thinned rhombus fractal array. The expansion factor of fractal array decides the number of elements and their positions. In this article,

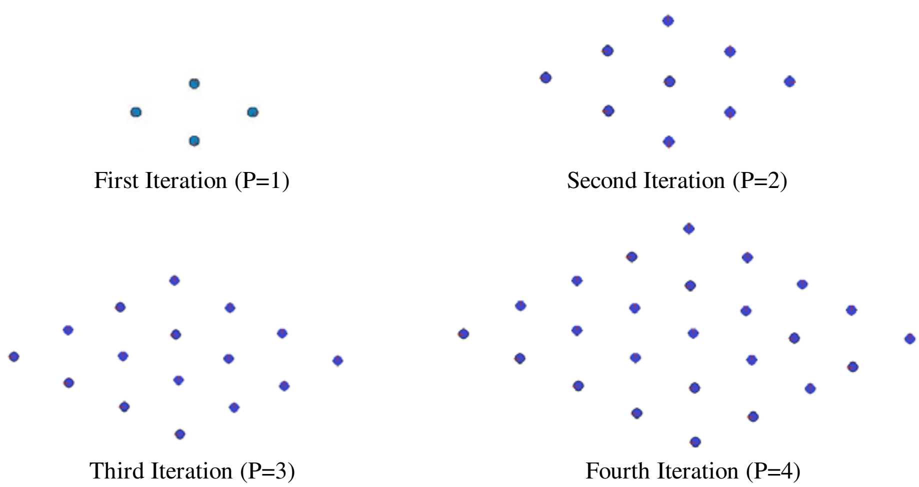

Figure 2. Concurrent iterations of rhombus fractal array antenna for an expansion factor of one.

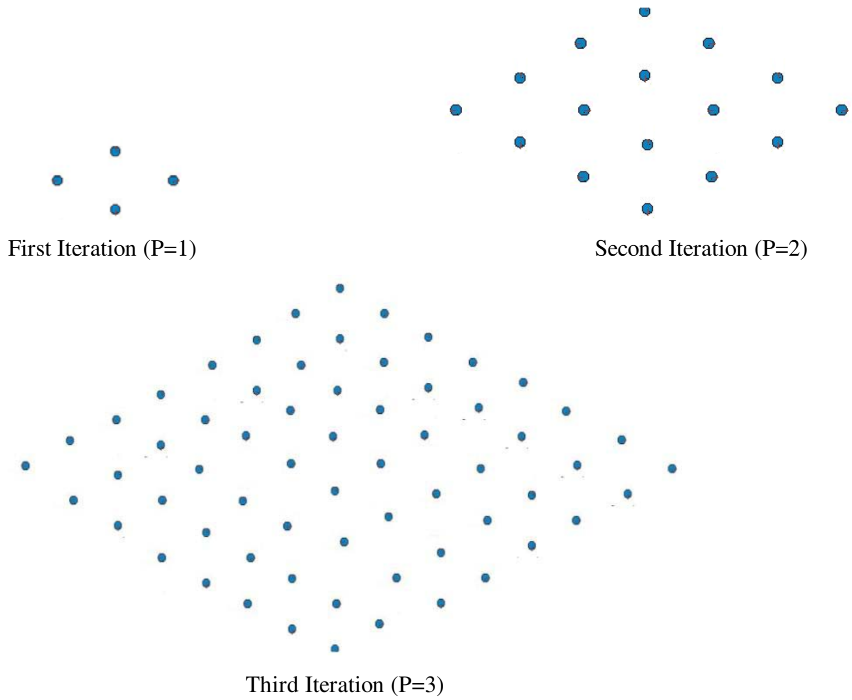

Figure 3. Concurrent iterations of rhombus fractal array antenna for an expansion factor of two.

two types of expansion factors are considered for expansion of the rhombus fractal array. The array factor of the proposed array will be changes depending on the expansion factor. Figure 2 illustrates the proposed array for an expansion factor of one. Nearly 20–30% elements thinned in second, third and fourth iterations. The array factor of this array is shown below,

$$A \cdot F_P(\theta, \phi) = \prod_{P=1}^{P} \left[\sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{jk(1)^{P-1}\Psi_{mn}} \right] \tag{4}$$

where I_{mn} is the current amplitudes, k the wave number, S the expansion factor, $S = 1$, P the iteration number, m the concentric rings, and n the number of elements. Array factor of the rhombus fractal array antenna for an expansion factor of two is illustrated in (6). Actually the difference between this array and the above array is expansion factor. Depending on the expansion factor, behavior of the array can be changed. Geometrical construction of the rhombus fractal array for an expansion factor of two is shown in Figure 3. Due to its expansion nature, no elements are thinned in this array configuration.

$$A.F_P(\theta, \phi) = \prod_{P=1}^{P} \left[\sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{jk(2)^{P-1}\Psi_{mn}} \right] \tag{5}$$

4. DISCUSSION ON RESULTS

This paper focuses on a new design geometric methodology named as concentric elliptical ring subarray geometric generator and examines performance of this methodology with a rhombus fractal planar array antenna. A basic array starts with four elements and can extend up to infinite iterations, but this paper examines up to four iterations for expansion factors of one and two. While increasing the number of iterations there is a simultaneous decrease observed in the resolution of both expansion factors. The proposed array antennas show better performance in all array factor properties than concentric circular geometric methodology. Figure 4(a) exemplifies the generation of multi-beam array factor configuration of the rhombus fractal array antenna of expansion one. Here, nearly half of the HPBW is reduced in all iterations compared with the square fractal array with wider side-lobe level angles. In the first iteration of this array, 92.4° wider side-lobe level is achieved with −9.8 dB side-lobe level. Tables 1 and 2 show that the behavior of this array is far better than the conventional rhombus array antenna. The basic difference between the proposed fractal arrays and conventional antenna array is logical geometrical construction, but the numbers of elements are the same. For example, the numbers of antenna elements in the second and third iterations of this array are nine and sixteen, respectively. Table 2 shows nine- and sixteen-element conventional rhombus antenna arrays having side-lobe levels of −11.5 dB and −15.2 dB with wider beam widths, respectively. But the proposed rhombus fractal array of expansion factor one with the same number of antenna elements has lower side-lobe level with narrower beam widths.

Figure 4(b) illustrates multi-beam radiation pattern of the proposed rhombus fractal array antenna of expansion factor two. In this case, side-lobe levels and beam widths decrease with respect to the number of increasing iterations. In this case, nearly the same type of side lobes is observed as in a square fractal array generated by concentric circular ring subarray generator, but nearly half of the beam width is reduced in all iterations. This array also shows better performance in all array factor properties than conventional rhombus array even though the number of elements is same. For example, the numbers of antenna elements in the third and fourth iterations of this array are sixty four and two fifty six, respectively. Table 2 shows 64 and 256 elements conventional rhombus antenna arrays having side-lobe levels of −18.9 dB and −22.3 dB with wider beam widths, respectively. But the proposed rhombus fractal array of expansion factor two with the same number of antenna elements has lower side-lobe level with narrower beam widths and wider side-lobe level angle. At the fourth iteration, HPBW is only 3.6° with a side-lobe level of −21.3 dB and side-lobe level angle of 8.1°.

Figure 6 shows an abated HPBW in both cases of the rhombus fractal array antenna compared with the quare fractal array antenna while maintaining the same or reduced side lobes. This indicates that the proposed array antenna can be useful for long distance, multi-beam satellite and radar communication systems. The separation between the main lobe and adjacent side lobes in degrees is generally called as side-lobe level angle. Wider side-lobe level angle with lower side-lobe level improves the signal to noise ratio and signal to inter symbol interference. Another improvement of the elliptical geometric

Table 1. Comparison of SLL, HPBW, SLL angle of both fractal arrays generated with concentric elliptical and circular sub array generators for expansion factors $S = 1$ and 2 up to four iterations.

	Rhombus Fractal Planar Array Antenna (Concentric elliptical ring sub array geometric generator)					
S	Expansion Factor $(S) = 1$			Expansion Factor $(S) = 2$		
Iteration (P)	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)
1	−9.8	31.1	92.4	−9.8	31.1	92.4
2	−17.6	22.0	89.1	−23.6	14.4	31.1
3	−26.4	18.0	90.0	−22.1	7.2	15.5
4	−35.2	15.6	91.5	−21.3	3.6	8.1
	Square Fractal Planar Array Antenna (Concentric circular ring sub array geometric generator) [10, 12]					
S	Expansion Factor $(S) = 1$			Expansion Factor $(S) = 2$		
Iteration (P)	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)
1	$-\infty$	62.1	-	$-\infty$	61.8	-
2	$-\infty$	44.2	-	$-\infty$	28.8	-
3	$-\infty$	35	-	−22.1	14.4	31.4
4	$-\infty$	31	-	−21.3	7.2	16.5

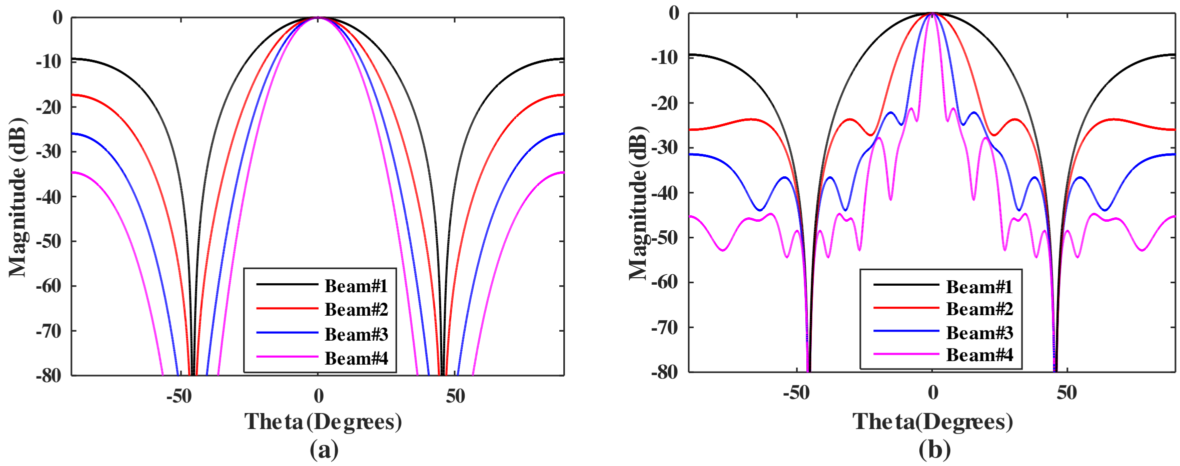

Figure 4. (a) Multi-Beam Array factor of rhombus fractal array antenna for $S = 1$ up to four iterations. (b) Multi-Beam Array factor of rhombus fractal array antenna for $S = 2$ up to four iterations.

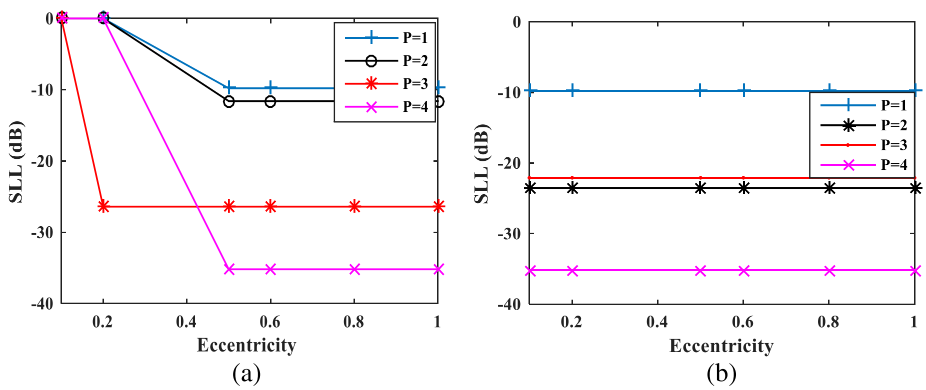

Figure 5. (a) Eccentricities versus SLL for expansion factor one. (b) Eccentricities versus SLL for expansion factor two.

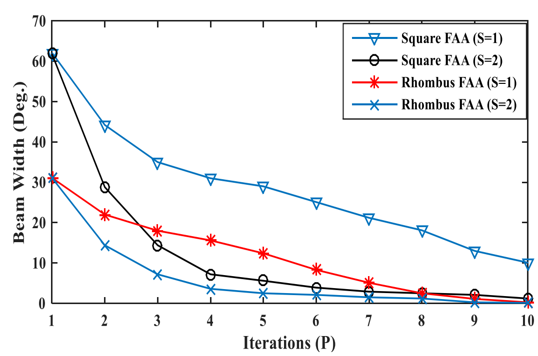

Figure 6. Number of iterations versus beam widths of square and rhombus fractal array antennas (FAA).

subarray generator is orientation, which means that the shape of the ellipse is based on major and minor axes. Depending on the space constraints, these axes can be interchanged. This is not possible in circular geometric generator because eccentricity of the circle is zero. SLL maintains constant for various eccentric values as shown in Figures 4 and 5.

Table 2. SLL, HPBW, SLL angle of conventional rhombus array antenna for different number of antenna elements (Here number of antenna elements considered according to number of antenna elements in each iteration of the proposed array antennas. *NA-Not Applicable) [3].

Rhombus Fractal Planar Array Antenna (Concentric elliptical ring sub array geometric generator)					
S	Ite. (P)	No. of Elements	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)
1	1	4	−9.8	31.1	92.4
	2	9	−17.6	22.0	89.1
	3	16	−26.4	18.0	90.0
	4	25	−35.2	15.6	91.5
2	1	4	−9.8	31.1	92.4
	2	16	−23.6	14.4	31.1
	3	64	−22.1	7.2	15.5
	4	256	−21.3	3.6	8.1
Conventional Rhombus Array Antenna					
S	Ite. (P)	No. of Elements	SLL (dB)	HPBW (Deg.)	SLL Angle (Deg.)
NA	NA	4	−9.8	31.1	92.4
	NA	9	−11.5	30.5	53.8
	NA	16	−15.2	25.3	50.6
	NA	25	−21.3	21.4	60.5
NA	NA	4	−9.8	31.1	41.8
	NA	16	−15.2	25.3	36.2
	NA	64	−18.9	19.5	22.1
	NA	256	−22.3	10.3	25.2

5. CONCLUSION

Concentric elliptical ring subarray geometric generator is a new type of geometric design methodology. It can be used for the generation of any polygon structure. This paper investigates a four-element rhombus fractal array antenna with different eccentric values and for expansion factors of one and two. This methodology shows better performance in orientation of the structure, multi-beam behavior, HPBW and SLL Angle than concentric circular ring subarray geometric generator and conventional rhombus array antenna. HPBW achieves a single-digit value at the fourth iteration for an expansion factor two of the proposed array. This design methodology and proposed array can be helpful for the generation of multiple beams with different array factor properties using a single fractal array antenna without any hardware complexity.

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